

## ERROR SEPARATION METHODS APPLIED TO FORM MEASUREMENT

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**Abstract.** The development of better measurement instruments has granted researchers and mechanical industry the possibility of working their products within extremely small tolerances. This condition impelled the utilization of artefacts and dedicated instruments to achieve more precise measurements. However, error presented by instruments has become not negligible if compared to parts error, given the current equivalence of their magnitudes. Error separation techniques allow decoupling measurement system errors from part errors. This paper aims to classify error separation techniques for dimensional metrology use with regard to application fields and nature of measurement. These measurement methods usually require special probe configurations and a certain number of transducers to accomplish measurement. The most widely used error separation techniques are the reversals and multi-probe or redundancy methods. The reversals are characterized by the mechanical manipulation of one degree of freedom of the system, except for the sensitive direction of the sensor used. Alternatively, data redundancy measurement methods require multiple sensorial configurations. Data must be collected along the part in a certain degree of overlapping. The utilization of several transducers allows decoupling between part and measuring process errors.

**Keywords.** Error separation, reversals, multi-probe systems.

### 1. Introduction

Most mechanical industries currently aim at the production of mechanical parts within small tolerance limits. In order to ensure product dimensional quality, industries must make investments on research and construction of measurement systems that comply with the required precision levels.

Today's finer manufacturing processes require better measurement accuracy. Instruments commonly used in measuring procedures, such as straightedges and reference gauges, present systematic errors (Whitehouse, 1976). Additionally, measuring procedure itself is influenced by random errors due to environmental changes and vibration.

For instance, a natural restriction on roundness error measurement is the reference rotation accuracy, either the spindle that contains the probe or the rotary table where the part is placed. However, the necessity of smaller absolute uncertainty can be replaced by the necessity of smaller repeatability uncertainty if systematic can be determined. Once determined, systematic errors can be compensated and roundness error can be displayed.

The introduction of coordinate measuring machines (CMM) technology allowed measurement of complex parts in short periods of time. Another advantage of CMM employment was the reduction of diversity of conventional instruments on metrology laboratories. However, in repetitive measurement runs of relatively simple parts, CMM may not be so effective and adequate as dedicated instruments. In this case, questioning about utilization and convenience of CMM may arise, for example, concerning the number of parts to be measured (Bosch, 1995).

The difficulties described above have motivated the development and enhancement of multi-orientation and multiprobe techniques for error decoupling and measurement without a mechanical reference.

Contributions by Tlustý (1957) and Bryan (1967) settled the starting point on the development of specific methods for spindle error measurement, as shown in Fig. 1(a) and (b). Arora et al (1977) present a comprehensive compilation of techniques for machine tool spindle assessment, including those implemented by Tlustý and Bryan.

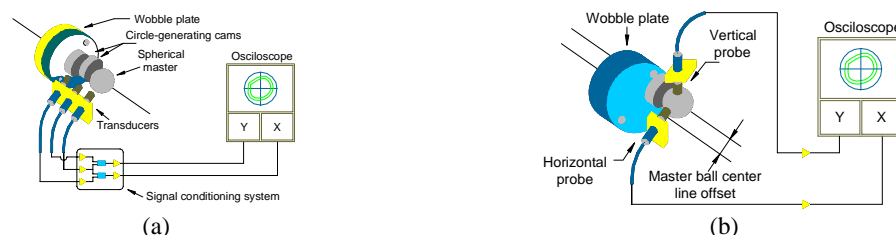


Figure 1. Spindle error measurement techniques.

## 2. Multi-orientation methods

Multi-orientation techniques, commonly known as reversal methods, are characterized by the mechanical manipulation of one degree of freedom of the system, except for the sensitive direction of the transducer. This operation inverts the sign of one component of the error (Evans et al, 1996).

Reversal is probably the most popular method for error separation.

Basically, this technique requires two measurement runs to be executed, involving usually a straightedge and the slide of a machine tool. The objective is to invert the sign of the slide error, allowing decoupling from straightedge error, which is always present in the system.

