Modal Analysis of Synthetic Graphite Beams

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***Abstract**: The objective of this study was to obtain the damping factor (ξ)as well as the elasticity modulus (E) of two kinds of synthetic graphite (HLM and ATJ), using the modal analysis technique. Prismatic beams of square section (~ 11 x 11 mm) and length over thickness ratio (L/t) of about 22.7 were tested in the free - free condition. The first four modes of vibration were taken into account in the analysis. In addition, numerical simulations were also carried out in this investigation. The agreement between the theoretical and the experimental results was quite good. The average values of E and ξ for the HLM graphite were 20% and 90% higher , respectively, than those presented by the ATJ graphite.*

***Keywords**: synthetic graphite, modal analysis, damping.*

1. Introduction

Graphite is a material that has shown a history of continually widening technological applications. Originally developed for use in electrodes, it now has many uses which include neutron moderation in nuclear power reactors; nozzle throats of rockets and space vehicles; brushes of electrical machines; crucibles to operate at high temperatures; and self lubricating bearings.

Natural graphite is an allotropic form of the element carbon that shows, in an ideal lattice, shown in Figure 1, a well developed layered structure, stacked parallel to each other, and to the plane (X,Y), in the sequence ABAB... , in which the atoms are hexagonally arranged forming the so called basal planes (Reynolds, 1968; Marsh, 1989). Due to the fact that the chemical bonds within the layers are covalent, but, on the other hand, those between the layers (i.e. in the direction Z) are weak forces of van der Waals, natural graphite is anisotropic. In the directions X and Y, for instance, the material is stiffer, stronger, and conducts electricity as well as heat. Along the direction Z, on the other hand, the material is an insulator of heat and electricity. So, it presents higher mechanical strength and stiffness in basal planes and, in comparison, poor mechanical behavior perpendicularly to the basal planes. Synthetic graphites manufactured in hydrostatic presses, which starts from a mixture of coke and binder (i.e. particulate composites), present a smaller degree of discrepancy among the mechanical properties, and so, macroscopically, they exhibit a mechanical behavior closer to the isotropic. However, synthetic graphites that are extruded, present some degree of anisotropy.

Synthetic graphites can be obtained from a mixture of coke and binder (e.g. pitch), in different proportions, which is subjected to a series of thermal and mechanical treatments that ends with a graphitization process at a temperature of approximately 3000 °C (Friedrich et al. 2002). Despite the fact that the mechanical properties of graphite, at about 20 °C, can be considered only moderate ($10 < E < 12$ GPa, tensile static strength of about 40 MPa), they can maintain such properties up to temperatures of about 2000 °C, in the absence of an oxidizing atmosphere (Polidoro, 1987; Pardini, et al., 2001). In addition, the specific mass of graphite is normally very low, typically below 2 g/cm^3, which make this material suitable for aerospace applications. Depending on the conditions of the treatments during processing, there are different kinds of synthetic graphite. In this investigation, the elasticity modulus (E) and the damping factor (ξ) of two kinds of synthetic graphites (HLM and ATJ) will be evaluated, using the technique of modal analysis. Such parameters (E and ξ), among others, are important for the design of nozzle throats of rocket engines which, in particular, are subjected to mechanical vibrations and high temperatures.

Despite the fact that details of the **static** behavior of graphites are available in some publications (Reynolds, 1968; Blackman, 1970; Marsh, 1989), no publications concerning with the **dynamical** behavior of these materials were found in the literature, so far. Graphites can be classified, according to the raw materials, as coarse, medium or fine grain, and, depending on their processing technique, as extruded or molded grades. In the present study, two kinds of commercial synthetic graphite will be investigated: (i) extruded graphite with coarse grains, designated as HLM; and (ii) a graphite which was subjected to isotropic compression during the manufacture process, presenting fine grains and designated as ATJ. In this context, one of the objectives of this research was to carry out experiments and numerical simulations, in order to obtain the damping factor as well as the elasticity modulus of prismatic beams of synthetic graphite.

2. Materials and Methods

The specimens used in this study consisted of prismatic beams of HLM and ATJ synthetic graphite, which were machined to the nominal dimensions 11 x 11 x 250 mm, and tested in the vertical position (direction Z), with a the free-free boundary condition, using two elastic strings at each extremity of the test specimens (four in total), as shown in Figure 2.

