COMPROMISE OPTIMIZATION AND META-MODELING FOR A FLEXIBLE 7 DOF MECHANICAL SYSTEM USING THE RESPONSE SURFACE METHOD

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Abstract. *The use of numerical optimization techniques has been largely explored in the literature concerning the optimization of mechanical systems. However, there are engineering design situations in which the complexity of the problem turns the modeling of the problem impossible. Consequently, it is necessary to build reduced models or meta-models, which are valid for a limited domain of analysis and consider only the most significant design variables by introducing simplifying hypothesis to the problem. The present contribution deals with the multicriteria optimization of a 7 d.o.f. mass lumped mechanical system. The following optimization criteria are taken into account: the minimization of the displacement of one of the masses and the maximization of the difference between the third and the fourth natural frequencies. The design variable involves three mass elements and a stiffness element. Quadratic response surfaces are obtained from factorial design techniques in order to represent the various criteria for optimization purposes for a given design space. Genetic algorithms are used together with compromise optimization techniques in order to obtain the optimal design of the 7 d.o.f. mechanical system..*

Keywords: *Meta-modeling techniques, response surface methods, compromise optimization.*

1. Introduction

The aim of the techniques of numerical optimization is to minimize or maximize the value an objective function or a performance criterion associated with a given engineering system. Thus, these techniques lead to the determination of which configurations of the design variables correspond to the optimal situation, provided that a set of constraint functions are respected. Consequently, optimization techniques are widely used to solve direct and inverse problems in engineering and find applications in a variety of problems (Vanderplaats, 1999). In the field of the dynamics and vibrations, these techniques can be applied to a number of problems due to the possibility of obtaining reliable discretized models, as those based on the Finite Element Method (FEM) (Butkewitsch and Steffen, 2001). However, there are situations in which a mathematical model is not available and the objective function must be optimized by using statistical parameters that are obtained experimentally (Montgomery, 1991). In other cases, modeling involves excessively complex operations and it is necessary to generate an approximate model (empirical model) by using computational experiments. From this approximate model, the optimization procedure can be conducted. Another problem in dynamics is the case in which the number of design variables to take into account is extremely high. Consequently, any optimization procedure would be very time consuming or even impossible. Thus, it is necessary to build a so-called meta-model as a function of a reduced number of variables which can be used as design variables for optimization purposes (Butkewitsch and Steffen, 2002).

It is also important to mention that a phenomenon called interaction between the variables is frequently found in the optimization process. Then, it is necessary to vary them according to statistical standards during the search for the optimum solution (Barros, Grandson et all, 1995).

The experimental design techniques cited by Barros, Grandson et all (1995) use statistical methods as in Fonseca (1982), in order to reduce modeling time, number of operations involved and costs of experiments. A side benefit one can take from this methodology is that the different experiments give new insight to the engineer about the influence of the variables upon the dynamical behavior of the system. However, it is important to mention that the previous technical knowledge of the design engineer about the process is always useful.

The objective of the present study is to revisit the principles of design of experiments and to use them together with compromise optimization techniques as a robust tool for meta-model generation aiming at optimizing the dynamical behavior of mechanical systems. As an application, the dynamical behavior of a seven d.o.f. mechanical system is optimized.

2. Meta-Model of a 7DOF system

A 7d.o.f. flexible undamped structure is presented in Fig. (1). An unbalanced engine is connected to mass 3 in such a way that forced vibrations result.