The first measurement run is carried out as follows: a displacement transducer is attached to a carriage that moves along the slide and straightedge. The transducer provides a reading that corresponds to the addition of the individual straightedge and slide straightness errors, as shown in Fig. 2(a).

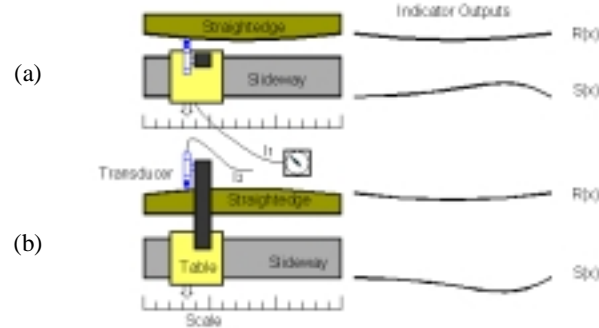


Figure 2. Straightedge reversal.

Let  $S(x)$  the slide straightness error and  $R(x)$  the straightedge straightness error, the first indicator output can be written as:

$$I_1(x) = R(x) + S(x) \quad (1)$$

Next, straightedge and transducer must be reversed and a second measurement run is performed, as shown in Fig. 2(b):

$$I_2(x) = R(x) - S(x) \quad (2)$$

The system composed by equations (1) and (2) can be solved and straightedge and slide straightness errors can be obtained separately:

$$\begin{aligned} R(x) &= \frac{I_1(x) + I_2(x)}{2} \\ S(x) &= \frac{I_1(x) - I_2(x)}{2} \end{aligned} \quad (3)$$

Indicator outputs  $I_1$  and  $I_2$  can be expanded in a polynomial form:

$$I_n(x) = A_n + B_n x + C_n x^2 + \dots \quad (4)$$

The first term represents initial displacement registered by the sensor and the second term represents misalignment between straightedge and slide. Both can be removed by means of fitting procedures, which always introduce uncertainties. In practice, however, the largest source of uncertainty in straightness reversal arises from environmental effects, such as temperature and vibration (Evans et al, 1996).

Reversals can also be employed to verify machine tool spindle errors. Ball reversal, proposed by Donaldson (1972) is exactly analogous to straightedge reversal.

This technique consists of the manipulation of an experimental set as shown in Fig. 3 (a) and (b). An arbitrary initial position between sensor and reference is taken. The arrangement (spindle and reference) is turned  $360^\circ$  and the

displacement sensor indication is registered. The indication corresponds to the spindle error motion, given by  $e(\theta)$ , added to the reference roundness error,  $r(\theta)$ .

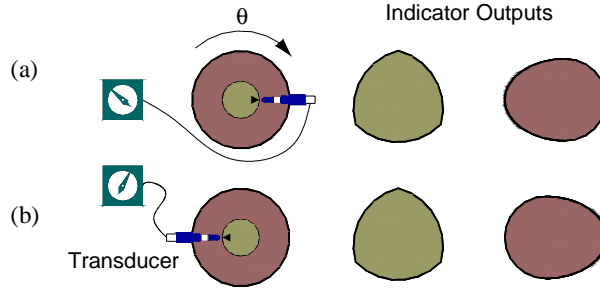


Figure 3. Ball reversal.

The second configuration requires that sensor and reference are turned 180° around the machine rotation axis and a new indication is recorded.

The indicators outputs are:

$$\begin{aligned} I_1(\theta) &= r(\theta) + e(\theta) \\ I_2(\theta) &= r(\theta) - e(\theta) \end{aligned} \quad (5)$$

Hence, spindle error motion and reference roundness error can be obtained separately by:

$$\begin{aligned} r(\theta) &= \frac{I_1(\theta) + I_2(\theta)}{2} \\ e(\theta) &= \frac{I_1(\theta) - I_2(\theta)}{2} \end{aligned} \quad (6)$$

It must be emphasized that in both cases, indication of reference error remains constant if the error presents repeatability. However, indication of spindle error is inverted when sensor and transducer are reversed, allowing error separation.

This simple method presents some limitations. Perhaps the most important is the transducer manipulation during reversal. It is also necessary to stop spindle rotation during evaluation.

