HEAT TRANSFER WITH ABLATION IN BODIES SUBJECTED TO HIGH HEAT FLUXES

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Abstract. *In regions of high heating in space vehicles the use of conventional materials must be quite restricted, because they would suffer catastrophic flaws due to the thermal degradation of their structures. However, the same materials can be quite suitable when protected by the well-known ablative materials. A mathematical model with non-linear contour condition is presented to solve the thermal protection problem involving space vehicles during the atmospheric re-entry. The process, that involves the ablative phenomenon, is complex and could involve the whole or partial loss of the material that is sacrificed due to the absorption of energy. The unknown moving boundary, which is caused by the phase change, yields to a non-linear process. The analysis of the ablative process in a blunt body with revolution geometry was done at the stagnation point area that was modeled as a one-dimensional flat plate problem. In this work, the Generalized Integral Transform Technique (GITT) was employed to solve the non-linear system of coupled partial differential equations that models the phenomenon. The solution of the problem is obtained by transforming the non-linear partial differential equation system in to a system of coupled first order ordinary differential equations and then solving it by using well-established numerical routines. The results of interest such as the depth and the removing rate the ablative material are presented and compared with data available in the literature.*

Keywords. *Thermal degradation, thermal protection, ablation, GITT, high heating*

1. Introduction

A phenomenon that is in study in the present days is the aerodynamic heating caused in the atmospheric re-entry of space vehicles. The aerodynamic heating is the heating of an object as a result of high speed air flow on the same, van Driest (1956).

When a space vehicle approaches the atmosphere of the Earth, it initially suffers an increase of speed due to the gravity action (rarefied atmosphere). However, when approaching the terrestrial atmosphere the drag force begins to exercise its influence, due to the interaction with the more dense layers of the terrestrial atmosphere, Pessoa-Filho (1997). This effect causes a significant decrease of speed. This speed reduction is only caused by the friction between the vehicle and the terrestrial atmosphere. From the physical point of view a conversion of kinetic energy into thermal energy (heat) takes place, Hatori (1999).

Due to this conversion of energy arises the need to protect thermally the space vehicles. Several kinds of thermal protection have been proposed (Hatori & Pessoa-Filho, 1998; Sutton, 1982; Steg & Lew, 1962). Among them the most used is that using ablative materials.

The ablation phenomenon is complex, involving heat and mass transfer, physical evaporation or pyrolysis, chemical reactions, etc (Lacaze, 1967; Kreith, 1973). Due to the complexity of the phenomenon, a convenient proposal to model the problem is to use a model that involves phase change with moving boundary and partial or total loss of mass.

In this work the resulting phenomenon of the heat transfer process is evaluated in the stagnation point of a blunt body (revolution), which can be considered as a problem of heat transfer in a flat plate, Kurokawa et al (2001).

The process of heat transfer with ablation is inherently non-linear with moving boundary initially unknown (Chung & Hsiao, 1985; Zien, 1978; Chung et al, 1983; Zien, 1981).

A technique that is being used to obtain of exact solutions of complex problems, where there are no analytical solutions, is the Generalized Integral Transform Technique (GITT) (Cotta & Özisik, 1987; Cotta, 1993a; Cotta, 1993b; Diniz et al, 1993a, Diniz et al, 1993b).

The GITT will be used in the analytical development where the problem solution is a coupled system of ordinary differential equations. This system will be numerically solved by known numerical techniques and using the IMSL library (IMSL, 1979).

The parameters of interest are the ablative thickness and the speed.

2. Formulation

In the formulation of this problem it is considered one-dimensional heat transfer in the stagnation point of a revolution body, which can be approached by the geometry of a flat plate. Initially the plate is at the uniform temperature T_o. The plate is subject to a prescribed transient heat flux in one of the extremities and isolated in the other.

The hypothesis of one-dimensional heat transfer is based on the fact that the ratio between the thickness of the ablative material and the nose curvature radius is small, Fig. (1). This problem of heat transfer in ablative materials is better described if the process is divided in to two periods: a preablative and an ablative.

