ON THE APPLICATION OF THE DANG VAN CRITERION IN FRETTING FATIGUE. PART I: ASSESSMENT OF CONTACT SIZE EFFECTS

José Alexander Araújo
Universidade de Brasília – UnB, Departamento de Engenharia Mecânica – ENM, Brasília – DF CEP:70910-900
alex07@unb.br

Edgar Nobuo Mamiya
Universidade de Brasília – UnB, Departamento de Engenharia Mecânica – ENM, Brasília – DF CEP:70910-900
mamiya@unb.br

Rodolfo Vivacqua Castro
Universidade de Brasília – UnB, Departamento de Engenharia Mecânica – ENM, Brasília – DF CEP:70910-900
rvc@iname.com

***Abstract.** This work concerns the application of the Dang Van criterion to predict fretting fatigue crack initiation. To validate the analysis a number of experiments published in the literature are considered. These experiments, which were carried out in a high strength Aluminium alloy, revealed there is a contact size effect in fretting fatigue life. The results show that the Dang Criterion can correctly predict the initiation of fretting cracks for larger contact configurations. It is concluded that the reason for the poor performance of the criterion in predicting failure at smaller contacts may well be related to attributed to the effect of the stress gradient, a variable not accounted for in the mesoscopic criterion proposed by Dang Van.*

***Keywords.** fretting fatigue, multiaxial fatigue, mesoscopic scale approach, contact size*

1. Introduction

Fretting fatigue is a term used to describe the process of crack nucleation, eventually leading to the failure of components in a mechanical assembly. Such phenomenon invariably occurs when the fixing is under some sort of vibration and one or both of the components of the assembly is also subjected to a bulk fatigue load. The vibrational load usually provokes a small relative displacement between contact surfaces leading to the formation of micro flaws. These will further propagate should the fatigue load be sufficiently severe. Riveted or bolted lap joints, spline couplings and fan blade connections are just a few among many other examples of practical structures that have shown fretting cracks (Shaffer and Glaeser, 1994).

Fretting cracks nucleate in a region where the stress field is multiaxial and the principal stress magnitudes and directions vary with time. It therefore seems appropriate to seek to apply multiaxial fatigue damage models that can accommodate these patterns to predict the initiation of such cracks, though there are criticisms in such approach. For instance, Mugadu et al. (2000) suggest that fretting initiation life must be influenced by the amount of damage in the slip zones, a variable not considered by the multiaxial fatigue parameters. We accept that slip amplitude will affect the fretting process at an asperity scale. However, the reader should be reminded that, as suggested by Miller (1993), the initiation life may well be dominated by the need for a crack to overcome the strongest microstructural barrier (e.g. a grain boundary). If this is the case, the fretting problem can also be approached as a plain fatigue problem subject to a localized stress concentration (see Giannakopoulos et al., 2000).

Returning to the multiaxial fatigue approach, many different methodologies have been proposed to model the fatigue phenomenon under complex stress states. Most of them have been extensively reviewed in the literature (You and Lee, 1996, Papadopoulos, 1997 and Araújo, 2000). Among these methodologies the one that considers the fatigue problem at the mesoscopic level (e.g. Dang Van, 1989, and Papadopoulos, 1994) have gained increasing interest due to the good predictions of fatigue strength associated with a strong physical interpretation of the phenomenon. In this article we will assess the use of the Dang Van model to the fretting case.

2. The Dang Van Criterion

Suppose that a volume of material containing a number of grains with arbitrary orientations is subjected to a cyclic elastic stress state, which is assumed to be known at any time t. Although the stress state is macroscopically elastic, localized plasticity may take place within some favourably oriented grain. Under high cycle fatigue conditions, this localized plastic deformation in the grain is contained by the bulk material, which remains elastic. According to Dang Van et al. (1989) crack initiation will take place if a state of plastic shakedown is achieved within this grain. Hence, the boundary between infinite life and failure corresponds to conditions of elastic shakedown at mesoscopic level. In this setting, Dang Van proposes that an important variable controlling the fatigue process is the microscopic deviatoric stress tensor, s(t), which is given as:

$$\mathbf{s}(t) = \mathbf{S}(t) - \rho \,, \tag{1}$$

where **S**(t) is the deviatoric stress tensor at any time *t*, and ρ is the stabilized deviatoric residual stress tensor after the state of shakedown was achieved. There is a number of well established techniques to determine ρ, space only preclude us to present any of them here but an interested reader should consult Dang Van et al. (1989), Araújo (2000) or Bernasconi (2001). Another important variable governing the initiation of a crack is the hydrostatic pressure p_h, since a positive p_h, will encourage crack opening. A simple fatigue criterion relating these variables was then proposed as:

$$\tau(t) + m p_h(t) - n = 0, \tag{2}$$

where $\tau(t)$ is

$$\tau(t) = \frac{1}{2}\left(s_{1p}(t) - s_{3p}(t)\right), \tag{3}$$

being $s_{1p}(t)$ and s_{3p}(t) the maximum and minimum principal microscopic deviatoric stresses at each instant *t*.

The material constants m and n are evaluated by considering the fatigue limits in bending (σ_{fl}) and torsion (τ_{fl}), yielding:

$$n = \tau_{fl}, \tag{4}$$

and

$$m = \frac{6\tau_{fl} - 3\sigma_{fl}}{2\sigma_{fl}}, \tag{5}$$

Notice that Eq. (1) leads to two intersecting lines in the τ versus p_h plane (Fig.1). If the local stress cycle, plotted on this plane stays within these lines the component is safe, if it crosses one or both of the lines failure will result. The use of Eq. (3) assures that the loading path is always above the p_h axis. From the foregoing a crack nucleation risk factor can be defined for the Dang Van criterion.

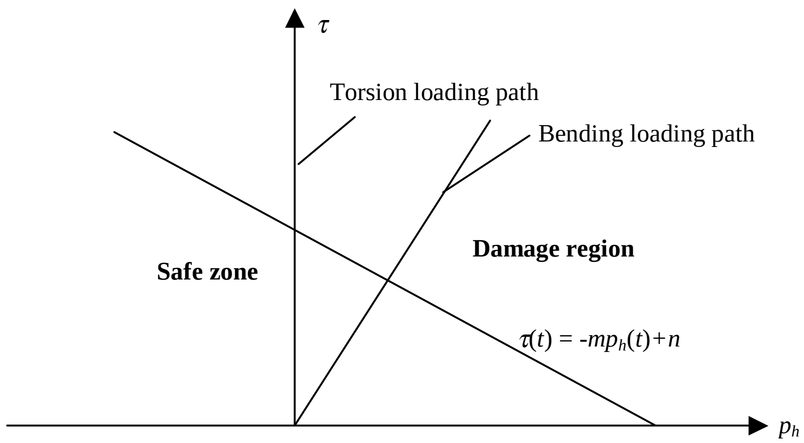

Figure 1– τ x p_h plane showing the torsion and bending load paths.

$$DV = Max\left[\frac{\tau(t)}{\tau_{fl} - m p_h(t)}\right]. \tag{6}$$

The maximization is to be carried out on time (*t*), and if *DV* is greater than 1 fatigue failure is expected to occur.

3. Experimental Results

To validate the analysis, experimental data published by Nowell (1989) will be considered. A schematic diagram of the experimental geometry is shown in Fig. 2. The main specimen is a “dog’s bone” tensile test piece held between two movable jaws. This is clamped by two cylindrical pads subjected to a normal load per unit specimen width, *P*. The specimen was then subjected to an oscillatory load $\sigma_0 sin(wt)$. This causes extension and contraction of the specimen. As the pads are in contact with the specimen and are restrained by springs a cyclic tangential fretting force, $Qsin(wt)$, is also developed. Five series of tests were carried out under this configuration. Within each data series the tests were conducted so that the contact size, a, was varied while the peak contact pressure, p_0, was held constant. This was possible because of the Hertzian nature of the experimental configuration, where the peak contact pressure, p_0, is proportional to $\sqrt{(P/R)}$ and the contact semi-with, a, is proportional to $\sqrt{(PR)}$. Hence, it is possible to vary P and R keeping their ratio constant. The advantage of conducting such tests is that it was possible to produce a data series where the magnitude of the stress field was held constant on the surface but it varied in extent from test to test. The fretting pads and specimens were made of Al4%Cu (HE15-TF). In order to provide a range of contact sizes, eight pairs of fretting pads were chosen with radii of curvature varying from 12.5 mm to 150 mm. An average of eight tests were run within each data series keeping the salient parameters p_0, Q/P and σ_0 constant. Table 1 summarizes the characteristic contact parameters for the data series analyzed here while Table 2 records the average total life and corresponding contact size and pad radius for each test condition. From Table 2 it is clear that within each data series small contacts produced a life greater than 10^7 cycles, whereas larger contacts produced lives in the region of 10^5 to 10^6 cycles. For Al series 2 data, the salient parameters gave rise to a condition of reverse slip. This requires a more complex approach to raise the subsurface stress field; hence this data series will not be considered in this work.