Two alternative approaches to Donaldson's method may be described. Fourier analysis can be applied to the measured signals so that harmonic content deficiencies of the output can be depicted. An algorithm that provides adequate signal treatment may compensate deficient harmonics and synthesizes the results of roundness error measurement.

The first manner to consider this issue is to simultaneously use two diametrically opposite transducers around the spindle. Sensor indications are:

$$\begin{aligned} I_1(\theta) &= s(\theta) + e(\theta) \\ I_2(\theta) &= s(\theta - \pi) - e(\theta) \end{aligned} \quad (7)$$

If the signals from the transducers were split in discrete harmonic components ( $n=0, 1, 2, \dots$ ) where  $n=1$  corresponds to one undulation per circumference of the part, the results of Fourier analysis for  $I_1$ ,  $I_2$ ,  $e$  and  $s$  are  $F_1$ ,  $F_2$ ,  $F_e$  and  $F_s$ , respectively:

$$\begin{aligned} F_1(n) &= F_s(n) + F_e(n) \\ F_2(n) &= F_s(n)e^{-jn\pi} - F_e(n) \end{aligned} \quad (8)$$

From Eq. (8) it is possible to eliminate the frequency component relative to the systematic error  $e$ . Considering  $I=I_1+I_2$ , it yields

$$F_I(n) = F_s(n)(1 + e^{-jn\pi}) \quad (9)$$

The term  $(1 + e^{-jn\pi})$  is a weighting function and has magnitude  $2\cos(n\pi/2)$ . This feature allows only even harmonics identification.

The second approach implies two orientations of the part with relation to spindle, and one sensor, only. The orientations configure  $180^\circ$  around the spindle. Indications of the transducers are:

$$\begin{aligned} I_1(\theta) &= s(\theta) + e(\theta) \\ I_2(\theta) &= s(\theta - \pi) + e(\theta) \end{aligned} \quad (10)$$

The associate Fourier transforms are:

$$\begin{aligned} F_1(n) &= F_s(n) + F_e(n) \\ F_2(n) &= F_s(n)e^{-jn\pi} + F_e(n) \end{aligned} \quad (11)$$

Elimination of  $e(\theta)$  can be accomplished, yielding:

$$F_I(n) = F_s(n)(1 + e^{-jn\pi}) \quad (12)$$

Weighting function has magnitude  $2\sin(n\pi/2)$ , and only odd harmonic information can be detected (Whitehouse, 1976). Additional information about other applications of reversal techniques can be found at Evans et al, 1996.

### 3. Multiprobe methods

Multisensorial error separation methods are also extensively employed in industry, mainly in cases where *in situ* measurement is required. Usually, it is not practical to remove the workpiece from the machine in order to have it measured. A problem arises, that is to measure the part without a formal datum, as a master ball, for instance. Also, the part should be turned in a relatively rough manner and variations originated from this handling may be considered as spindle random movements with relation to the measuring device.

For roundness error measurement, sensor configuration must be determined so that any random movements of the workpiece are not detected and do not deteriorate the signal. According to Mitsui (1982), the most fundamental method for spindle error motion assessment is measuring the movement of the workpiece attached to the machine tool spindle using a displacement transducer. However, transducers output signals take into account not only the spindle radial movement, but also the roundness error of the part.

If this error is known at the cross section where measurement is taken, it is possible to decouple spindle error motion and roundness error.

A measuring device that contains three displacement transducers can be built, and the *Three Points Method* can be applied to detect roundness error and the orthogonal components of the spindle error motion simultaneously.

The sensors are placed so that the central sensor is the reference for the others, located at angles  $\alpha$  and  $\beta$  from the reference, as shown in Fig. 4.

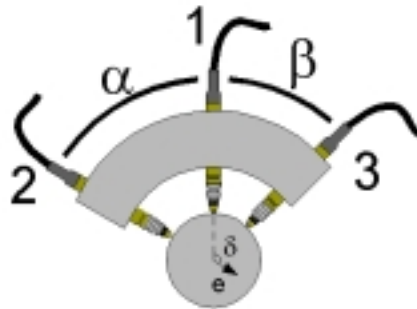


Figure 4. Three points method.