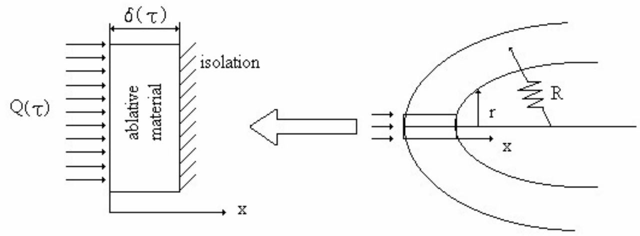


Figure 1. Schematic representation of ablative process in the stagnation point.

The governing equations in the dimensionless form are:

- ✓ Preablative period:

$$\frac{\partial\theta(x,\tau)}{\partial\tau} + L\theta(x,\tau) = 0 \qquad \begin{matrix} 0 < \tau < \tau_f \\ 0 < x < 1 \end{matrix} \tag{1}$$

where $L = -\frac{\partial^2(\ )}{\partial x^2}$ is a dimensionless parameter. The initial and boundary conditions are:

$$\theta(x,\tau) = 0 \qquad \tau = 0 \text{ e } 0 < x < 1 \tag{2}$$

$$\frac{\partial\theta(x,\tau)}{\partial x} = 0 \ ; \ x = 1 \tag{3}$$

$$\frac{-\partial\theta(x,\tau)}{\partial x} = Q(\tau) \ ; \ x = 0 \tag{4}$$

- ✓ Ablative period:

$$\frac{\partial\theta(x,\tau)}{\partial\tau} + L\theta(x,\tau) = 0; \qquad \begin{matrix} \tau_f < \tau < \infty \\ S(\tau) < x < 1 \end{matrix} \tag{5}$$

with initial and boundary conditions:

$$\theta(x,\tau) = \theta_f(x,\tau); \qquad \tau = \tau_f \text{ e } 0 < x < 1 \tag{6}$$

$$\frac{\partial\theta(x,\tau)}{\partial x} = 0, \ \mathrm{x} = 1 \tag{7}$$

$$\theta(x,\tau) = 1, \ \mathrm{x} = \mathrm{S}(\tau) \tag{8}$$

with the following constraint condition, resulting from the energy balance at the interface:

$$\frac{-\partial\theta(x,\tau)}{\partial x} + \nu\frac{dS(\tau)}{d\tau} = Q(\tau); \ x = \mathrm{S}(\tau) \tag{9}$$

where $S(\tau)$ and ν are the position of the boundary and the inverse of Stefan number respectively.

3. Analytical solution

For the solution of the preablative period, the temperature distribution $\theta(x,\tau)$ and the time at the beginning of the ablative period are obtained.

The GITT is applied to both the preablative and the ablative periods. For appling this technique it is defined an appropriate eigenvalue auxiliary problem.

For the preablative period the auxiliary problem is:

$$\mu^2\psi(x) = L\,\psi(x) \tag{10}$$

whose solution yields the following eigenvalues and eigenfunctions, respectively:

$$\mu_i = i\pi \tag{11}$$

$$\psi_i = \cos \mu_i x \tag{12}$$

After some mathematical manipulations of Eqs. (1) and (10) the following expression is obtained:

$$\frac{\partial}{\partial \tau}\int_0^1 \psi_i(x)\theta(x,\tau)dx + \int_0^1\left[\frac{\partial^2\psi_i(x)}{\partial x^2}\theta(x,\tau) - \frac{\partial^2\theta(x,\tau)}{\partial x^2}\psi_i(x)\right]dx + \mu_i^{\,2}\int_0^1 \psi_i(x)\theta(x,\tau)dx = 0 \tag{13}$$

Defining:

$$g_i(\tau) = -\int_0^1\left[\frac{\partial^2\psi_i(x)}{\partial x^2}\theta(x,\tau) - \frac{\partial^2\theta(x,\tau)}{\partial x^2}\psi_i(x)\right]dx \tag{14}$$

and developing the integral, one obtains that:

$$g_i(\tau) = Q(\tau) \tag{15}$$

Defining the integral transform and the inverse pair as:

$$\tilde{\theta}_i(\tau) = \int_0^1 \psi_i(x)\theta(x,\tau)dx \tag{16}$$

$$\theta(x,\tau) = \sum_{i=0}^{\infty}\frac{\psi_i(x)}{N_i}\tilde{\theta}_i(\tau) \tag{17}$$