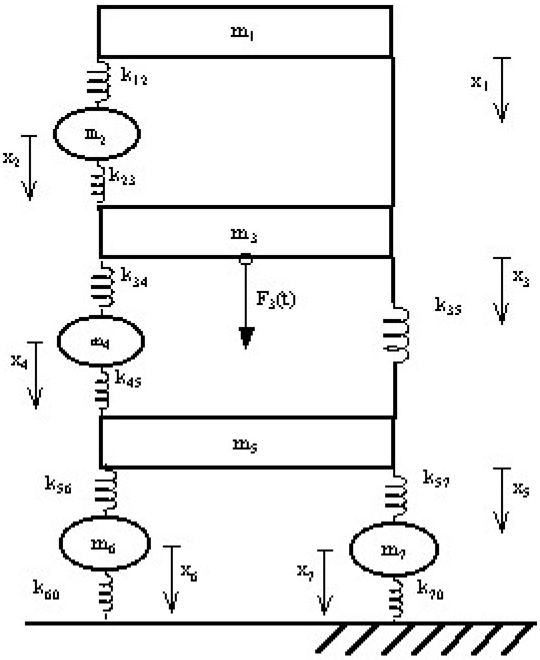

Figure 1. 7 d.o.f. mechanical system

Two aspects of the dynamic behavior of the system are to be optimized: the difference between the third and the fourth natural frequencies has to be maximized and, simultaneously, the displacement of floor 3 has to be minimized. Thus, the resulting optimization problem is considered as a multi-objective one. To facilitate the analysis it has been considered that all stiffness k of the model are equal and the masses of floors 3 and 5 are the same. The masses of floors 2, 4, 6 and 7 are fixed previously. Therefore three design variables are considered, namely the stiffness k and the masses m1 and m3.

The dynamic equations of motion can be derived by using Newton's second law, resulting Eqs. (1) to (7):

$$m_1 \ddot{x}_1 + k_{12}x_1 - k_{12}x_2 = 0 \tag{1}$$

$$m_2 \ddot{x}_2 - k_{12}x_1 + (k_{12} + k_{23})x_2 - k_{23}x_3 = 0 \tag{2}$$

$$m_3 \ddot{x}_3 - k_{23}x_2 + (k_{23} + k_{34} + k_{35})x_3 - k_{34}x_4 - k_{35}x_5 = F_3(t) \tag{3}$$

$$m_4 \ddot{x}_4 - k_{34}x_3 + (k_{34} + k_{45})x_4 - k_{45}x_5 = 0 \tag{4}$$

$$m_5 \ddot{x}_5 - k_{35}x_3 - k_{45}x_4 + (k_{35} + k_{45} + k_{56} + k_{57})x_5 - k_{56}x_6 - k_{57}x_7 = 0 \tag{5}$$

$$m_6 \ddot{x}_6 - k_{56}x_5 + (k_{56} + k_{60})x_6 = 0 \tag{6}$$

$$m_7 \ddot{x}_7 - k_{57}x_5 + (k_{57} + k_{70})x_7 = 0 \tag{7}$$

or, alternatively, in the matrix form as shown in Eq. (8).

$$\begin{bmatrix} m_1 & & & & & & \\ & m_2 & & & 0 & & \\ & & m_3 & & & & \\ & & & m_4 & & & \\ & & & & m_5 & & \\ & 0 & & & & m_6 & \\ & & & & & & m_7 \end{bmatrix} \bullet \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \\ \ddot{x}_6 \\ \ddot{x}_7 \end{Bmatrix} +$$

$$\begin{bmatrix} k_{12} & -k_{12} & & & & & \\ -k_{12} & k_{12}+k_{23} & -k_{23} & & & 0 & \\ & -k_{23} & (k_{23}+k_{34}+k_{35}) & -k_{34} & -k_{35} & & \\ & & -k_{34} & k_{34}+k_{45} & -k_{45} & & \\ & & -k_{35} & -k_{45} & (k_{35}+k_{45}+k_{56}+k_{57}) & -k_{56} & -k_{57} \\ & 0 & & & -k_{56} & k_{56}+k_{60} & 0 \\ & & & & -k_{57} & 0 & k_{57}+k_{70} \end{bmatrix} \bullet \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ F_3(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (8)$$

For simulation purposes, the excitation force is considered as F_3 (t) = 150cos(ωt) + 250cos(2ωt) + 80cos(4ωt), and the rotation of the engine is n=1500 rpm.

Table 1. Complete factorial design 3^3 for the 7 d.o.f. mechanical system.