The multiprobe arrangement provides the following indications:

$$\begin{aligned} I_1(\theta) &= s(\theta) + e \cos \delta \\ I_2(\theta) &= s(\theta + \alpha) + e \cos(\delta + \alpha) \\ I_3(\theta) &= s(\theta - \beta) + e \cos(\delta - \beta) \end{aligned} \quad (13)$$

where  $\theta$  is the reference angle of sensor 1,  $e$  is the eccentricity imposed to the part and  $\delta$  is the angle that determines its direction. Error due to part movement can be removed by means of a proper signal combination so that the terms of  $e$  and  $\delta$  become zero. This condition can be met if sensors are placed  $120^\circ$  from each other around the part. However, this configuration is disadvantageous because the part would be completely enclosed by the transducers, causing difficult access to the part. Moving the workpiece in such a spatial configuration affects at least two probes in the opposite direction. Therefore, if one of the sensors is moved  $180^\circ$  from its original position and its sensitivity direction inverted, equivalent results can be obtained and the part remains accessible. This arrangement is highly desirable for measurement of roundness error of relatively large and hardly accessible spindles (Whitehouse, 1976).

Mitsui (1982) built an experimental apparatus to implement the three points method. It employed three capacitive displacement sensors, rigidly attached to the structure of a lathe. The utilization of a rotational encoder presenting resolution of 2048 pulses per revolution, attached to the other end of the spindle, allowed angle detection. The highest frequency that can be detected by the displacement sensors goes up to 100 kHz. Given that the sensor touching tip has a diameter of 8 mm, relatively short wave roundness error components, such as surface roughness, are filtered out.

Experiments compared the results from the three points method against measurement using a master sphere. The master sphere was attached at one end of a 50 mm diameter, 150 mm long steel workpiece. Rotation speed was set at 600 rpm and measurement was conducted simultaneously using both methods. Examination of the results has demonstrated that the three points method was less susceptible to vibration effects.

A frequency component analysis of the signals showed that peaks related to vibration effects presented much lower amplitude when using the three points method and peaks relative to master ball roundness error were not present. However, they were detected by the conventional method. In other words, spindle error motion could be more precisely evaluated by the three points method, while measurement with a master ball was influenced by its roundness error.

Shinno et al (1987) have applied the three points method to a roundness error measurement procedure for ultra precision aerostatic bearing system. Three capacitive displacement transducers were employed. Spindle rotating movement was measured with a photoelectric rotary encoder. Output pulses from the encoder were used to trigger an A/D converter to allow data recording. The obtained values for roundness error ranged from 13 to 17 nm.

Gao et al (1996) present an alternative multisensorial method for roundness error measurement, namely the *Mixed Method*. This method uses two displacement transducers and one angle transducer to separate the roundness error from the spindle error completely and to capture high frequency components. Differential output of transducers cancels spindle radial motion and manipulation of differential data yields roundness error. Figure 5 illustrates the mixed method.

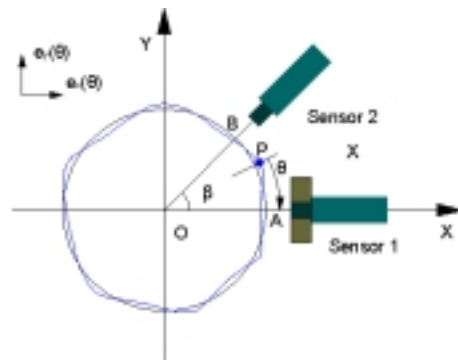


Figure 5. Mixed method.

However, the sensors employed in this application consist of optical devices, such as Laser heads, mirrors and lenses that limit operating performance on improperly controlled environments.

The mixed method had posterior enhancements. Gao et al (1997) verified that sensor arrangement provided better harmonic response if the displacement sensor was placed on a right angle with respect to the angular sensor. Therefore, one displacement sensor was removed from the previous configuration. The new arrangement was referred to as the *Orthogonal Mixed Method*. Optical sensors were used, in the same way as the original method.