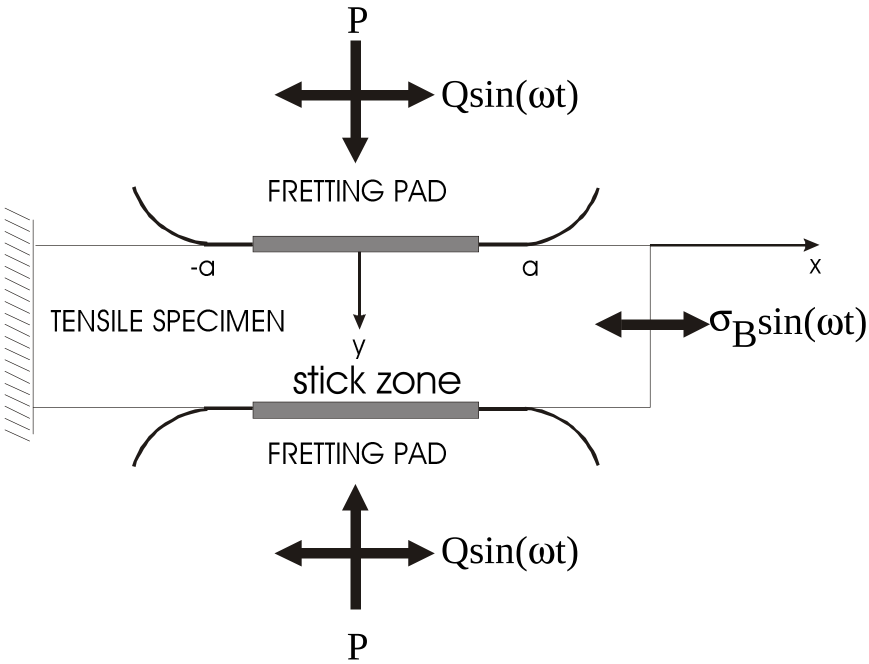

Figure 2 – Schematic configuration of the fretting fatigue tests.

4. Subsurface Stress Field

In order to apply the Dang Van criterion to the fretting problem it is first necessary to evaluate the cyclic stress field developed under the contact in the experimental configuration. These quantities may be evaluated using a well-established technique and only an overview will be presented here. The first step towards a solution for the subsurface stress field is to solve the contact problem itself, i.e., to find the magnitude and distribution of the surface tractions. The shear tractions will not cause any disturbance in the Hertzian pressure distribution if the contacting surfaces are elastically similar. Thus, since the normal load is constant, the normal traction will also be independent of time. The cyclic shear load on the other hand will give rise to history dependent tractions as described by Cattaneo (1938) and Mindlin (1949). Once the normal and shear tractions have been found, the cyclic stress field can be obtained at each load step by using Muskhelishvili's potential theory (Muskhelishvili, 1953).

Table 1. Contact parameters for the experimental series analyzed.

Series No.	p_0 (MPa)	σ_0 (MPa)	Q/P	Friction coefficient
1	157	93	0.45	0.75
3	143	93	0.45	0.75
4	143	77	0.45	0.75
5	120	62	0.45	0.75

Table 2 – Experimental fretting fatigue total life of Al4%Cu for each theoretical contact size, *a*, and pad radius, *R*.