m_1	m_3	K	Δω	y
-1	-1	-1	0.03290115254438	-0.00000400026145
-1	-1	0	0.03290115254438	0.00076450574870
-1	-1	1	0.03290115254438	0.00006242138917
-1	0	-1	0.03062806672919	-0.00052186998396
-1	0	0	0.03062806672919	0.00025107763174
-1	0	1	0.03062806672919	-0.00044656512224
-1	1	-1	0.02993995523440	-0.00034164109350
-1	1	0	0.02993995523440	0.00043574812775
-1	1	1	0.02993995523440	-0.00025745302068
0	-1	-1	0.03125583693753	-0.00047363510735
0	-1	0	0.03125583693753	0.00027327501031
0	-1	1	0.03125583693753	-0.00045040524172
0	0	-1	0.02902874559220	-0.00048431743723
0	0	0	0.02902874559220	0.00026703428597
0	0	1	0.02902874559220	-0.00045220436050
0	1	-1	0.02838662856727	0.00020309884585
0	1	0	0.02838662856727	0.00095889217460
0	1	1	0.02838662856727	0.00024409513368
1	-1	-1	0.03004480756068	-0.00026935856191
1	-1	0	0.03004480756068	0.00045595566325
1	-1	1	0.03004480756068	-0.00028932048127
1	0	-1	0.02786371068521	0.00022714650083
1	0	0	0.02786371068521	0.00095690233154
1	0	1	0.02786371068521	0.00021606779257
1	-1	1	0.02726758813014	0.00142175017654
1	1	0	0.02726758813014	0.00215594761280
1	1	1	0.02726758813014	0.00141955467938

For the calculation of the response, classical modal analysis techniques were used. From the solution of the corresponding eigenvalue problem, the difference between the frequencies of the fourth and third modes was calculated. A factorial design 3^3 was made to generate the Meta-Model corresponding to the 7 d.o.f. mechanical system. The results for the computational experiments can be seen in the Tab. (1). Due to the nature of the problem, a quadratic model was used (both displacement of floor 3 and difference between the frequencies, as commented previously). To define the design space to be considered, the following bounds were assigned to the design variables: $10 \le m1 \le 20$; $10 \le m3$ and $m5 \le 20$; $10e5 \le k \le 10e6$.

The values of the angular coefficients of the regression curves (*bw* and *by*) were obtained for the curves representing the variation of the frequencies ($\Delta\omega$) and of the displacement (y), respectively, as shown in the Tabs. (2) e (3).

Table 2. Model parameters (*bw*) corresponding to the regression curve $\Delta\omega$

Term	Value
1	0.02902874559220
m1	-0.00138217802199
m3	-0.00143460418513
K	-0.00000000000000
$m1^2$	0.00021714311500
$m3^2$	0.00079248716020
k^2	0.00000000000000
m1 * m3	0.00004599446986
m1 * k	-0.00000000000000
m3 * k	-0.00000000000000

Table 3. Model parameters (*by*) corresponding to the regression curve *y*.

Term	Value (* 1.0e-003)
1	0.26703428597122
m1	0.35291234990066
m3	0.34280858214848
K	0.01605653836501
$m1^2$	0.33695569567216
$m3^2$	0.34904930648227
k^2	-0.73529518484012
m1 * m3	0.50718739262576
m1 * k	-0.02159589249591
m3 * k	0.00444160554866

In Tabs. (4) and (5) the standard errors calculated for each one of the parameters for both responses of response of the system ($\Delta\omega$ and y) are presented.

Table 4. Standard errors for the response $\Delta\omega$.

Term	Standard Error (* 1.0e-004)
1	0.80029372383499
m1	0.37046400762413
m3	0.37046400762413
K	0.37046400762413
$m1^2$	0.64166248358057
$m3^2$	0.64166248358057
k^2	0.64166248358057
m1 * m3	0.45372389337282
m1 * k	0.45372389337282
m3 * k	0.45372389337282

As the standard error values can be considered small it can be assumed that the regression model corresponds to the original 7 d.o.f. mechanical system. To confirm this assumption, it is necessary to make an analysis of variance

(ANOVA). ANOVA results are presented in Tabs. (6) and (7), and the conclusion is that the quadratic model is adjusted to the system.

Table 5. Standard errors for the response *y*.

Term	Standard Error (* 1.0e-003)
1	0.51031687297423
M1	0.23623080912631
M3	0.23623080912631
K	0.23623080912631
$M1^2$	0.40916376371987
$M3^2$	0.40916376371987
K^2	0.40916376371987
M1 * m3	0.28932247194213
M1 * k	0.28932247194213
M3 * k	0.28932247194213

Table 6. ANOVA for the 7d.o.f. system ($\Delta\omega$ response).

Source of Variation	Quadratic Sum	Quadratic Average
Regression	7.550960262831032e-005	7.550960262831032e-005
Residual	2.470384457008718e-008 * (n-2)	2.470384457008718e-008
Total	7.612719874256250e-005	

Table 7. ANOVA for the 7d.o.f. (*y* response).