In order to evaluate the dynamic parameters of the beams, the experimental modal analysis was carried out exciting the structure, a prismatic beam in a free-free boundary condition and endowed with eleven equidistant nodes along the longitudinal direction, with an instrumented hammer (PCB Piezotronic), according the set up represented in Figure 2. The hammer had a sensibility of 0.18 mV/N and its plastic tip was chosen in order to generate a well-defined spectrum in the band varying from 0 to 5 kHz, considered in this study. The average mass of the tested beams was about 55,2 grams, and the chosen accelerometer (Piezotronics 353B16, sensibility of 10.31 mV/g) had a mass of 1.5 grams. The ratio of the mass of the accelerometer over the average mass of the tested beams was 0.027, which corresponds to less than 3% of the mass of the specimens.

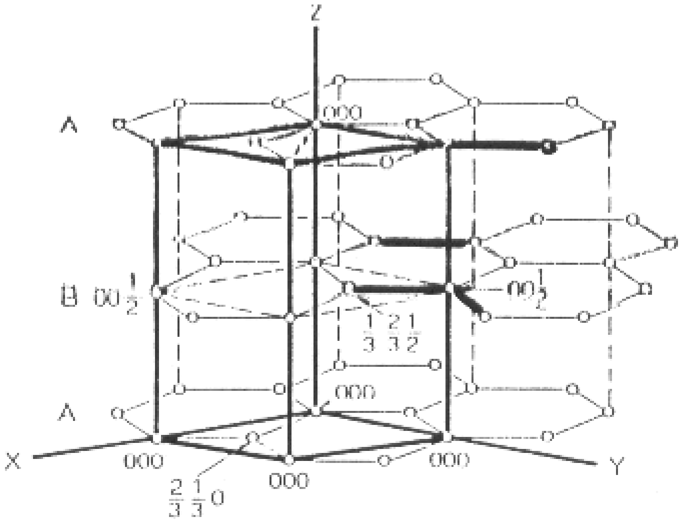

Figure 1 – Representation of an ideal lattice of a graphite

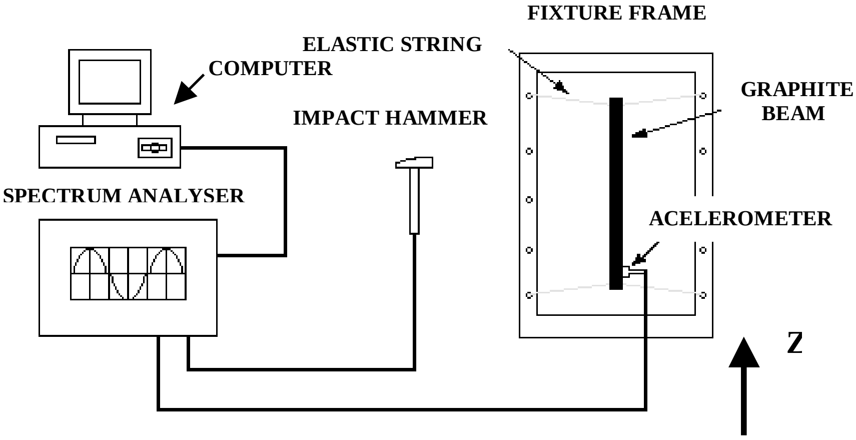

Figure 2 - General view of the test assembly.

Taking into account the relative dimensions of the accelerometer and the graphite beams, the number of drive points adopted in the experiments for the dynamical measurements was eleven. Briefly, for each one of the 11 drive points, successively, the following steps were executed during the course of the experiments:

1. Fixation of the accelerometer in the first drive point, with its longitudinal axis perpendicularly to the surface of the beam, using wax;

2. Excitation of the first test point with the instrumented hammer, verifying after the impact, by inspection of the temporal response signal, if multiple impacts did not occur;

3. Repeat step 2 five times;

4. Processing of the Frequency Response Function (FRF) and the coherence between excitation signal of the test point (i.e. the force from the hammer) and the acceleration from the drive point, based on the average of the five impacts (step 3); and

5. Repetition of steps 2, 3 and 4, for the remaining 10 test points.

The processing of the data obtained from the tests was executed by the commercial software STARmodal from Structural Measurements Systems Inc. The dynamical analyzer used in the experiments the Hewlett Packard (HP) model 35665A and the first four modes of the beams were considered in this study. For the initial validation of the experiments, the test results were compared with classical solutions for the dynamical behavior of beams found in the literature (Rao, 1995). In addition, the results were also compared with numerical simulations of the problem using the finite element (FE) code ANSYS/ED, using an eight nodes plane stress element of constant thickness. In such simulations it was possible to obtain the natural frequencies and the modes of vibration. These two spectral parameters were analyzed and also compared with those obtained experimentally.

