

## COMMANDING MOBILE ROBOTS WITH CHAOS

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**Abstract.** *The work presented in this paper deals with the problem of commanding a mobile robot either in unknown or known environment. We study the implementation of chaotic behavior on a mobile robot: a simulated version of KHEPERA mini-robot. Chaotic trajectories are executed using state variables of the Lorenz system as input for a robot controlled through two independent active wheels. The high unpredictability of motion trajectory and the fast scanning of the entire robot workspace make this control strategy interesting to deal with the problem of exploration of terrain for vigilance, search or cleaning purposes. The mathematical models of the Lorenz system and the mobile robot are detailed, and the control strategy is described. Results of simulation tests and its analysis, and concluding discussions close the paper.*

**Keywords** *mobile robot, trajectory control, chaotic behavior, vigilance robot, terrain scanning.*

### **1. Introduction**

The development of mobile robots in the past twenty years has opened new applications perspectives for these machines, including intelligent capacities to accomplish difficult tasks. The technological stage of motion mechanisms, sensor devices, and embedded electronic systems allow pointing the researches to the development of control strategies to solve complex problems that the robot must deal with. For instance, we can see recent works related to these problems in Kim and Tsiotras (2002), Maaref and Barret (2002), Berlanga et al. (2002) and Hoseninnezhad et al. (2003). In the present work, we are interested on the specific problem of exploration of terrain for vigilance, search or cleaning purposes. In the case of the two first applications, the most suitable features are the high unpredictability of motion trajectory and the fast scanning of the entire robot workspace. In the case of floor cleaning, if a terrain map is not available, the trajectory planning for complete scanning becomes a difficult task, consequently a non-planned motion consist one interesting solution to this problem.

Nakamura and Sekiguchi (2001) proposed an approach to solve this problem based on chaotic systems. They proposed in their work, to impart a chaotic behavior to the robot, an integrated system model that connects the equations of the two subsystems, the robot motion and the Arnold dynamical system, where the control variables are functions of the Arnold subsystem variables. In other words, an open loop control system is defined using state variables of the chaotic subsystem. Chaos is a typical behavior of nonlinear dynamical systems, and has been studied deeply in terms of its mathematical model and different applications. Chaotic behavior is desirable in mobile robotics designed for this problem, since a chaotic motion guarantees the scanning of the whole connected workspace without a terrain map or motion plan. In our control scheme we are implementing chaotic behavior on mobile robots based on the Lorenz system, a well-known dynamical system that produces chaos. We consider the locomotion of a robot controlled through two independent active wheels using state variables of a previously simulated Lorenz system. We have been validating the control approach with numerical simulations using a mobile robot simulator.

The paper presents the Lorenz system description and the results of its numerical simulations, where a chaotic behavior can be identified. In the sequence, the adopted mobile robot simulator, including the robot mathematical model, is detailed. The subsequent sections present the motion control strategy, the approach validations using the KHEPERA simulator tests for chaotic inputs, and the results analysis. Conclusions and perspectives of future related works close the article.

## 2. The Lorenz system

A chaotic system can be defined as a nonlinear deterministic system having complex and unpredictable behavior. Such systems have sensitive dependence on initial condition and on the system parameter variation, two main features of the chaotic behavior. Several chaotic systems have been developed and thoroughly analyzed over the past two decades, however the Lorenz system remains one of the most classical examples of autonomous system with chaotic behavior. One significant example of Lorenz system application is the synchronization of chaotic circuits for communication purposes (Cuomo et al., 1993; Liao, 1998).

The Lorenz dynamical system is the mathematical model of a thermodynamic system (Lorenz, 1963). It consists of three nonlinear ordinary differential equations that depend on three positive real parameters, and it models an atmospheric fluid convection problem, where a flat fluid layer is heated from below and cooled from above. Such system is considered as the first canonical chaotic attractor in a simple three-dimensional autonomous system, which was mathematically confirmed to exist. Lorenz system can be described by

$$\begin{cases} \dot{x}_1(t) = c_1[x_2(t) - x_1(t)] \\ \dot{x}_2(t) = [c_2 - x_3(t)]x_1(t) - x_2(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - c_3x_3(t) \end{cases} \quad (1)$$

where  $x_1$ ,  $x_2$  and  $x_3$  are the state variables,  $c_1$ ,  $c_2$  and  $c_3$  are positive constants that work as parameters of the dynamical behavior of this system.