Al Series 1								
Pad Radius, *R* (mm)	12.5	25	37.5	50	75	100	125	150
Contact size, *a* (mm)	0.10	0.19	0.28	0.38	0.57	0.76	0.95	1.14
Life (10^6 cycles)	>10	>10	>10	1.29	0.67	0.85	0.73	0.67
Al Series 3								
Pad Radius, *R* (mm)	12.5	25	37.5	50	75	100	125	150
Contact size, *a* (mm)	0.09	0.18	0.27	0.36	0.54	0.72	0.9	1.08
Life (10^6 cycles)	>10	>10	4.04	1.50	0.80	0.61	1.24	0.69
Al Series 4								
Pad Radius, *R* (mm)	12.5	25	50	75	100	125		
Contact size, *a* (mm)	0.09	0.18	0.36	0.54	0.72	0.9		
Life (10^6 cycles)	>10	>10	1.2	1.42	0.61	1.24		
Al Series 5								
Pad Radius, *R* (mm)	25	37.5	50	75	100	125	150	
Contact size, *a* (mm)	0.14	0.21	0.28	0.42	0.57	0.71	0.85	
Life (10^6 cycles)	>10	>10	>10	>10	>10	1.57	1.23	

5. Results

The Dang Van criterion is now used to assess the crack nucleation risk for Nowell's experiments with Al4%Cu. Application of the criterion requires the knowledge of the fatigue limit in bending, σ_{fl}, and torsion, τ_{fl}.. The endurance limit in bending for this alloy is reported to be σ_{fl}=124 MPa (Nowell, 1988). Specific torsional data has not been found for such material, however τ_{fl} can be predicted approximately from the ratio τ_{fl}/σ_{fl}, which is available in the literature for a wide range of Aluminum alloys (Forrest, 1962, Sauer and Lemmon, 1949, Nishihara and Kawamoto, 1953, Mathaes, 1932, Ludwik, 1931,). These data reveals that the average value of τ_{fl}/σ_{fl} for wrought aluminum alloys is 0.55. This yields τ_{fl}=68.2 MPa.

The cracking risk *DV* was evaluated on and under the surface at spatial intervals $\Delta x/a = \Delta y/a$ = 0.01. At each point sixteen different load steps in the complete cycle were examined. The analysis revealed that the larger values of *DV* were found on the surface. Fig. 3 depicts the variation of the parameter on the contact surface for Al series 1 data. This plot shows that the maximum global value of the parameter, DV_{max}, is reached on the surface at *x*/*a*=-1. This is at the trailing edge of the contact. As the parameter exceeds unity at this position high cycle fatigue failure is predicted.

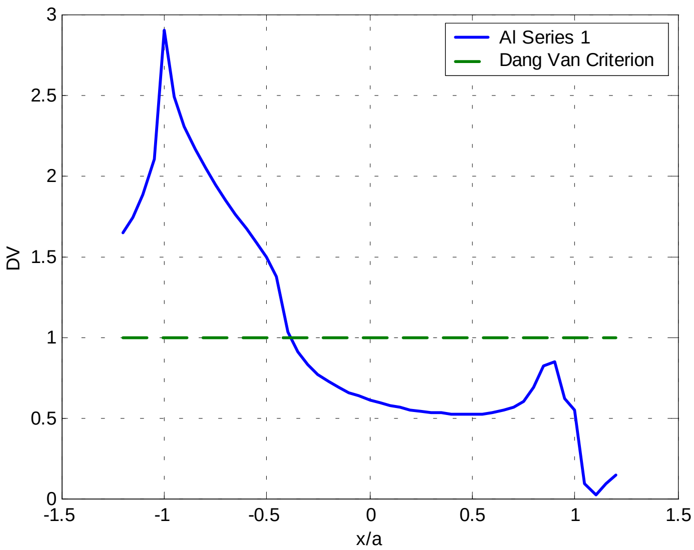

Figure 3 – Surface variation of the Dang Van cracking risk parameter *DV* for Al series 1.

Perhaps a better way to visualize the application of the fatigue criterion is to plot on the same graphic the history of the local stress state and the cracking risk line dividing the safe life and damage regions. This is depicted in Figs. 4 and 5. The curves shown are for Al series 1 and 5 data respectively. A summary of the application of the Dang Van criterion to all tests with Al is presented in Table 3. This records the maximum global value of the cracking risk parameter DV_{max} and its corresponding location of occurrence on the surface, $(x/a)_{max}$, for each data series. Notice that application of the criterion for each different contact size within the same data series is not necessary once the magnitude of stress field does vary for tests under the same p_0.