Source of Variation	Quadratic Sum	Quadratic Average
Regression	1.211071128472091e-005	1.211071128472091e-005
Residual	1.004489913248459e-006* (n-2)	1.004489913248459e-006
Total	3.722295911593240e-005	

At this point the response surfaces are obtained from the factorial design (Tab. (1)). The response surfaces are illustrated in Figs. (2) and (3). Another alternative to obtain the response surfaces could consider a linear model for the regression curves for the responses ($\Delta\omega$ e *y*). For this purpose a smaller interval should be taken and the modeling steps should be repeated until the best design space is obtained. As two conflicting objectives are involved, it is possible that different design spaces are obtained. Because of that, it was decided to work on a larger design space and, consequently, a more complex model was defined to obtain the response surfaces. Figs (2) and (3) illustrate the response surfaces for the two objective functions ($\Delta\omega$ and *y*) and their confidence intervals (95% in this case).

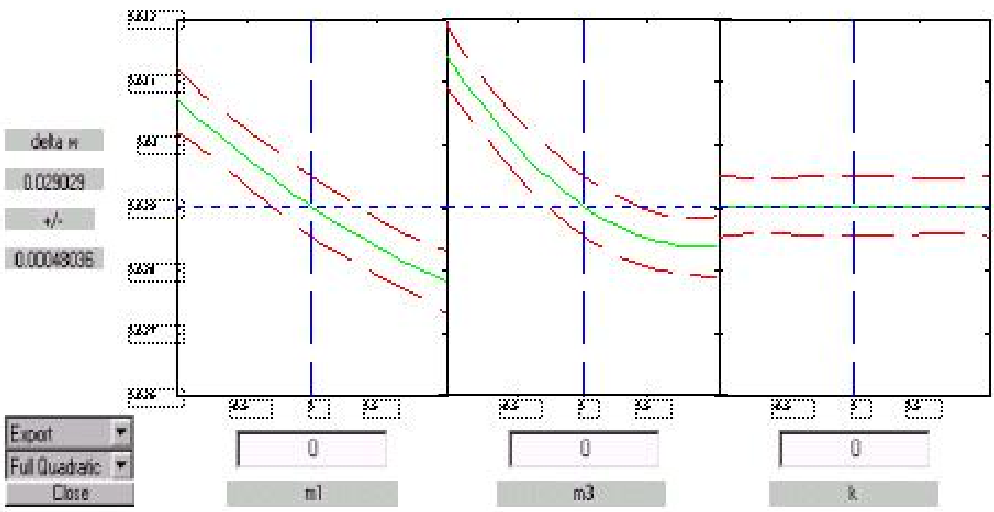

Figure 2. Response surfaces for the three parameters (m1, m3 e k) – Objective function $\Delta\omega$.

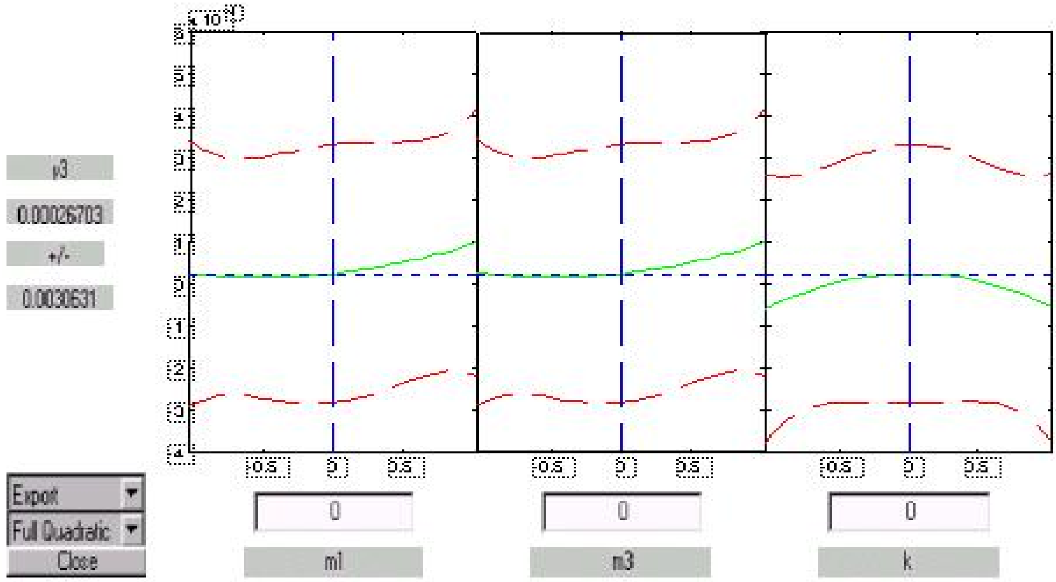

Figure 3. Response surfaces for the three parameters (m1, m3 e k) – Objective function *y*.