To show the typical dynamics of the system, we simulated numerically its behavior starting from a point near the origin of the 3D Cartesian space, using the parameters values  $c_1 = 10$ ,  $c_2 = 28$  and  $c_3 = 8/3$ ; values chosen based on simulations and results observations. The  $x_1x_2$  and  $x_1x_3$  phase plans can be seen in Fig. (1) and Fig. (2), respectively. The chaotic behavior is here clearly illustrated, and the presence of two strange attractors can be easily observed.

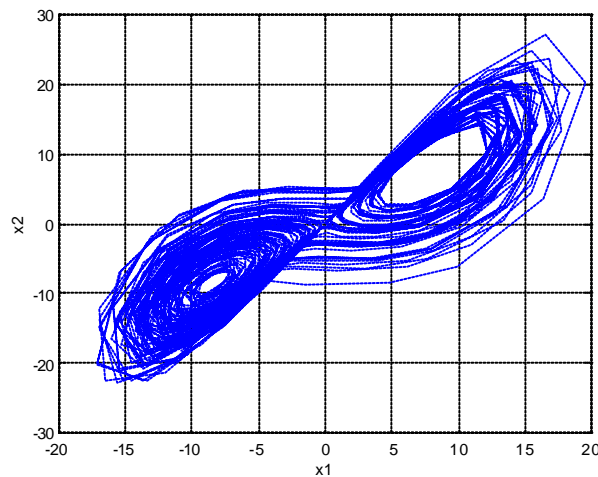


Figure 1: The  $x_1x_2$  phase plan of Lorenz system.

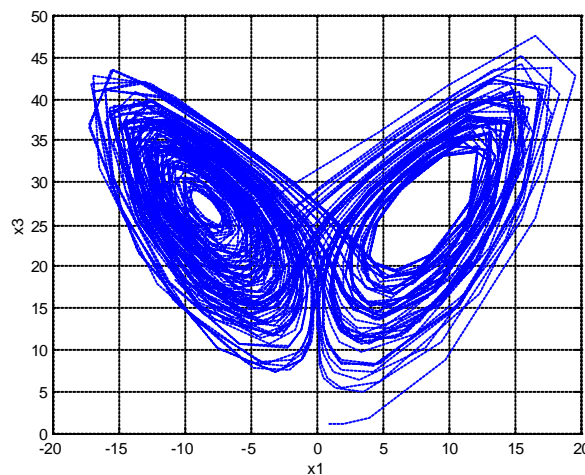


Figure 2: The  $x_1x_3$  phase plan of Lorenz system.

The control strategy of the KHEPERA mobile robot takes the results of this system simulation to provide the motion command and, consequently, to impart a chaotic dynamics behavior to the robot trajectory. The details of the use of Lorenz system state variables acting as control variables are discussed in Section (4).

### 3. Mobile robot's modelling

The mobile robot used in our control strategy validations is a simulated version of KHEPERA mini-robot, a simulation software of a largely applied experimental platform for mobile robotics researches, e.g. see applications of this robot in Kim and Tsiotras (2002), Maaref and Barret (2002) and Hoseninnezhad et al. (2003). The robot simulator, available on the KHEPERA Simulator Homepage (Olivier, 1997), works on LINUX environment and the programs use C language. This KHEPERA simulator has also been used in mobile robots researches (Berlangu et al., 2002). We decided to adopt this simulated mobile robot in the present stage of our work because it offers the emulation of the real robot main features, and it allows analyzing the properties of the commanded robot motion. The robot consists basically of two wheels and eight proximity sensors (simulating a kind of infrared device). The wheels are controlled on velocity and sense of turning. The sensors provide the distance data of possible obstacles in short range. This model is shown in Fig (3). Six sensors are located on the front side of the robot, and two others are located in the opposite side to allow back maneuvers.

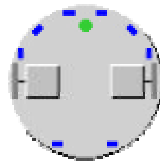


Figure 3: The simulated KHEPERA mobile robot.

The robot motion is obtained by driving the independent active wheels. The resultant motion is described by the linear velocity  $v(t)$  and the direction  $\mathbf{q}(t)$ , providing a instantaneous linear motion of the medium point of the wheel axis, and a rotational motion (rotational velocity  $\mathbf{w}(t)$ ) of the robot body over this same point. Figure (4) shows this motion scheme on 2D plan.

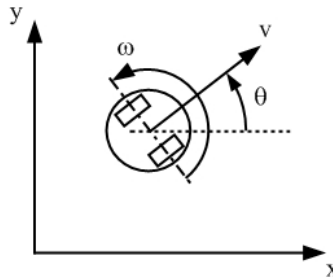


Figure 4: Description of the robot motion on Cartesian plane.