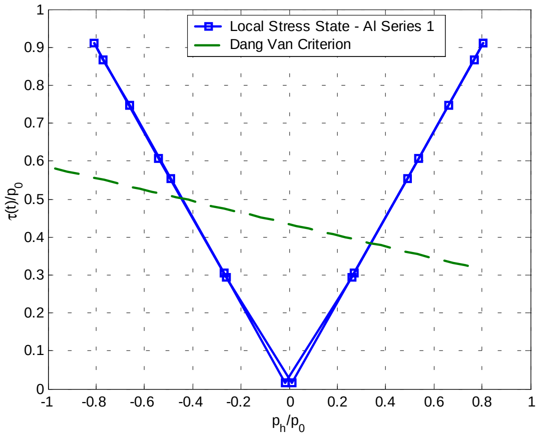

Figure 4. History of local stress state and Dang Van fatigue criterion for Al series 1 data at $(x,y)=(-1, 0)$.

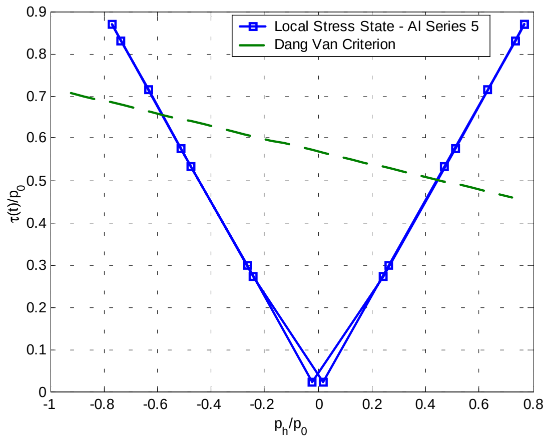

Figure 5. History of local stress state and Dang Van fatigue criterion for Al series 5 data at $(x,y)=(-1, 0)$.

Values greater than one are predicted for DV_{max} in all the different series data assessed. However, as previously reported the experimental data show that for smaller contact sizes infinite lives were achieved while for larger contacts failure occurred. Here the reader should be reminded that smaller contacts present a more rapidly varying stress field than larger contacts if they are subjected to the same peak pressure. On the other hand, the Dang Van's cracking risk parameter assumes that fatigue damage is controlled by local microscopic stresses of superficial critically stressed points, hence, the Dang Van criterion can not account for the presence of sharp stress gradients.

Table 3. Summary of the Dang Van cracking risk predictions for tests with Al4% Cu

Series	DVmax	x/a
1	2.90	-1.0
3	2.67	-1.0
4	2.44	-1.0
5	1.91	-1.0

6. Discussion And Conclusions

The analysis of the results previously presented reveals that the Dang Van criterion can not explain the pad size effect in Nowell's experiments with Al4%Cu (Nowell, 1988) if the evaluation of the cracking risk parameter is based on the microscopic local stress state of single superficial severely stressed points. The reason for such phenomenon may be associated with the steeper stress gradient present at smaller contacts. As previously discussed, the tests carried out by Nowell (1988) were designed so that the contact size could be varied whilst the peak pressure was kept constant. This essentially means that the stress field for smaller contacts decline more rapidly than for larger ones though they are equal in magnitude on the surface. A direct consequence of this is that a crack growing under smaller contacts will be less severely stressed as it moves away from the interface and therefore it is likely that it will propagate slower than a crack of the same size growing under a larger contact. On the other hand, the Dang Van mesoscopic criterion suggests that the fatigue initiation process is a punctual phenomenon, which will start and be controlled by the level of microscopic stresses at the most severely stressed point of the component. The aforementioned results provide us with strong evidences that this is not the case when steep stress gradients are present. Therefore, a modification of the Dang Van criterion or the development of a new multiaxial fatigue model which can incorporate such characteristic is of fundamental importance. The authors intend to address this issue in future work.

7. References

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