Based on k natural frequencies obtained experimentally, the elasticity modulus of a prismatic beam can be estimated by equation (1).(Rao, 1995; and Adorno, 2002)

$$E = \frac{\rho \cdot A \cdot \omega_i^2 \cdot l^4}{I \cdot (\beta \cdot l)_i^4} \qquad (1)$$

where: ρ is the specific mass; A is the cross section area; ω_i is the natural frequency associated to the order of the vibration mode; l is the length of the beam; I is the moment of inertia; and ($\beta \bullet l$) is the constant associated to the order of the vibration mode and the boundary condition. For the free-free boundary condition, the ($\beta \bullet l$) constants are equal to 3.9266 in the 1st mode, 7.0685 in the 2nd mode, 10.2101 in the 3rd mode, and 13.3517 in the 4th mode.

The typical characteristics of the beams used in the experimental study are show in Table 1. The density of the HLM and the ATJ graphite beams was estimated using their dimensions and the weights, while the variation range of the elasticity modulus (E) was obtained in the literature (Pardini et al., 2001).

Table 1 – Characteristics and Elasticity Modulus of the synthetic graphite beams.

Graphite	Length (mm)	Height (mm)	E (GPa)	Density (kg/m^3)
HLM	250	11.10	10 < E < 12	1731.43
ATJ	250	11.45	10 < E < 12	1740.83

In order to evaluate the quality of the experimental results, the degree of correlation between the experimental modal shapes and those obtained numerically were compared using the Modal Assurance Criterion (MAC), which is also known as Mode Shape Correlation Coefficient (MSCC) (Sreenivas, 1997; Ewins, 1984). This criterion generates a parameter that indicates the degree of correlation between the mode j of the first test and mode k of the second test; varying their values from 0 to 1.0 with 0 for no correlation and 1 for total correlation. Let ϕ_A and ϕ_B be the first and the second modes of vibration written as $\phi_A[n \times m]$ and $\phi_B[n \times m]$, where m is the number of modes and n is the number of nodal points analyzed. The MAC is then defined for j and k as follows:

$$MAC(j,k) = \left(\sum_i^n {}_i\phi_{Aj} \cdot {}_i\phi_{Bk}\right)^2 \Big/ \left(\sum_i^n \left({}_i\phi_{Aj}\right)^2 \cdot \sum_i^n \left({}_i\phi_{Bk}\right)^2\right) \qquad (2)$$

where: ${}_i\phi_{Aj}$ is the coordinate i of the column j of $\boldsymbol{\phi_A}$ and ${}_i\phi_{Bk}$ the coordinate i of column k of $\boldsymbol{\phi_B}$.

As one can observe in Equation (2) the MAC is based on the scalar product between two unitary vectors. So, if such measure is equal to zero it can be assumed that the vectors are orthogonal. If the MAC between two modal shapes is equal to 1.0, then such vectors are perfectly correlated. In practice, any value between 0.9 and 1.0 is considered to be a good correlation (HP Application Note, 1986]. If the value of the coefficient is smaller than 0.9, then there will be a degree of inconsistency which will greater the smaller is the value of the coefficient. So, the MAC is important to evaluate the correlation between the numerical and experimental modal shapes.

In addition to the MAC, the Modal Scale Factor (MSF) (HP Application Note 243-3, 1986; Ewins, 1984) was also used in order to evaluate the agreement between the estimates experimental and numerical natural modes of vibration. Put in a simple way, the MSF represents the inclination of the best-fitted straight line that represents the points of the graphic. The quantification of this parameter can be obtained from equation (3):

$$MSF(x,p) = \frac{\sum_{i=1}^{N} (\phi_X)_i (\phi_p)_i^*}{\sum_{i=1}^{N} (\phi_p)_i (\phi_p)_i^*} \qquad (3)$$

3. Results and Discussions

The experimental and numerical (FE) natural frequencies, as well as the experimental damping factor of the HLM and ATJ graphite beams, for the first four vibration modes, are presented in Tables 2 and 3, respectively. The damping factors presented in these Tables suggests that the damping characteristics of the HLM graphite, in the range of frequencies covered in the experiments (from about 440 to 4300 Hz), are better than those presented by the ATJ graphite. The average damping factor of the HLM graphite (0.724 ± 0.135 %) was about 90% higher than the average damping factor of the ATJ graphite (0.331 ± 0.061 %). The HLM graphite was also stiffer than the ATJ and, quantitatively, in the experiments carried out so far, presented an elasticity modulus which, in average, was 20% higher.