The robot motion control can be done providing the two independent wheels velocities,  $\mathbf{w}_l(t)$  and  $\mathbf{w}_r(t)$ , or the body linear and angular velocities,  $v(t)$  and  $\mathbf{w}(t)$ , which can be converted in terms of each wheel velocity and vice-versa. Taking a mathematical modeling for this motion, we can consider two input variables:  $v(t)$  and  $\mathbf{w}(t)$ , and three state variables: the robot position and orientation ( $x(t)$ ,  $y(t)$ ,  $\mathbf{q}(t)$ ):

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\mathbf{q}}(t) \end{pmatrix} = \begin{pmatrix} \cos \mathbf{q}(t) & 0 \\ \sin \mathbf{q}(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ \mathbf{w}(t) \end{pmatrix} \quad (2)$$

The input variables are also called control variables, and constitute the way to command the robot to provide the desired motion. The robot motion history, i.e. the executed trajectory, is recorded to plot graphics that allow easier visualization of motion topological properties.

### 4. Robot motion control

In this work, we propose a robot motion controller capable of producing a chaotic motion, i.e. the goal is to impart the topological transitivity property to the robot motion. Such procedure can provide complete scanning of the

connected workspace without terrain maps nor planned trajectories. The unpredictability feature of this motion points to vigilance tasks applications, and also all kind of search strategies without a priori information.

The control scheme can be described as an open loop control law, where the input variables are previously defined and available under the format of a vector containing the discrete-time constant control values. These values are used to command the independent wheel velocities. This control law may, however, command the robot towards a physical obstacle. Therefore, the controller still must solve the problem of obstacle and workspace boundary avoidance.

The robot must change its trajectory when it goes into a physical obstacle or a workspace boundary. In this case, the control loop adopts a reflexive motion. When the robot depth sensors detect a obstacle approximation, it changes the direction of movement respecting the strategy described by the Fig (5), i.e. it takes a reflexive trajectory that requires the measurement of the local normal to the obstacle when the sensors senses it coming closer.

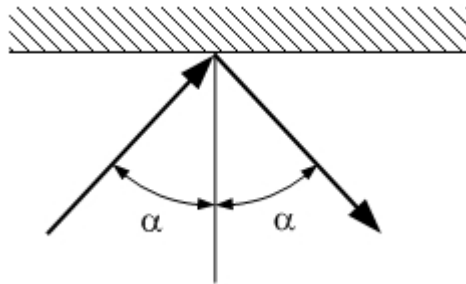


Figure 5: Geometric description of the obstacle avoidance strategy.

## 5. Simulations results

The control strategy was tested through simulations of KHEPERA robot motion in a limited connected workspace. The simulator console is shown in Fig. (6): in the left side, we see the “world” editor and robot motion viewer appears, and in the right side, we see the sensors and the wheels velocities console. The motion command was provided by the two independent wheels velocities,  $w_l(t)$  and  $w_r(t)$ , formatted on two vector of consecutive discrete-time instants of control application. It works like an open loop control system with piecewise constant value control inputs. These values of velocity were previously obtained by Lorenz system numerical simulation (Section 2), and scaled to the KHEPERA inputs range. Notice that in a real robot application, the Lorenz system simulation can turn simultaneously to the control routine on the controller computer.

