NON-LINEAR ANALYSIS OF BEND STIFFENERS

Murilo Augusto Vaz
Ocean Engineering Department, Federal University of Rio de Janeiro, Brazil
murilo@peno.coppe.ufrj.br

Carlos Alberto Duarte de Lemos
Petrobras R&D Centre, Subsea Engineering, Brazil
caolemos@cenpes.petrobras.br

Abstract. *This paper presents a mathematical formulation and a solution for the geometrical and material non-linear analysis of bend stiffeners employed to protect the upper terminations of flexible pipes and umbilical cables. The governing equations result from considering geometrical compatibility, equilibrium of forces and moments and material constitutive relations, which can be linear elastic or non-linear elastic asymmetric. In this latter case, the bending moment versus curvature is expressed by a power series expansion. Hence, a set of four non-linear ordinary differential equations may be written and four boundary conditions are defined in both ends. A shooting method is employed and a solution is presented for a case study where linear and non-linear constitutive models are compared and discussed. It is shown that an accurate analysis of the bend stiffener depends on a precise assessment of the material property.*

Keywords*. Bend stiffeners, flexible pipes, umbilical cables.*

1. Introduction

Bend stiffeners are ancillary devices employed to provide a smooth transition between very flexible and stiff structures such as, respectively, a flexible riser (or umbilical cable) and a platform at the upper connection. This region has been long recognized as one of the most susceptible for failure from accumulation of fatigue damage or from excessive bending. Usually, the main design concern has been the integrity of pipe/umbilical itself, but recent failures on bend stiffeners have motivated further investigation. In addition, the intensive use of floating production ships and monobuoys (which present more severe motions than semi-submersibles) in deep waters (i.e., subjected to higher tensions) have also justified additional research on the structural integrity of those devices.

The design, installation and operation of flexible pipes have offered diverse themes for research. In terms of flexible pipe local behaviour, axisymmetric and bending models have been proposed, see Féret and Bournazel (1987), Saevik (1992), Witz and Tan (1992), Féret et all (1995) and Custódio and Vaz (2002). The design of bend stiffeners is strongly influenced by the flexible line global dynamic analysis, once the forces (intensity and direction) at the upper end determine the bend stiffener response. A local pipe cross-section analysis indicates its minimum allowable bending radius. Bend stiffeners are designed to support extreme loads and cyclic excitation from the wave induced floating production unit motions. The flexible pipe analysis usually considers the top end boundary condition hinged and the bend stiffener is excluded once its influence on the global response is small. After the bend stiffener design is completed, the global analyses considering the bend stiffener could be redone, but this is not the usual practice. Instead, an analysis of the bend stiffener with a small portion of the pipe is carried out.

Bend Stiffeners are usually made from polymeric materials, such as a polyether thermoplastic polyurethane. This class of material may exhibit an asymmetric behaviour in tension and compression, and this effect on the bend stiffener local response will be addressed in this paper. It is usual to carefully consider the operational and environmental temperatures once the material elastic properties may be greatly affected. The system stiffness, and consequently the geometric configuration and curvature distribution are obviously strongly dependent on the polyurethane young modulus. It is also known that if the stiffness is too large the bend stiffener may fail by shear in the connection otherwise it may fail by excessive bending.

The bend stiffener design is initially carried out with the aid of programs based on Boef and Out (1990) formulation for large deflection of slender rods. A finite element program may be employed if stress concentration points, such as the interface between the stiffener and its metal support, are sought. The material is assumed linear or sometimes non-linear elastic but symmetric in tension and compression. Lane et all (1995) work also address the bend stiffener design.

2. Mathematical Formulation of the Problem

The analysis of bend stiffeners subjected to terminal loads was proposed by Boef and Out (1990), who employed a slender rod model as schematically shown in Fig. 1a. The following aspects are considered: large deflections are accepted, it is a pure bending problem, the cross-section is variable, the self-weight and external forces are disregarded and the material is assumed linear elastic. Next, a similar methodology is developed, however the material is assumed elastic but non-linear asymmetric, hence exhibiting a tension-compression skew behaviour. The moment-curvature non-linear characteristic is numerically calculated and expressed by a power series expansion. The mathematical formulation derives from considering geometrical compatibility, equilibrium of forces and moments and constitutive relations yielding a system of four first order non-linear ordinary differential equations, which describe the bend stiffener large displacement configuration.

