WHIRLING FREQUENCY CALCULATION USING FINITE ELEMENTS METHOD.

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Abstract. *The gyroscopic effect changes the natural frequencies of rotator machines, which induce the appearance of a pair of frequencies, called whirling frequencies. The highest whirling frequencies value of a pair presents a rotation in same direction of the rotor and the other, the lowest, rotate in the other direction. The occurrence of the highest or lowest frequencies depends on the rotation and on the initial condition of the problem. When the velocity of rotation has a whole multiple of the value of the whirling frequencies, there is a violent vibration that may damage the machine. In the design of rotator machine, the operation closer these points must be avoided. In this paper, computer software to determine the whirling frequencies using finite elements technique, considering the effects of the rotatory inertia and shaft-bending shear, has been developed. The discretization process has considering three basic elements: shaft, disc and bearings. This modeling has assembled the damping, stiffness, mass and gyroscopic matrices. The rank these matrices depend of the number of nodes, and for every node there are four degrees of freedom. The problem has been described in the space state, in order to obtain the classic eigenvalue and eigenvector problem. We realized the simulation of the multi disk rotor in order to obtain the Campbell diagram.*

Keywords*. Whirling Frequencies, Finite Elements, Vibration Analysis.*

1. Introduction

The time of high velocity was opened in final of the nineteen century, when Carl Gustaf of Laval developed a butter separator powered by steam turbine that reached high velocities, up 30,000 rpm, (Dimaragonas, 1976). Nowadays the modern turbo machines reach high rotation velocities, about 50,000 rpm, for example the propulsion jet turbines used in airplanes. This is the reason, because is very important to study the movements of rotate machines covering concepts such as: critics velocities, whirling frequency and gyroscopic effects.

The critics velocities appear when in same rotation the rotor is excited, going in resonant state and having catastrophic vibration, therefore is indispensable that this critics velocities taken been in consideration in the design of this machines. Gyroscopic effect is associated with the variation of direction of angular rotation momentum, that do the machine shown a precession, this phenomenon can be watched in a automobile doing a curve, the engine turn around of its crankshaft been this movement damping by dashpot, (Mabie, 1980). These precession effects do that each natural frequency been double, this one highest, called of forward, and another one smallest, called backward. This couple of frequencies is called as whirling and the direction depends of the initial conditions.

2. Basic elements of rotors

The basic elements of a good model are: disk, shaft and bearing elements. As following, this elements will be described through of its strain and kinetic energy.

2.1. The disk element

Using form function that satisfy the boundary conditions, in the generic form are given by, (Oliveira, 1999):

$$f(y) = ay + \frac{by^2}{L^2} + \frac{cy^3}{L^3} \tag{1}$$

Where L (m) is the disk dimension, a, b, c are constants and y is the position along of disk or shaft. So the kinetic energy T_D of disk element can be written, generalized coordinates form, like this

$$T_D = \frac{1}{2}\left[M_D f^2(l_1)\left(\dot{q}_1^{\,2} + \dot{q}_2^{\,2}\right) + I_{DX} f^2(l_1)\left(\dot{q}_1^{\,2} + \dot{q}_2^{\,2}\right)\right] - I_{DY}\Omega \frac{df}{dy}^2 (l_1)\dot{q}_1 q_2 + \frac{1}{2} I_{DY}\Omega^2 \tag{2}$$

Where M_D is the disk mass (kg), $\dot{q}_1$, $\dot{q}_2$ are velocity (m/s), I_{DX}, I_{DY} are inertia momentum ($kg.m^2$). and Ω rotation (rad/s).

2.2. The shaft element

The element shaft has both kinetic and strain energy due elastic deformation, so the kinetic energy is:

$$T_S = \frac{1}{2}\left[\rho S\int_0^L f^2(y)dy + \rho I\int_0^L\left(\frac{df}{dy}\right)^2 dy\right]\left(\dot{q}_1^2 + \dot{q}_2^2\right) - \rho IL\Omega^2 + \left[\rho I\Omega^2\int_0^L\left(\frac{df}{dy}\right)^2 dy\right]\dot{q}_1 q_2 \tag{3}$$

Where ρ is the density (*kg/m³*), *I* is area momentum (*m⁴*) and *S* is the shaft cros section area (*m²*), so the strain energy is:

$$U_S = \frac{EI}{2}\left[\int_0^L\left(\frac{d^2f}{dy^2}\right)^2 dy\right]\left(q_1^2 + q_2^2\right) \tag{4}$$

Where *E* is the Young module (*Pa*).