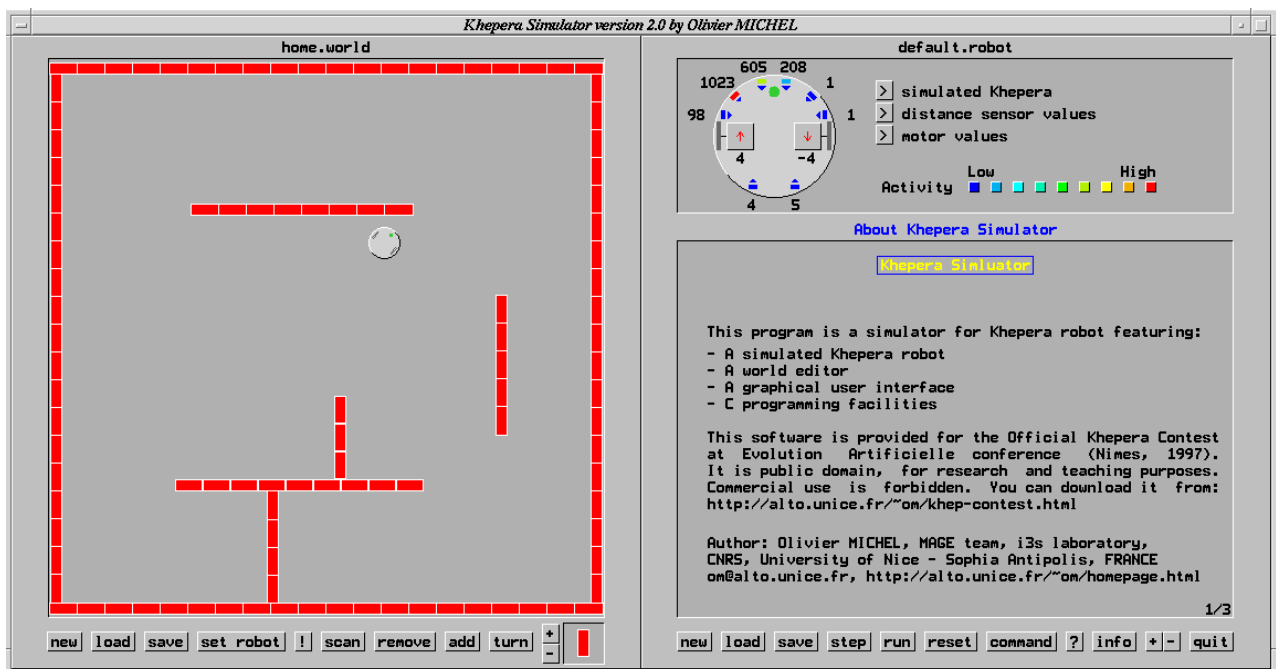


Figure 6: The KHEPERA simulator console.

The simulated terrain map, called “world” in KHEPERA simulator, to be explored using the control strategy is showed in Fig. (7).

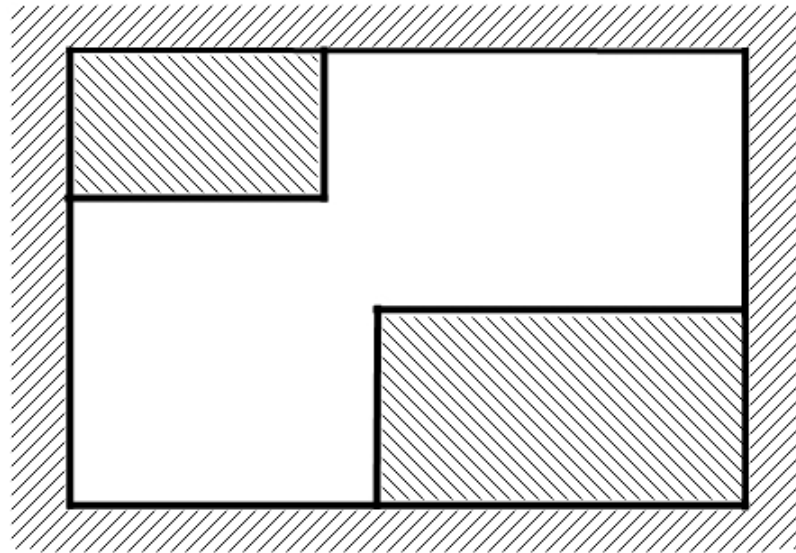


Figure 7: The simulated terrain.

Figures (8)-(10) show the trajectory of the robot commanded using the combination of the three Lorenz system state variables two-by-two, i.e. taking  $x_1x_2$  plane in Figure (8),  $x_2x_3$  phase plane in Figure (9), and  $x_1x_3$  phase plane in Figure (10).