Therefore the transformed Eq. (1), with the initial condition, yields:

$$\frac{d\tilde{\theta}_i(\tau)}{d\tau} + \mu_i^{\,2}\tilde{\theta}_i(\tau) = Q(\tau) \tag{18}$$

$$\tilde{\theta}_i(0) = \int_0^1 \psi_i(x)\theta(x,0)dx = 0 \tag{19}$$

Assuming a polinomial heat flux $Q(\tau) = a + b\tau + c\tau^2$, the solution of Eq. (18) is:

$$\tilde{\theta}_i(\tau) = \frac{1}{\mu_i^{\,2}}\left[a + b\left(\tau - \frac{1}{\mu_i^{\,2}}\right) + c\left(\tau^2 - \frac{2\tau}{\mu_i^{\,2}} + \frac{2}{\mu_i^{\,4}}\right) - \left(a - \frac{b}{\mu_i^{\,2}} + \frac{2c}{\mu_i^{\,4}}\right)e^{-\mu_i^{2}\tau}\right] \tag{20}$$

Therefore, the temperature distribution in the preablative period is given by:

$$\theta(x,\tau)=\theta_{av}(\tau)+\sum_{i=1}^{\infty}\frac{\psi_i(x)}{N_i}\tilde{\theta}_i(\tau) \tag{21}$$

where $\theta_{av}(\tau)$ is the medium potential. Because the boundary condition is of the second type corresponding to index $i=0$, Mikhailov & Özisik (1984):

$$\theta_{av}(\tau)=a\tau+\frac{b\tau^2}{2}+\frac{c\tau^3}{3} \tag{22}$$

Therefore the temperature distribution is:

$$\theta(x,\tau)=\left(a\tau+\frac{b\tau^2}{2}+\frac{c\tau^3}{3}\right)+\sum_{i=1}^{\infty}2\cos(\mu_i x)\frac{1}{\mu_i^2}\left\{\left[a+b\left(\tau-\frac{1}{\mu_i^2}\right)+c\left(\tau^2-\frac{2\tau}{\mu_i^2}+\frac{2}{\mu_i^4}\right)\right]+\right.$$
$$\left.-\left[a-\frac{b}{\mu_i^2}+\frac{2c}{\mu_{i^4}}\right]e^{-\mu_i^2\tau}\right\} \tag{23}$$

For $\tau>\tau_f$ the phase change period begins, designated ablative period. A variable change in Eqs. (5) to (8) is convenient for turning the problem homogeneous. Defining:

$$\theta^*(\eta,\tau)=\theta(x,\tau)-1;\ \mathrm{S}(\tau)<\eta<1;\ \tau>\tau_f \tag{24}$$

where $\eta=1-x$, we obtain that:

$$\frac{\partial\theta^*(\eta,\tau)}{\partial\tau}+L\theta^*(\eta,\tau)=0 \quad \tau>\tau_f,\ 0<\eta<\eta_{\mathrm{b}}(\tau) \tag{25}$$

with initial and boundary conditions:

$$\theta^*(\eta,\tau_f)=\theta(1-\eta,\tau_f)-1 \tag{26}$$

$$\theta^*(\eta,\tau)=0;\ \eta=\eta_{\mathrm{b}}(\tau) \tag{27}$$

$$\frac{\partial\theta^*(\eta,\tau)}{\partial\eta}=0;\ \eta=0 \tag{28}$$

And the constraint equation, Eq. (9), in the new variable becomes:

$$\frac{\partial\theta^*(\eta,\tau)}{\partial\eta}-\nu\frac{d\eta_b(\tau)}{d\tau}=Q(\tau);\ \eta=\eta_{\mathrm{b}}(\tau) \tag{29}$$

For the ablative period, it was adopted a new auxiliary problem of appropriate eigenvalue that results in the following eigenvalues and eigenfunctions, respectively:

$$\mu_i=\frac{(2i-1)\pi}{2\eta_b(\tau)} \tag{30}$$

$$\psi_i(\eta,\tau)=\cos[\mu_i(\tau)\eta] \tag{31}$$

Defining a normalized eigenfunction:

$$K_i(\eta,\tau)=\frac{\cos[\mu_i(\tau)\eta]}{\sqrt{0{,}5\eta_b(\tau)}} \tag{32}$$