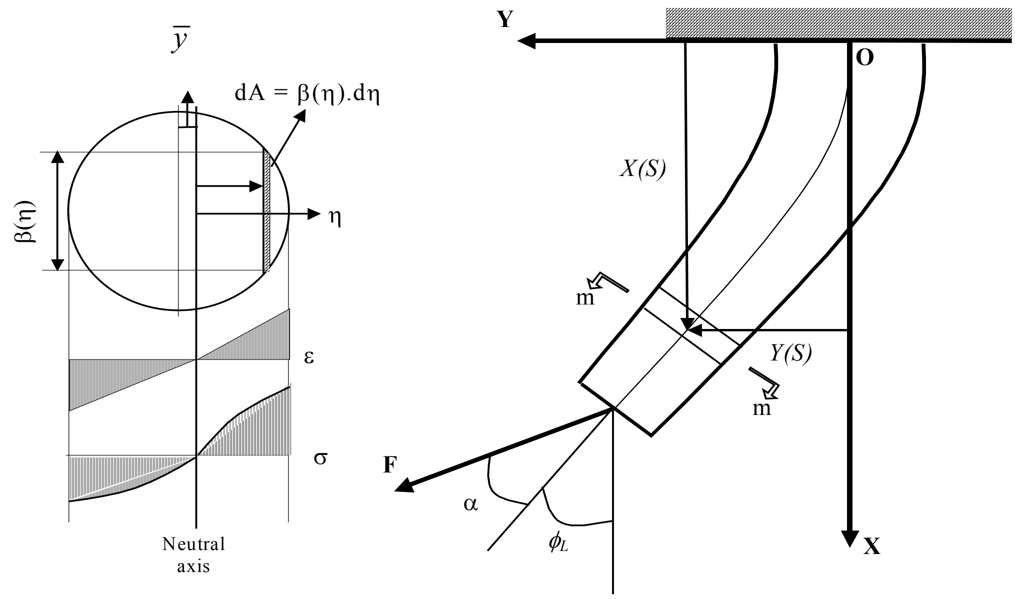

Figure 1a. Schematic of the Bend Stiffener as a Beam Model.

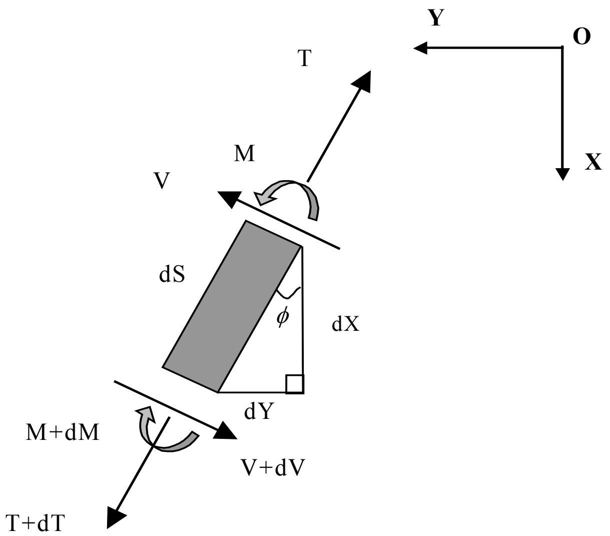

Figure 1b. Infinitesimal Element of the Bend Stiffener.