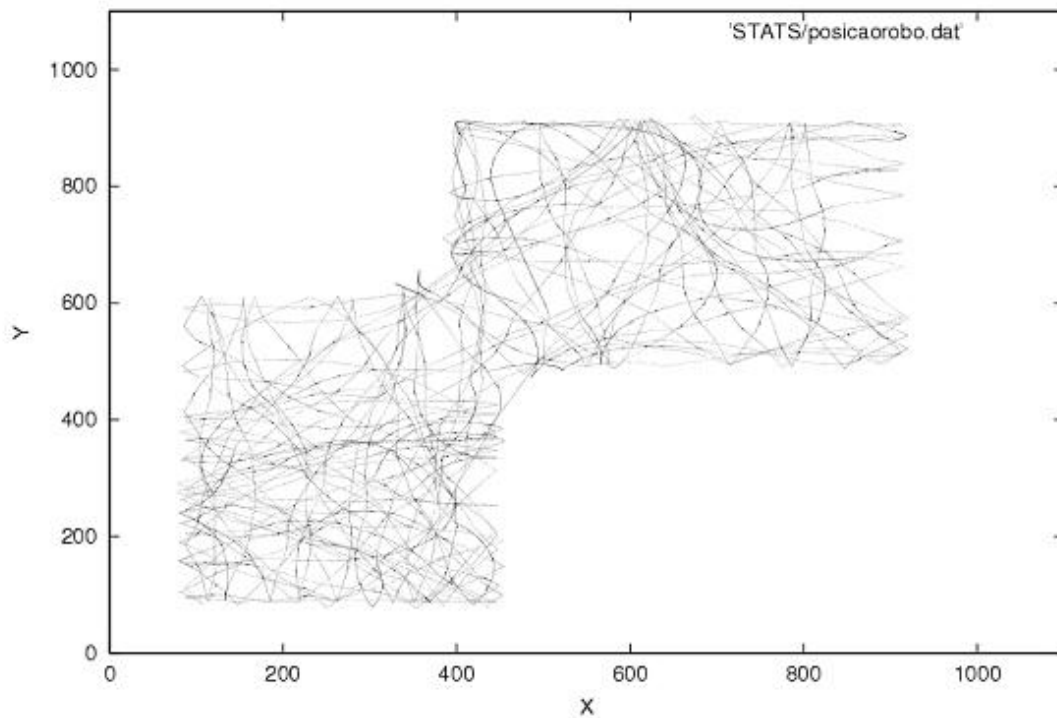


Figure 8: The KHEPERA robot trajectory using Lorenz system state variables  $x_1$  and  $x_2$  as control inputs. Position coordinates ( $X$  and  $Y$ ) are expressed in millimeters.

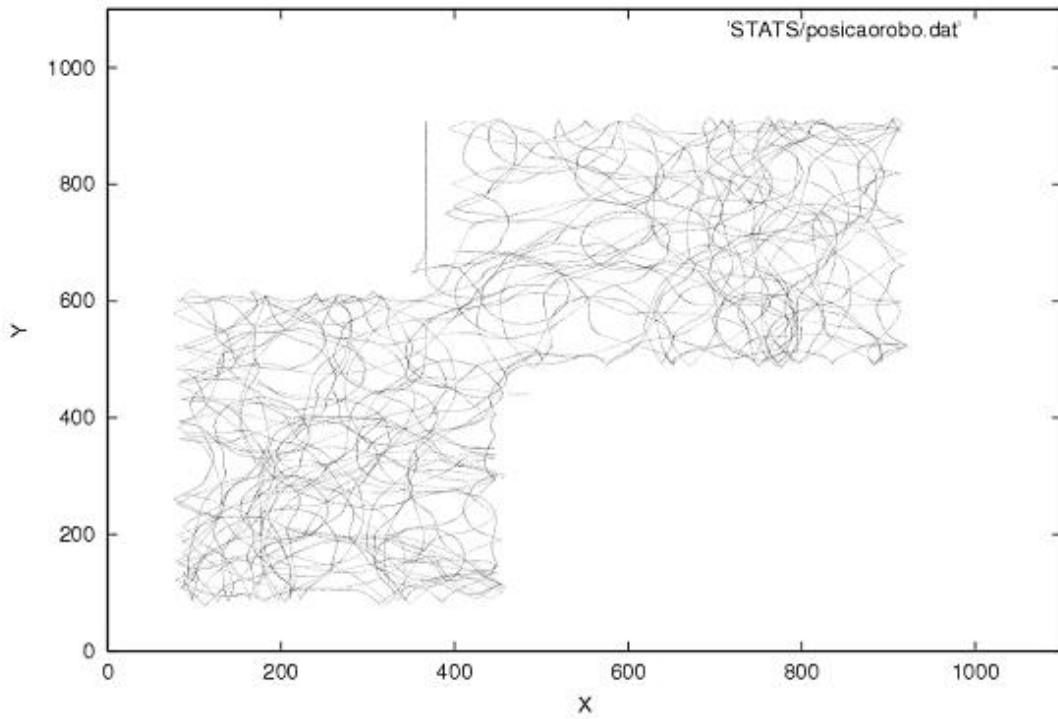


Figure 9: The KHEPERA robot trajectory using Lorenz system state variables  $x_1$  and  $x_3$  as control inputs. Position coordinates ( $X$  and  $Y$ ) are expressed in millimeters.