After some mathematical manipulations we obtain an expression of the type:

$$\int_0^{\eta_b(\tau)} K_i(\eta,\tau)\frac{\partial\theta^*(\eta,\tau)}{\partial\tau}d\eta + \mu_i^2(\tau)\int_0^{\eta_b(\tau)} K_i(\eta,\tau)\theta^*(\eta,\tau)d\eta = \int_0^{\eta_b(\tau)}\left[\frac{\partial^2\theta^*(\eta,\tau)}{\partial\eta^2}K_i(\eta,\tau) - \frac{\partial^2 K_i(\eta,\tau)}{\partial\eta^2}\theta^*(\eta,\tau)\right]d\eta \tag{33}$$

Defining the integral transform and the inverse pair as:

$$\tilde{\theta}_i^*(\tau) = \int_0^{\eta_b(\tau)} K_i(\eta,\tau)\theta^*(\eta,\tau)d\eta \tag{34}$$

$$\theta^*(\eta,\tau) = \sum_{i=1}^{\infty} K_i(\eta,\tau)\tilde{\theta}_i^*(\tau) \tag{35}$$

and substituting Eq. (34) in Eq. (33), we obtain:

$$\frac{d\tilde{\theta}_i^*(\tau)}{d\tau} + \sum_{j=1}^{\infty} A_{ij}(\tau)\tilde{\theta}_j^*(\tau) + \mu_i^2(\tau)\tilde{\theta}_i^*(\tau) = 0 \tag{36}$$

with initial condition:

$$\tilde{\theta}_i^*(\tau_f) = \frac{2\sqrt{2}}{(2i-1)\pi}(-1)^{i+1}\left[\theta_{av}(\tau_f) - 1\right] + \frac{4\sqrt{2}}{\pi}(2i-1)(-1)^{i+1}\sum_{K=1}^{\infty}\frac{\tilde{\theta}_k(\tau_f)}{(2i-1)^2 - 4k^2} \tag{37}$$

where:

$$\tilde{\theta}_k(\tau_f) = \frac{1}{\mu_k^2}\left[a + b\left(\tau_f - \frac{1}{\mu_k^2}\right) + c\left(\tau_f^2 - \frac{2\tau_f}{\mu_k^2} + \frac{2}{\mu_k^4}\right) - \left(a - \frac{b}{\mu_k^2} + \frac{2c}{\mu_k^4}\right)e^{-\mu_k^2\tau_f}\right]$$

$$A_{ij} = \begin{cases} 0;\ para\, i = j,\ (Diniz, 1996) \\ \dfrac{1}{2\eta_b(\tau)}\dfrac{\partial\eta_b(\tau)}{\partial\tau}\dfrac{(2j-1)(2i-1)(-1)^{i+j}}{(i^2-i)-(j^2-j)};\ para\, i \neq j \end{cases}$$

The transformed constraint equation is obtained substituting the inverse transformed equation in the constraint equation, resulting in:

$$\frac{d\eta_b(\tau)}{d\tau} = \frac{\sqrt{2}}{2\nu}\frac{\pi}{\left[\eta_b(\tau)\right]^{3/2}}\sum_{j=1}^{\infty}(2j-1)\tilde{\theta}_j^*(\tau)(-1)^j - \frac{Q(\tau)}{\nu} \tag{38}$$

4. Numerical analysis

The system formed by Eqs. (36) and (38) is the solution system for one-dimensional heat transfer problem in bodies with ablative thermal protection. These equations form an infinite system of ordinary differential equations coupled to the constraint equation, with Eq. (37) being the initial condition.

The calculation of the values of interest was made through the implementation of an algorithm in Fortran. For doing that it was necessary to transform this infinite system in a finite system of order N. Making N sufficiently great we obtain the solution of the coupled ordinary differential equation system, providing the values of the ablative thickness and speed.

In the numerical analysis it was used the available subroutines of IMSL (1979), mainly the routines DGEAR and DECADRE, that meet the exigency of this type of problem.

5. Results

The ablative thickness and speed were obtained by analytic-numeric hybrid analysis through the application of GITT for prescribed dimensionless heat flux on the surface illustrated in Figs. (2) and (3). The prescribed heat flux $Q = 10\tau$ was chosen as boundary condition. The choice of the heat flux as boundary condition is arbitrary. This heat flux was chosen because it can represent a normal operation condition more realistically than a constant heat flux.