2.3. The bearing element

The virtual work executed by bearing forces, δ is a variation, that act in the shaft is given by:

$$\begin{aligned}\delta W = &-k_{xx}(l_2)\delta u(l_2) - k_{xz}w(l_2)\delta u(l_2) - k_{zz}w(l_2)\delta w(l_2) - k_{zx}u(l_2)\delta w(l_2) - \\ &c_{xx}\dot{u}(l_2)\delta u(l_2) - c_{xz}\dot{w}(l_2)\delta u(l_2) - c_{zz}\dot{w}(l_2)\delta w(l_2)\delta w(l_2) - c_{zx}\dot{u}(l_2)\delta w(l_2)\end{aligned} \tag{5}$$

Where k_{xx}, k_{zz}, k_{xz}, k_{zx} are the stiffness constants (*N/m*). The c_{xx}, c_{zz}, c_{zx} and c_{xz} are the damping constants (*N.s/m*).
We can see the adopted referencials in the Fig. (1).

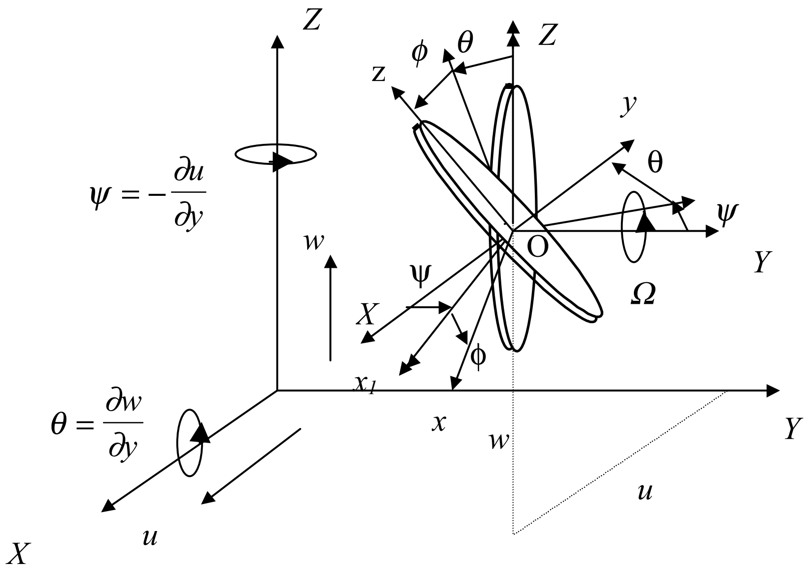

Figure 1 Adopted referential

3. Discretization method fundaments

Starting with the Hamilton equation, we have:

$$\int_{t_1}^{t_2}\delta(T - V)dt + \int_{t_1}^{t_2}\delta w_{nc}dt = 0 \tag{6}$$

Where T is the kinetic energy, V is the strain energy, w_{nc} is the virtual work of no conservatives forces and δ is a variation.

We can get the Lagrange equation for discreet system with n degree of freedom, described by independent displacement q_1 , q_2,..., q_n :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)+\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i, \quad i=1,2,\ldots,n \tag{7}$$

Where Q_i is the generalized force.

4. Transfer matrix from locals coordinates to global

The transfer matrixes in the global coordinates are like this:

$$[M]=\sum_{e=1}^{n}[a]_e^T[m]_e[a]_e \tag{8}$$

$$[K]=\sum_{e=1}^{n}[a]_e^T[k]_e[a]_e \tag{9}$$

$$[C+G]=\sum_{e=1}^{n}[a]_e^T[c+g]_e[a]_e \tag{10}$$

Where [M], [K] and [C+G] are the mass, stiffness, damping and gyroscopic global matrix respectively. The [m], [k] and [c+g] are the mass, stiffness, damping and gyroscopic local matrix respectively. The [a] is the transfer matrix, compound of "1" and "0".The transfer matrix dimension is 8x8n for shaft element and 4x8n for bearing and disk elements.

5. Equation solution

The numbers of degrees of freedom of the rotor, chosen of model, has *5* nodes with four degrees of freedom each node, therefore *20* degrees of freedom. This way all equationing of the rotors was modeled through of *4n* matrix dimension. So the dimensions of matrix [M], [C+G] and [K] are *20x20*, therefore the dimension of matrix [A] and [B], eigenvalue problem, are *40x40*. This way, we obtained *40* eigenvalues double in pairs, that is, *20* natural frequencies.