As the response surfaces were obtained according to a full-scale process, the complete design space can be explored for optimization purposes. The optimization of the objective function that is represented by the surface responses is made by using the program GAOT®, written in MATLAB® and developed at North Caroline State University (Houck et all., 1995), which is based on genetic algorithms (Michalewicz, 1996). As two conflicting objectives are competing in the multicriteria optimization problem, compromise optimization techniques, as given by Eq. (9) will be used. Therefore, the $F_k^*(X)$ terms were calculated separately (see Figs (4) and (5)). On the other hand, the values of $F_k^{Worst}(X)$ correspond to not desired values for each objective function.

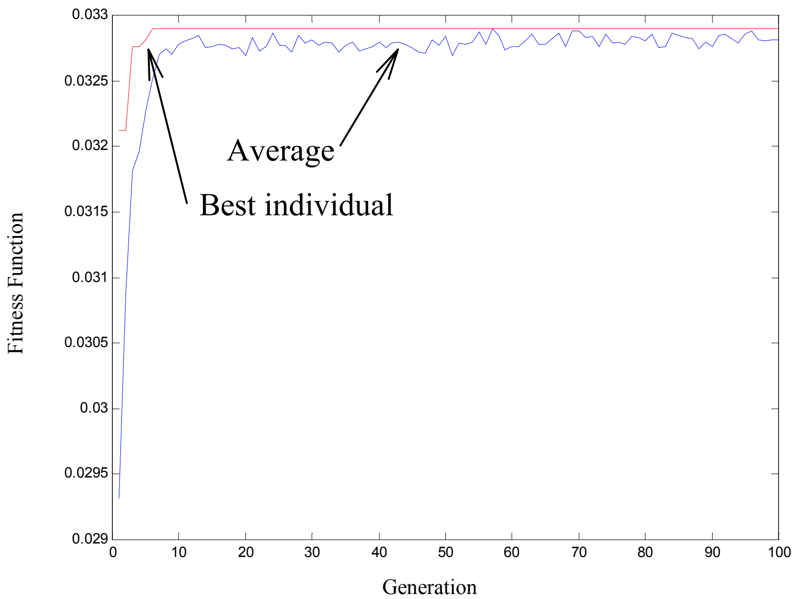

Figure 4. Meta-Model Optimization for F_1^*

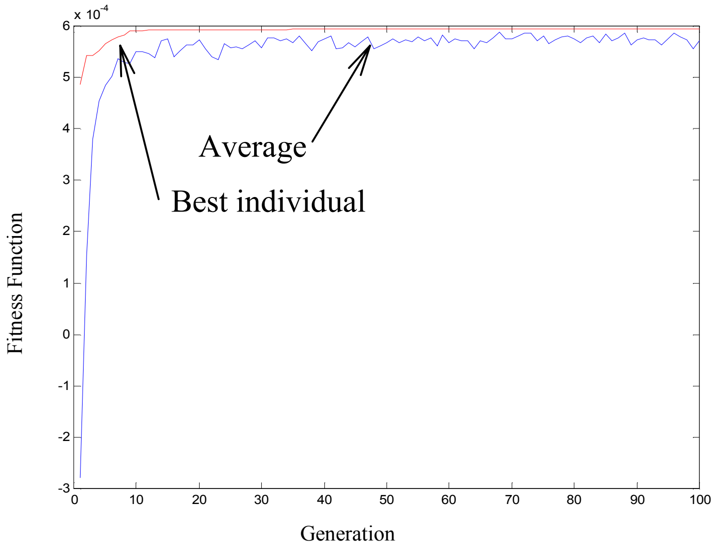

Figure 5 Meta-Model optimization for F_2^*

The optimization program returned the following results:

For F_1^*:

SOLUCTIONS

-1.00000000000000 -1.00000000000000 -0.89838690421774

OBJETIVE FUNCTION: $F_1^*(X)$

0.03290115254438

For F_2^*:

SOLUTIONS

-0.42064440321213 -0.17901856283499 -1.00000000000000

OBJETIVE FUNCTION: $F_2^*(X)$

5.934256647274693e-004

$$F(X) = \left\{ \sum_{k=1}^{|K} \left[\frac{W_k \{F_k(X) - F_k^*(X)\}}{F_k^{Worst}(X) - F_k^*(X)} \right]^2 \right\}^{1/2} \tag{9}$$

where W_k is the weighting factor on *k-th* objective function, $F_k(X)$ is the *k-th* objective function, $F_k^*(X)$ is the *k-th* objective function target and $F_k^{Worst}(X)$ is the worst known value of the *k-th* objective function.

Fig. (6) illustrates the maximization of the multiobjective function encompassing the difference of frequencies given by $\Delta\omega$ and the displacement y by using genetic algorithms.

The optimization program returned the optimal values for the design variables, as well as the optimum value of the multiobjective function:

SOLUTIONS

1.00000000000000 1.00000000000000 -0.00074632580532

MULTIOBJECTIVE FUNCTION

1.374768458672033e-004

From the above results, the displacement of floor 3 and the difference between the frequencies can be determined:

$y = 0.01119345$ m

$\Delta\omega = 13.073$ Hz

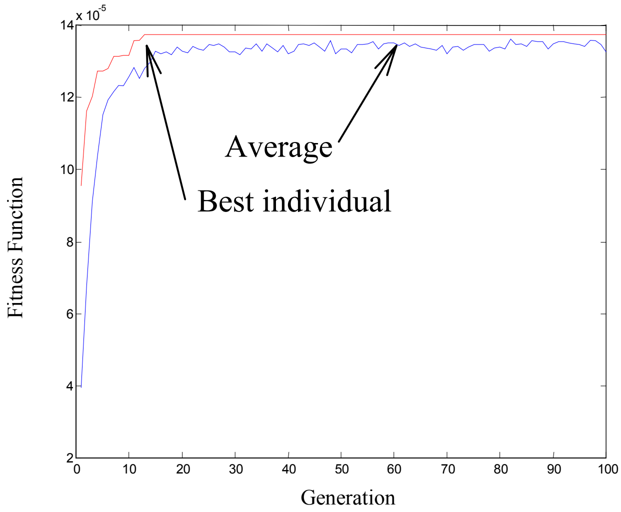

Figure 6. Multicriteria optimization of the global meta-model.

The same optimization problem was also solved by using the original analytical model, for comparison purposes. The value of the error was 0.00157344733867, that is the difference between the optimal multiobjective function obtained from the metal model is approximately 0,157 % different from the one obtained from the analytical 7 d.o.f. model. This means that the meta model is a good representation of the original system.

3. Conclusions

In this work statistics and design of experiment techniques were applied to improve the dynamical behavior of a representative mechanical system. As stated before, the goal was to demonstrate these techniques are a very useful design tool in dynamics. It can be pointed out that design of experiments is a quite new approach in this field. A meta-modeling approach can effectively reduce the computational effort as well as find solutions in the case the mathematical model is very complicated or even impossible to be obtained. In the present application the meta-modeling approach was used directly to the analytical model of a 7 d.o.f. undamped mechanical system. The general methodology can be easily extended to more sophisticated applications. It is also important to point out that, in the present case, only one experiment for each association of design variables was made (in real experimental cases, more experiments for the same variable configuration would be necessary because experimental procedures are obvious more likely to errors).

Through this contribution, it was possible to verify that genetic algorithms together with compromise optimization techniques were successfully used in meta-modeling optimization of dynamic systems by using response surfaces.

Finally, it should be added that, in general, dynamic system parameters do not vary significantly in the design procedure. Otherwise, the overall dynamic behaviour of the system could not be handled for design purposes. Complete models (analytical or discretized ones) can be used to determine the influence of parameter variations on the dynamical behaviour of the system, provided that computational costs can be afforded. In optimization contexts this problem becomes even more important, particularly in the case that a large number of parameters have to be taken into account. As in the meta-models the design variables are generally restricted to a small design space, the complexity of the optimization procedure is significantly reduced by using the methodology presented in this paper.

4. Acknowledgement

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