Other multiprobe technique derived from the three points method is the *Four Points Method*, proposed by Zhang and Wang (1993). The authors considered that the angle between sensors on the three points method could produce extremely small transfer coefficients for some harmonic components. Once the sensor arrangement is set, it is also determined the transfer coefficients of the process. If the sensor layout is unsatisfactory for detecting certain harmonic components, the addition of an extra probe will allow identification of the defective frequency range by increasing sensitivity of the transfer function.

For straightness error measurement, however, a representation in terms of Fourier coefficients is not the most suitable one to express random variations that must be removed. Variations can occur as follows: variations in the linear

distance between part and datum, variations in the tilt between the two and relative curvature variations. Whitehouse (1976) discussed a generalized sensor configuration for straightness error measurement.

Concerning the variations mentioned above, the distance  $y$  between the part and the datum could be expressed as a random variable in time. Thus,

$$y = d(t) + m(t)x + c(t)x^2 \quad (14)$$

where  $d$ ,  $m$  and  $c$  are independent random variables representing average separation, tilt and curvature, respectively, and  $x$  is distance.

To eliminate this number of random variables, in the general case, four transducers are needed. The sensors would have sensitivities of 1,  $a$ ,  $b$  and  $c$ , placed at distances  $l_1$ ,  $l_2$ ,  $l_3$  and  $l_4$  from the centre of the measuring system, as shown in Fig. 6.

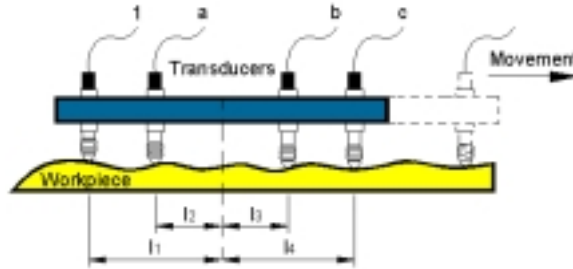


Figure 6. Generalized multiprobe configuration.

Three equations need to be satisfied in this case:

$$1 + a + b + c = 0 \quad (15)$$

$$-l_1 - al_2 + bl_3 + cl_4 = 0 \quad (16)$$

$$l_1^2 + al_2^2 + bl_3^2 + cl_4^2 = 0 \quad (17)$$

Equation (15) relates to average separation, Eq. (16) takes account of the tilt and Eq. (17) considers the quadratic terms. The negative terms in Eq. (16) are due to odd symmetry about the midpoint of the sensors carriage.

Solving Eq. (15), (16) and (17) shows that numerical differentiation algorithms can be used in multiprobe measuring systems. Considering the case where  $l_1 = -2h$ ,  $l_2 = -h$ ,  $l_3 = h$  e  $l_4 = 2h$ , given these restrictions, the probe combination signal that satisfy Eq. (15), (16) and (17) are:

$$S = V_1 - 2V_2 + 2V_3 - V_4 \quad (18)$$

and  $a=-2$ ,  $b=2$ ,  $c=-1$ .

Equation (18) corresponds to the third numerical differential where the measured ordinates  $f_1$ ,  $f_2$  etc have been replaced by transducers. Thus,

$$h^3 f''' = \frac{1}{2}(f_{+2} - 2f_1 + 2f_{-1} - f_{-2}) \quad (19)$$

In this case, there is a gap of  $2h$  between  $V_2$  and  $V_3$ , and  $h$  corresponds to the spacing between probes. If probes are taken to be equidistant, it yields that  $a=-3$ ,  $b=3$ ,  $c=1$ , which reduces the overall distance from  $4h$  to  $3h$ .

A system as described has a harmonic weighting function  $W_h$  given by:

$$W_h = \left[ e^{\left(-j2\pi l_1/\lambda\right)} + a e^{\left(-j2\pi l_2/\lambda\right)} + b e^{\left(-j2\pi l_3/\lambda\right)} + c e^{\left(-j2\pi l_4/\lambda\right)} \right] \quad (20)$$

The weighting function refers to the harmonic distortion that is introduced by the method and which, because it is known, can be removed by relatively simple computational algorithms (Whitehouse, 1994).