6. Finite elements model

6.1. Disk element

In each node, the rotors has four degrees of freedom: two linear displacements u and w and two angular displacements around of the X and Y axes, θ and ψ, respective. So the displacements nodal vector, q of disk center mass are:

$$\{q\}=\{u \quad w \quad \theta \quad \psi\}^T \tag{11}$$

Using Lagrange equation to disk element, we get:

$$[m_D]=\begin{bmatrix} M_D & 0 & 0 & 0 \\ 0 & M_D & 0 & 0 \\ 0 & 0 & I_{DX} & 0 \\ 0 & 0 & 0 & I_{DY} \end{bmatrix} \tag{12}$$

$$[g_D]=\Omega.\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_{DZ} \\ 0 & 0 & I_{DZ} & 0 \end{bmatrix} \tag{13}$$

6.2. Shaft element

The shaft was modeled such as a bar with transversal section constant. So finite element used has two nodes, with four degrees of freedom in each node. Therefore the matrixes are dimension equal eight, including four linear displacements and four angular displacements, according to the relationship:

$$\theta = \frac{\partial w}{\partial z} \text{ e } \psi = -\frac{\partial u}{\partial z} \tag{14}$$

The nodal displacements are given by:

$$\{q\} = [u_1, w_1, \theta_1, \psi_1, u_2, w_2, \theta_2, \psi_2]^T \tag{15}$$

Using Lagrange equation to shaft element, we get:

$$[m_1] = \frac{\rho SL}{420}\begin{bmatrix} 156 & 0 & 0 & 22L & 54 & 0 & 0 & 13L \\ 0 & 156 & 22L & 0 & 0 & 54 & -13L & 0 \\ 0 & 22L & 4L^2 & 0 & 0 & 13L & -3L^2 & 0 \\ -22L & 0 & 0 & 4L^2 & -13L & 0 & 0 & -3L^2 \\ 54 & 0 & 0 & -13L & 156 & 0 & 0 & 22L \\ 0 & 54 & 13L & 0 & 0 & 156 & -22L & 0 \\ 0 & -13L & -3L^2 & 0 & 0 & -22L & 4L^2 & 0 \\ -13L & 0 & 0 & -3L^2 & 22L & 0 & 0 & 4L^2 \end{bmatrix} \tag{16}$$

$$[m_2] = \frac{\rho I}{30L}\begin{bmatrix} 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ -3L & 0 & 0 & 4L^2 & 3L & 0 & 0 & -L^2 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \\ 3L & 0 & 0 & -L^2 & 3L & 0 & 0 & 4L^2 \end{bmatrix} \tag{17}$$

The mass matrix of the shaft element is given by $[m_s] = [m_1] + [m_2]$. While that the gyroscopic matrix, associated with the damping effect, is given by:

$$[g_S] = \frac{\rho I\Omega}{15L}\begin{bmatrix} 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 3L & 0 & 0 & -4L^2 & -3L & 0 & 0 & L^2 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 3L & 0 & 0 & L^2 & -3L & 0 & 0 & -4L^2 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \end{bmatrix} \tag{18}$$

The stiffness matrixes are:

$$[k_1]=\frac{EI}{(1+a)L^3}\begin{bmatrix} 12 & 0 & 0 & -6L & -12 & 0 & 0 & -6L \\ 0 & 12 & 6L & 0 & 0 & -12 & 6L & 0 \\ 0 & 6L & (4+a)L^2 & 0 & 0 & -6L & (2-a)L^2 & 0 \\ -6L & 0 & 0 & (4+a)L^2 & 6L & 0 & 0 & (2-a)L^2 \\ -12 & 0 & 0 & 6L & 12 & 0 & 0 & 6L \\ 0 & -12 & -6L & 0 & 0 & 12 & -6L & 0 \\ 0 & 6L & (2-a)L^2 & 0 & 0 & -6L & (4+a)L^2 & 0 \\ -6L & 0 & 0 & (2-a)L^2 & 6L & 0 & 0 & (4+a)L^2 \end{bmatrix} \tag{19}$$

$$[k_2]=\frac{F}{30L}\begin{bmatrix} 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ -3L & 0 & 0 & 4L^2 & 3L & 0 & 0 & -L^2 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \\ -3L & 0 & 0 & -L^2 & 3L & 0 & 0 & 4L^2 \end{bmatrix} \tag{20}$$

Where E (Pa) is the Young module and a is given by:

$$a=\frac{12EI}{GSL^2} \tag{21}$$

Where G (Pa) is the tranverse elasticity module, it is given by:

$$G=\frac{E}{2(1+\nu)} \tag{22}$$

Where ν is the Poisson coeficient, so the stiffness matrix resultant is $[k_S] = [k_1] + [k_2]$.