In order to evaluate the dynamic parameters of the beams, the experimental modal analysis was carried out exciting the structure with an instrumented hammer (PCB Piezotronic), according the set up represented in Figure 2. The hammer had a sensibility of 0.18 mV/N and its plastic tip was chosen in order to generate a well-defined spectrum in the band varying from 0 to 5 kHz, considered in this study. The average mass of the tested beams was about 55,2 grams, and the chosen accelerometer (Piezotronics 353B16, sensibility of 10.31 mV/g) had a mass of 1.5 grams.

Table 2 – Dynamical characteristics estimated for the HLM graphite beam.

Vibration Modes	ω_n (Hz)		ξ - *(%)*	E [GPa] *Based on Equation* (*1*)
	Exp.	*FE*		
1° mode	512.44	475.52	0.62274	12.50
2° mode	1371.00	1296.00	0.81372	11.78
3° mode	2745.45	2503.00	0.56600	10.99
4° mode	4318.18	4059.00	0.89503	11.26

Table 3 – Dynamical characteristics estimated for the ATJ graphite beam.

Vibration Modes	ω_n (Hz)		ξ - *(%)*	E [GPa] *Based on Equation* (*1*)
	Exp.	*FE*		
1° mode	463.93	441.74	0.45056	10.25
2° mode	1229.09	1203.60	0.34874	9.47
3° mode	2464.55	2321.8	0.29796	9.91
4° mode	3927.27	3761.00	0.42598	9.21

Comparing the numerical and the experimental results of the first four natural frequencies of the graphite beams, shown in Tables 2 and 3, it can be verified that the maximum difference observed between them is equal to 9% , while the average difference was about 6%. This indicated that the finite element simulation model was fairly accurate in the range of frequencies investigated in this study, which included the fist four modes of vibration of the graphite beams. Based on Equation (1) and the frequencies presented on Tables (2) and (3) the mean values of the elasticity modulus of the graphite HLM and ATJ were estimated, respectively, equal to 11.9 and 9.7 GPa.

The MAC and MSF values between the numerical and experimental modal vectors for the HLM and the ATJ beams are presented in Tables 4. And 5, respectively. According to the data presented in these Tables, there was an excellent correlation between the vectors, since the MAC and MSF values in all the analyzed modes are very close to 1.0. It was also observed that the largest deviations from the best straight line matching the experimental points and the ideal theoretical straight line obtained numerically occur for the modes 2 and 4. However, the good correlations obtained so far were the result of an extensive series of experiments and simulations. Initially, the measurements of all the eleven nodes of the tested beams, for all the four vibration modes taken into account, were included in the modal analysis, and the estimations of the elasticity modulus of the graphites HLM and ATJ were not in good agreement with those obtained from the literature. It was observed that, along the direction Z shown in Figure 2, each vibration mode presents points of relatively large amplitudes, and points of stagnation. Since the points of higher amplitudes tend to present a more favorable signal to noise ratio, and for the points of lower amplitude the opposite trend takes place, the points of stagnation (e.g. points 1, 6 and 11, in Figure 3) were eliminated from the analysis. As a result, when only the points of so called “good displacement” (e.g. points 2, 5, 7 and 10, in Figure 3) were considered in the modal analysis, the correlations involving the comparison of the experimental elasticity modulus with those from the literature improved significantly. In Figure 3, which refers to the fourth vibration mode of the beam, the points 2, 5, 7 and 10 were considered points of “good displacement”, whereas the points 1, 6 and 11, on the other hand, were treated as points of stagnation and were eliminated from the analysis.