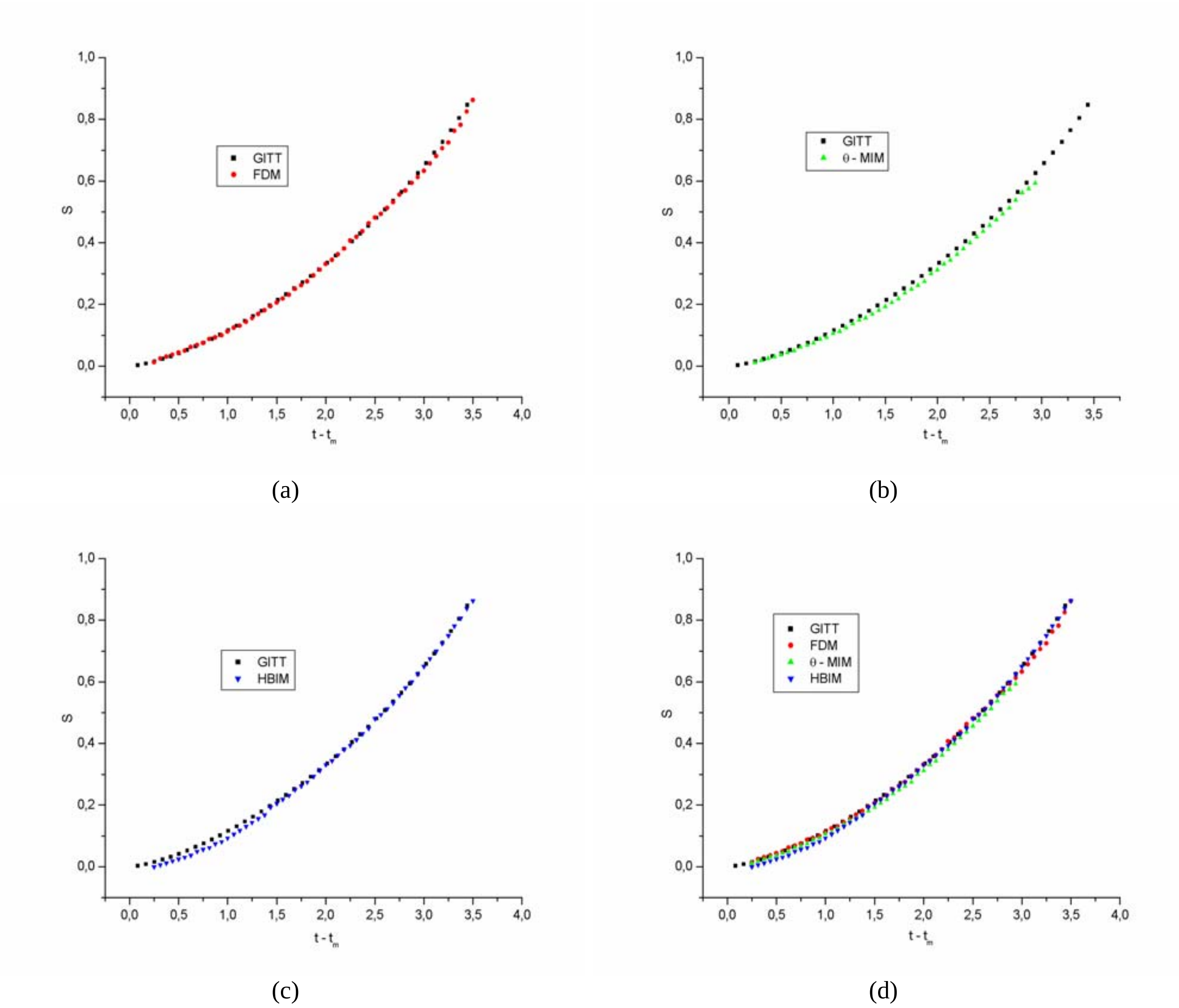


Figure 2. Comparison of ablative thickness for various methods.