6.3. Bearings elements

The model of forces applied by bearing element is given by:

$$\begin{bmatrix} F_u \\ M_\theta \\ F_w \\ M_\psi \end{bmatrix} = -\begin{bmatrix} k_{xx} & 0 & k_{zx} & 0 \\ 0 & 0 & 0 & 0 \\ k_{zx} & 0 & k_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u \\ \theta \\ w \\ \psi \end{bmatrix} - \begin{bmatrix} c_{xx} & 0 & c_{zy} & 0 \\ 0 & 0 & 0 & 0 \\ c_{zy} & 0 & c_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{w} \\ \dot{\psi} \end{bmatrix} \tag{23}$$

Where F_u is the force along the X axe, F_w is the force along the Z axe (N), M_θ is the moment force around the X axe and M_ψ is the moment force around the Z axe ($N.m$).

7. State space model

The state space formulation was used of way to get the classic eingenvalue and eigenvector problem. So through of geral equation as following:

$$[M]\{\ddot{q}\}+([C]+[G])\{\dot{q}\}+[K]\{q\}=\{0\} \tag{24}$$

In the free case, the solution must be:

$$\{q\}=\{\Delta\}e^{rt} \tag{25}$$

Replace eq. (25) in eq. (24), we get:

$$\left(r^2[M]+r([C]+[G])+[K]\right)\{\Delta\}=\{0\} \tag{26}$$

Using identity form, we can write the equation (26) the better way. This way is the state space model, so we get:

$$\begin{bmatrix} [0] & [M] \\ [M] & ([C]+[G]) \end{bmatrix}\begin{bmatrix} r\{\Delta\} \\ \{\Delta\} \end{bmatrix}=\frac{1}{r}\begin{bmatrix} [M] & [0] \\ [0] & -[K] \end{bmatrix}\begin{bmatrix} r\{\Delta\} \\ \{\Delta\} \end{bmatrix} \tag{27}$$

We can see that the equation (27) is written in the eigenvalue and eigenvector form, like this:

$$[\tilde{A}]\{\vec{X}\}=\lambda[\tilde{B}]\{\vec{X}\} \tag{28}$$

Where:

$$[\tilde{A}]=\begin{bmatrix} [0] & [M] \\ [M] & ([C]+[G]) \end{bmatrix} \tag{29}$$

$$[\tilde{B}]=\begin{bmatrix} [M] & [0] \\ [0] & -[K] \end{bmatrix} \tag{30}$$

$$\lambda=\frac{1}{r} \tag{31}$$

The eingenvalue and eigenvector solution, yield in *2n* conjugate complex eigenvalue and therefore *2n* conjugate complex eigenvector. The eigenvalue is given by:

$$r_i=-\zeta\omega_n+i\omega_n\sqrt{\left(1-\zeta^2\right)} \tag{32}$$

Where ζ is damping ratio.

7. Multi disk rotor simulation

The rotor simulated is multi disk (Lalanne, 1990), showed in the Fig. (2):