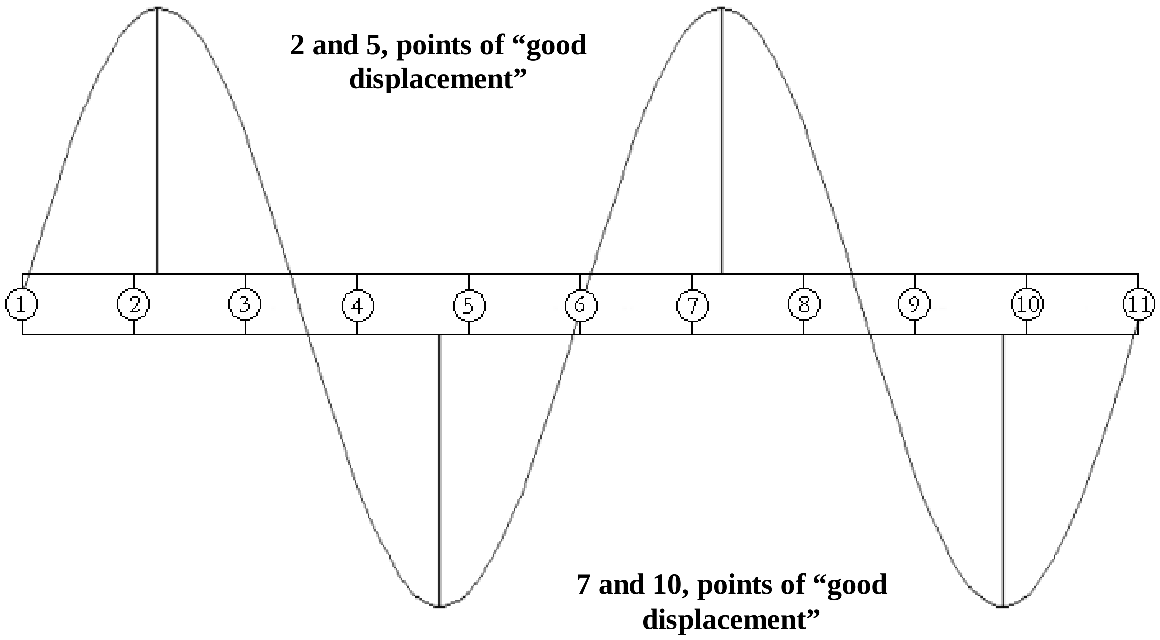

Figure 3. Points of “good displacement” for the fourth mode of vibration of the beam.

Table 4 – MAC and MSF values between the numerical and experimental vectors for HLM graphite beam.

MODE	MAC	MSF
1°	0.998115	0.999057
2°	0.981135	0.990523
3°	0.983119	0.991523
4°	0.978570	0.989227

Table 5 – MAC and MSF values between the numerical and experimental vectors for HLM graphite beam.

MODE	MAC	MSF
1°	0.998120	0.999060
2°	0.994070	0.997031
3°	0.979822	0.989860
4°	0.982981	0.991454

4. Main Conclusions

This work presented the results obtained using an experimental non-destructive technique known as modal analysis, which allows one to obtain the damping factor (ξ), as well as the elasticity modulus (E) of synthetic graphites. The elasticity moduli, alternatively, could also be measured with the use of strain gages in beams subjected to static known forces. However, graphite is a very brittle material and fractures with very low strains. So, it is not an easy task to obtain the elasticity modulus of a graphite beam, with good precision, from the measurement of its bending strains under the action of a known force, without destroying the test specimen in the first place. Cracks and pores are common features that are found in graphite microstructure, and graphite beams are prone to fail when they are subjected to very low strains. In addition, the modal analysis is cheaper, and allows one to obtain both properties, E and ξ, in a single test.

The experimental values of E, for both kinds of graphite, presented good correlation with data available in the literature (Polidoro, 1987; Pardini et al., 2001), and the scatter of the results, taking into account all the four vibration modes considered in the analysis, was always less than 5%. The HLM fine grain extruded graphite presented an average elasticity modulus (E) higher than the one concerned with the ATJ (coarse grain, obtained by isotropic compression) by a factor of 1.20. In addition, the average damping factor (ξ) of the HLM graphite was about 90% higher in comparison with the value obtained for the ATJ graphite. The scatter in the values of ξ was in the range from about 16 to 19%. However, since the synthetic graphites are porous particulate composites with presents some degree of residual stresses from the heat treatments which they are subjected (Reynolds, 1968; Marsh, 1989), such range of magnitudes in the scatter for the damping factors are not a surprise. As far as the values of E and ξ are concerned, the HLM graphite can be considered superior relatively to the ATJ graphite. However, an important detail must be pointed out, which is the fact that the HLM graphite is extruded, and was tested with the extrusion direction parallel to the longitudinal axis of the beam. Since this material is ortotropic, it is expected that in the transverse direction the elastic properties will be rather lower. The ATJ graphite obtained using the process of isotropic compression, on the other hand, is an isotropic material and should be expected to present the same elastic properties both, in the longitudinal a transverse direction. So, the apparent superior elastic properties of the HLM graphite beam must be treated with caution. In order to obtain both the longitudinal as well as the transverse properties of these two kinds of graphite, it is necessary to test plates of these materials and to carry out a 2-D analysis. However, this, at this moment, is still beyond the scope of the present 1-D analysis.

5. Acknowledgements

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