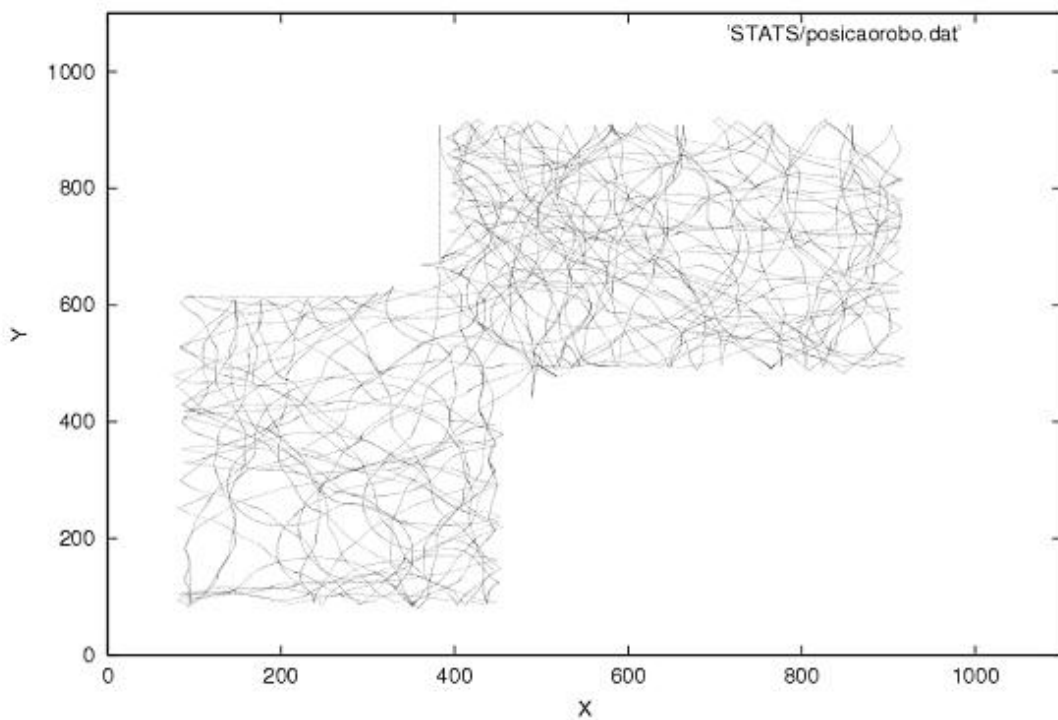


Figure 10: The KHEPERA robot trajectory using Lorenz system state variables  $x_2$  and  $x_3$  as control inputs. Position coordinates ( $X$  and  $Y$ ) are expressed in millimeters.

The motion trajectories obtained using chaotic variables, i.e. temporal series resulting from a chaotic dynamic system, as velocity inputs can be considered in accord with the control strategy objectives. The mobile robot has achieved a complete terrain scanning that allows the accomplishment of patrol, search or cleaning tasks.

Other possible solution to command the robot into a complete terrain scanning with unpredictable trajectories is to provide the two independent wheels velocities in terms of uniformly distributed random numbers. Such alternative approach has some undesirable properties like the discontinuities of trajectories, which are excessively frequent. Considering a real robot problem, these trajectory changes means difficulties on the motor driving. Moreover, the energy necessary to move the robot is augmented to cover the same terrain map.

## 6. Conclusion

In the domain of mobile robots, we have presented a particular methodology of commanding the robot motion either in unknown or known environment. The approach adopted consists of implementing a chaotic behavior on a simulated version of KHEPERA mini-robot. The high unpredictability of chaotic motion trajectories and the fast scanning of the entire robot workspace make this control strategy specially good for terrain exploration problems, where the robot must perform patrol, search or cleaning tasks.

Chaotic trajectories have been obtained using state variables of well-known Lorenz system as input of the simulated robot, controlled by two independent active wheels. Such input values are defined in previous numerical simulations of Lorenz system and are stored to be used by the open loop controller.

The results of the robot motion command show that such strategy can be applied in real robot control to accomplish such specific kinds of tasks. This way, we can define the perspectives of future works in this domain in order to obtain an experimental realization of the discussed ideas, and to explore the possibilities of this control strategy to study dynamic systems. The present research efforts point to the implementation of a mobile robot prototype and a simplified electronic circuit that emulates the Lorenz system.

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