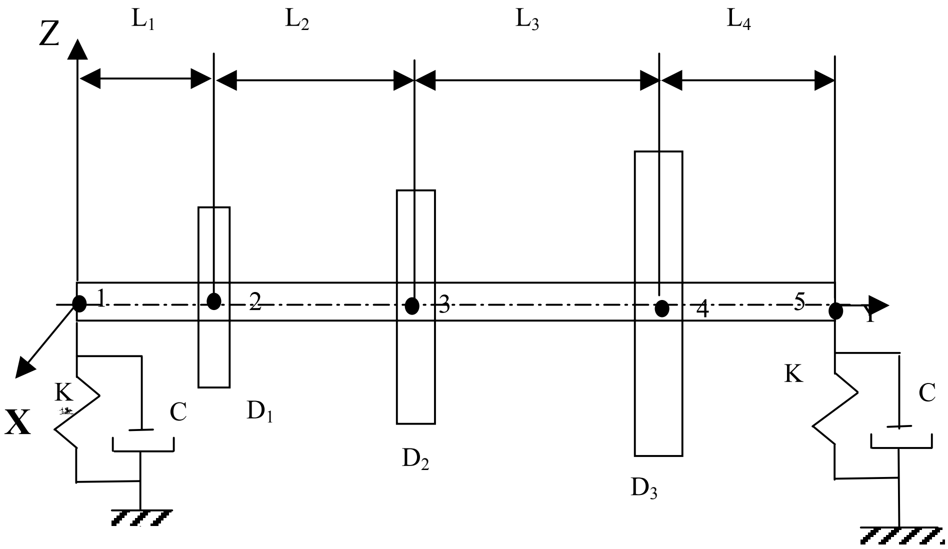

Figure 2 Multi disk rotor simulated

The constants used to simulate the multi disk rotor are in the Tab (1) above:

Table 1 inputs of multi disk rotor

Symbol	Value	Unit
E	2.10^{11}	N/m^2
υ	0.3	-
ρ	7800	Kg/m^3
S_{haft}	0.0079	m^2
$\mathit{S}_{\mathit{haft}}$	$4.9.10^{-6}$	m^4
L_1	0.2	m
L_2	0.3	m
L_3	0.5	m
L_4	0.3	m
I_{DY1}	0.1232	$kg.m^2$
I_{DY2}	0.9763	$kg.m^2$
I_{DY3}	1.1716	$kg.m^2$
$I_{Dx1} = I_{Dz1}$	0.0646	$kg.m^2$
$I_{Dx2} = I_{Dz2}$	0.4977	$kg.m^2$
$I_{Dx3} = I_{Dz3}$	0.6023	$kg.m^2$
k_{xx}	$5.7.10^7$	N/m
k_{xz}	0	N/m
k_{zx}	0	N/m
k_{zz}	7.10^7	N/m
c_{xx}	5.10^2	N.s/m
c_{xz}	0	N.s/m
c_{zx}	0	N.s/m
c_{zz}	7.10^2	N.s/m

8. Flow chart of developed software

The software was developed in Matlab© environment, because it is very reliable. The flow chart is showed in the Fig. (3).