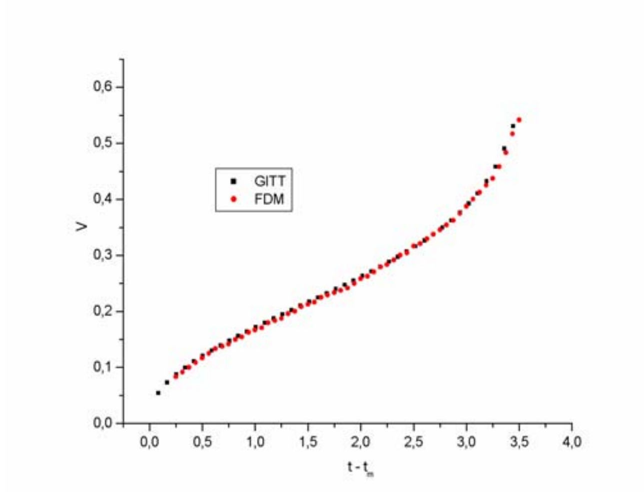
The thermal diffusivity, characteristic time, reference heat flux and the inverse of Stefan number are $\alpha = 0{,}1 ft^2/s$, $t = 10s$, $q_r = 10Btu/s - ft^2$ and $\nu = 1$, respectively. These numerical values are used in the work of Chung & Hsiao (1985), where the authors employed three different approaches: The Heat Balance Integral Method (HBIM), the θ - Moment Integral Method (θ - MIM) and Finite Difference Method (FDM).

Table (1) presents a comparison of the times starting of the ablative period obtained for boundary condition $Q = 10\tau$.

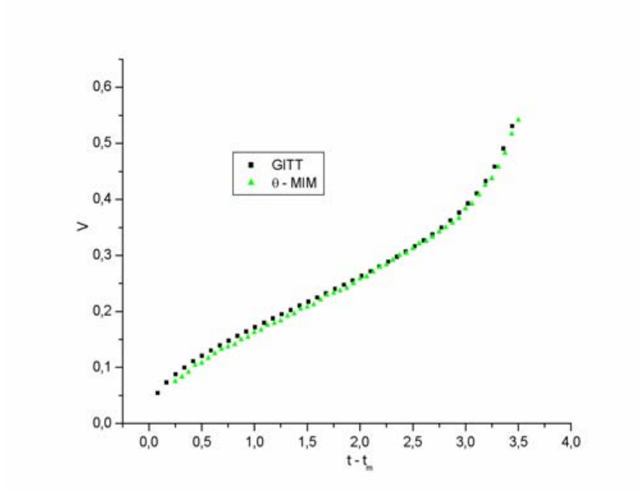
Table 1. Comparison of times starting of the ablative period.

	τ_m
FDM	0.26
θ - MIM	0.26
HBIM	0.261
GITT	0.261

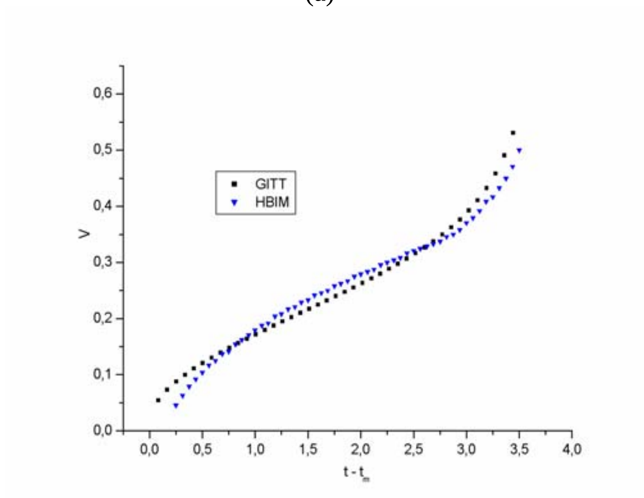
Figures (4) and (5) show the temperature field in the preablative and ablative phases for the boundary condition $Q = 10\tau$.