A method for straightness error measurement that employs relative displacement between tool and part was proposed by Tanaka et all (1981). Two displacement sensors are moved along the motion direction of the tool, scanning

the workpiece in regular intervals that correspond to the distance between the two probes. Eddy current type sensors are employed.

An estimation of the straightness error can be obtained by means of an algorithm in terms of the relative displacements. This technique is known as *TSP (Two Successive Points)*. A clear advantage of the TSP method over Laser interferometric systems is that straightness for both machine tool and workpiece can be obtained simultaneously.

Reliable straightness error estimations are specially needed to build slides of machine tools. Tanaka et al (1981) have applied the TSP method to evaluate the straightness error of the slides of a large boring and milling machine to investigate the practical aspects of the implementation of the technique. The TSP method requires the utilization of two transducers attached to the tool post that scans the surface of the part, as shown in Fig. 7.

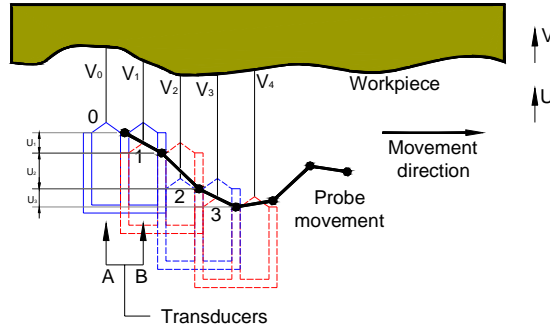


Figure 7. The Two Successive Points method.

After the initial measurement, the tool post is successively fed with the distance of the probes, and a set of relative displacement at two points can be obtained.

The straightness error of the movement of the probes is designated as  $U_1, U_2, U_3, \dots$  and the straightness error of the surface is designated as  $V_1, V_2, V_3, \dots$ . The relative displacement between probes and part is obtained by the sensors A and B, as follows:

$$D_{0,B} - D_{0,A} = V_1 \quad (21)$$

$$D_{1,A} - D_{0,A} = V_1 - U_1 \quad (22)$$

$$D_{1,B} - D_{0,B} = (V_2 - V_1) - U_1 \quad (23)$$

$$D_{2,A} - D_{0,A} = V_2 - U_2 \quad (24)$$

$$D_{2,B} - D_{0,B} = (V_3 - V_1) - U_2 \quad (25)$$

The general description of the equations above can be derived:

$$D_{(i-1),B} - D_{0,B} = (V_i - V_1) - U_{i-1} \quad (26)$$

$$D_{i,A} - D_{0,A} = V_i - U_i \quad (27)$$

Thus, the straightness error  $U_i$  at the  $i^{\text{th}}$  position of the tool post due to measured relative displacement is given as:

$$U_i = U_{i-1} + D_{(i-1),B} - D_{(i,A)} \quad (28)$$

$$V_i = U_i + D_{i,A} - D_{0,A} \quad (29)$$

Assuming  $U_0 = V_0 = 0$ , series of  $U_i$  e  $V_i$  can be determined, providing part and tool motion straightness errors.

A variation of the TSP method is discussed by Gao and Kiyono (1996). The *Combined Method* consists of an application of the TSP method associated to the generalized two points method, where the tool post is not fed with the distance between the transducers. The utilization of this combination and the acquisition of several data sets allowed measurement of profiles containing high frequency components, with wavelength shorter than the distance between transducers.

Multi-orientation and multiprobe methods, however, have restrictions to their applications. For multi-orientation techniques, environmental factors should be deeply considered. It is highly desirable that temperature, air flow and vibration are controlled on the workspace. Positioning accuracy of axial and angular orientations must also be observed.

Multiprobe techniques are inherently more susceptible to instrumentation than to environmental factors. For instance, all the transducers must touch the part at the same line of action, which is difficult to ensure. Yet, every sensor must have a valid output, that is, all the transducers must be in constant contact with the part, or the instantaneous compensation provided by the method will be lost.

#### 4. Conclusion

This paper aimed to provide a comprehensive review of measurement techniques, enlightening advantages and difficulties that may occur during practical application.

Some aspects still need deep investigation, such as better qualification of high frequency components on roundness error evaluation.

#### 5. Acknowledgment

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