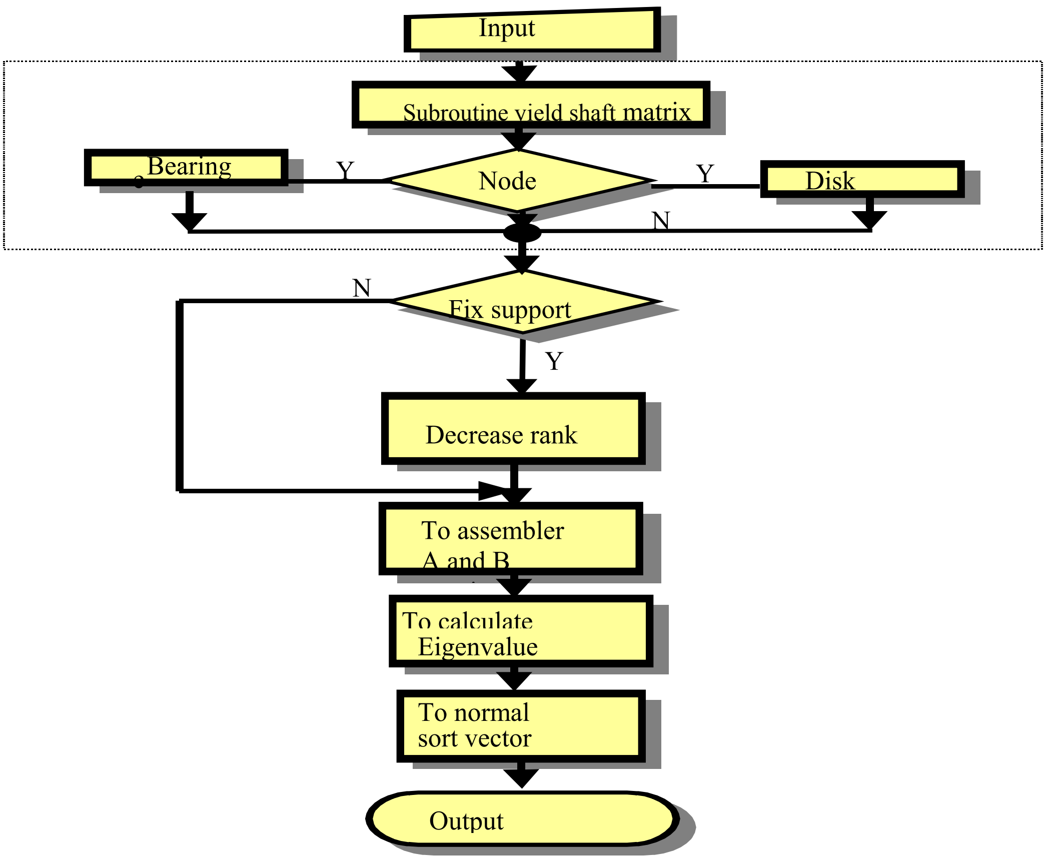

Figure 3 flow chart of software

Through this developed software, we yield the Campbell diagram of multi disk rotor such as showed in the Fig. (4).

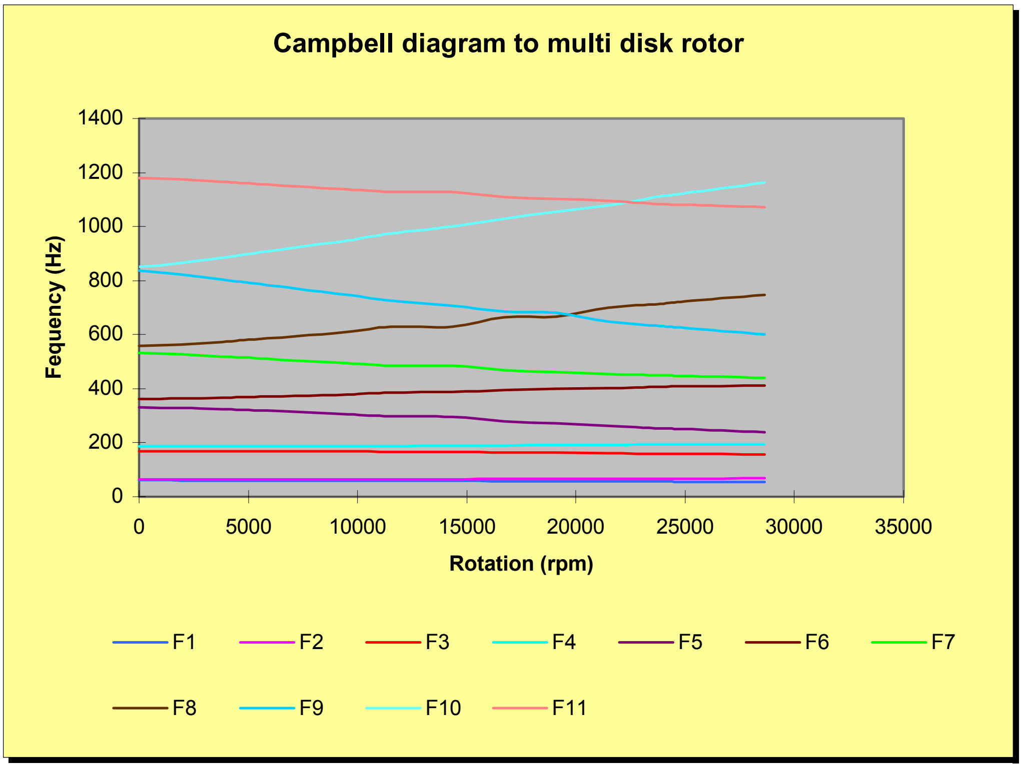

Figure 4 Campbell diagram to multi disk rotor

9. Conclusions

The developed software has showed a well performance in comparison with (Lalanne, 1990) results, because the proposed model has a good agreement with the rotor dynamic theory. The advantage this software is that it takes in consideration the gyroscopic effect on disk element, we don't consider the disk element such as shaft element like do another software. The shaft element was modeled according to Timoshenko beam formulation, including shear effect and rotatory inertia in five nodes. Moreover this software can be used in rotors where the stiffness of the bearings are very high, in this case they are modeled such as fix support and it is very cheaper than other specialized softwares. The Campbell show the gyroscopic effect on fhe natural frequencies, where the developed software calculate the whirling frequencies in rotation frequency function. The gyroscopic effect increase or decrease the natural frequencies, depends of system initial conditions. So the cross of 45° line with the whirling frequencies, are the critics velocities. This critics velocities must be avoid, in permanent regime case, in the turbomachine design.

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