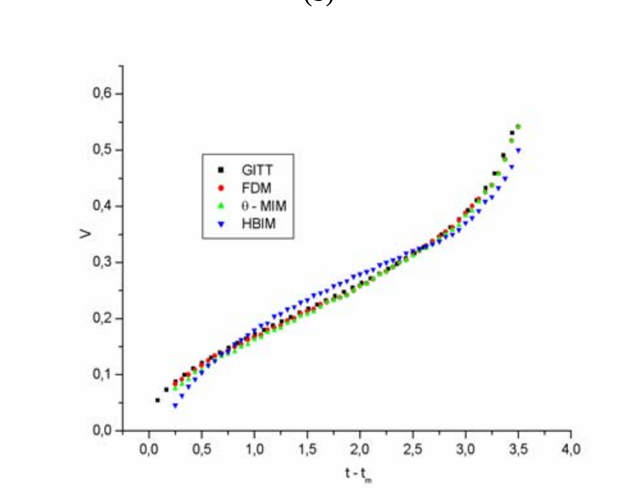
(a)



(b)



(c)



(d)

Figure 3. Comparison of ablative speed for various methods.

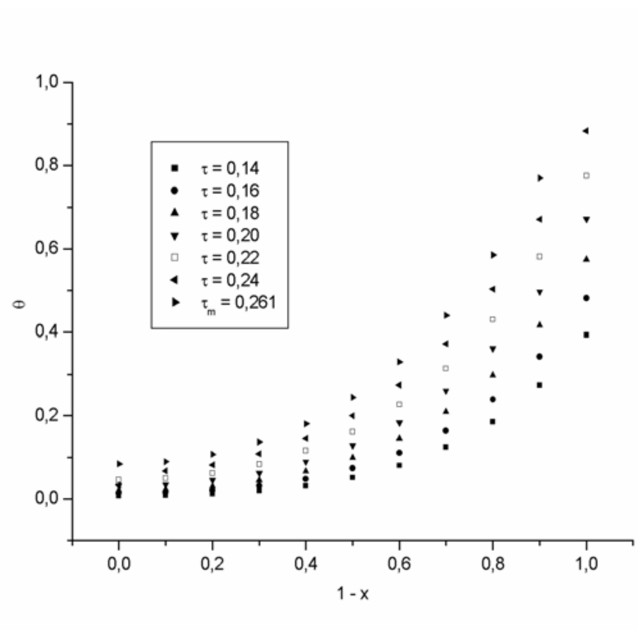


Figure 4. Temperature field in the preablative phase.

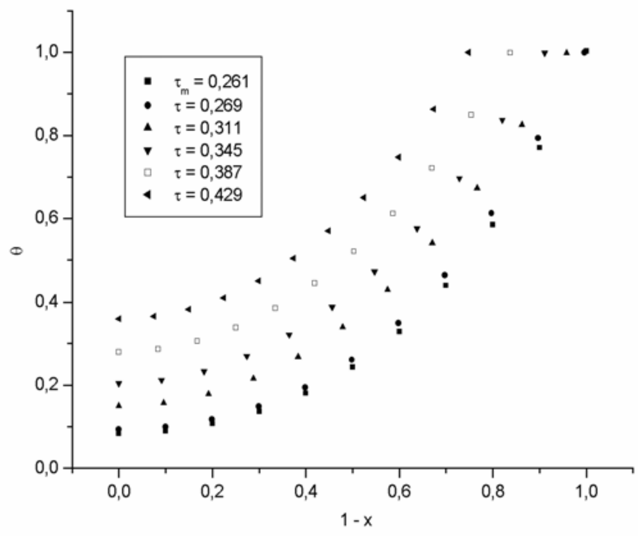


Figure 5. Temperature field in the ablative phase.

6. Discussion

From Tab. (1) we can observe that the starting times of ablative phase obtained by different methods are similar. In Fig. (2) a comparison of the data obtained by GITT with other methods for the ablative thickness is presented. We notice a good agreement of the data obtained by GITT. Fig. (3) shows a comparison for the ablative speed obtained by GITT with others methods. We notice that there is a disagreement of the data obtained by GITT when compared with HBIM.

The values obtained by GITT are more exact and consistent, because GITT is a hybrid analytic-numeric tool in which we transform the partial differential equation system in a coupled ordinary differential equation system.

Figures (4) and (5) present the temperature field for the preablative period as well as for the ablative period. In the preablative phase the temperature distribution presents convergence with elapsed time, up to the starting time of the ablation.

In this work a solution was presented for the problem of heat transfer in systems with ablative thermal protection, considering a simplification of the phenomenon assuming it is only a process of phase change.

7. Acknowledgments

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