2.1. Geometrical compatibility

Applying trigonometrical relations to the infinitesimal rod element dS (see Fig. 1b) yields:

$$\frac{dX}{dS} = \cos\phi \tag{1a}$$

$$\frac{dY}{dS} = sin\phi \tag{1b}$$

Where S is the rod arc-length ($0 \le S \le L$), $[X(S), Y(S)]$ are the Cartesian coordinates of the deflected rod and $\phi(S)$ is the angle between the tangent and the X - axis. Furthermore the curvature $\kappa(S)$ is given by:

$$\kappa = \frac{d\phi}{dS} \tag{1c}$$

2.2. Boundary conditions

A set of four boundary conditions must be specified for the *encastré* rod:

$$X(0) = Y(0) = \phi(0) = \phi(L) - \phi_L = 0 \tag{2}$$

Where ϕ_L is the angle at the rod free end.

2.3. Equilibrium of forces and moments

A schematic of the internal forces and moments in the rod infinitesimal element is shown in Fig. 1b. The self-weight and external forces are disregarded hence the equilibrium of horizontal and vertical forces and bending moments respectively yields:

$$\frac{d}{dS}\left(T\,sin\phi - V\cos\phi\right) = 0 \tag{3a}$$

$$\frac{d}{dS}\left(T\cos\phi + V\,sin\phi\right) = 0 \tag{3b}$$

$$\frac{dM}{dS} - V = 0 \tag{3c}$$

Where $M(S)$ is the bending moment, $V(S)$ and $T(S)$ are respectively the shear and axial forces. Integrating Eqs. (3a) and (3b) and applying the conditions $T(0) = T_0$ and $V(0) = V_0$ yields:

$$Tsin\phi - V\cos\phi = V_0 \tag{4a}$$

$$T\cos\phi + V\,sin\phi = T_0 \tag{4b}$$

Further manipulating the Eqs. (4a) and (4b) and applying the condition $V(L) = -F\ sin(\alpha)$ yields:

$$V = -F\ sin[\phi_L + \alpha - \phi] \tag{5}$$

2.4. Constitutive relations

The stress-strain ($\sigma - \varepsilon$) curve should be obtained experimentally. Generally a material such as polyurethane may present an asymmetric behaviour in tension and compression:

$$\sigma = f(\varepsilon) \tag{6}$$

And possibly $f(\varepsilon) \neq -f(-\varepsilon)$. Considering the Bernoulli-Euler formulation where plane cross-sections remain plane after bending, the strain ε at a distance η from the neutral axis is given by:

$$\varepsilon = \kappa\,\eta \tag{7}$$

The neutral axis position may be obtained from the equilibrium of forces in the cross-section, i.e.:

$$\int \sigma\, dA = 0 \tag{8a}$$

Where dA is an infinitesimal element of the area cross-section. The bending moment may be expressed by:

$$M = \int \eta\,\sigma\, dA \tag{8b}$$

If the width of the infinitesimal element of area dA, located at a distance η from the neutral axis (see Fig. 1a), is expressed by a function $\beta(\eta)$ then $dA = \beta(\eta)d\eta$ and substituting Eqs. (6) and (7) in (8a) and (8b) respectively results:

$$\int f(\varepsilon)\beta(\varepsilon/\kappa)\, d\varepsilon = 0 \tag{9a}$$

$$M = \int \frac{\varepsilon\, f(\varepsilon)\beta(\varepsilon/\kappa)}{\kappa^2}\, d\varepsilon \tag{9b}$$

The neutral axis position $\bar{y}$ is then a function of the curvature but it is usually very small (i.e., close to the circle center) for moderately asymmetric materials.

Linear Elastic Material

For linear elastic, homogeneous and isotropic materials $\sigma = E\,\varepsilon$, and considering the state of pure bending the neutral axis coincides with the cross-section centroid of area ($\int \eta \;\; dA = 0$) and the Eq. (9b) results:

$$M = EI\;\kappa \tag{10}$$

Where E is the modulus of Young and I is the cross-section second moment of area. The total bending stiffness distribution is then given by $EI(S) = EI_{pipe} + EI(S)_{stiffener}$.

Therefore, substituting the Eq. (10) in (3c) and using (5) yields:

$$\frac{d\kappa}{dS} = \frac{-1}{EI}\left\{\kappa \cdot \frac{dEI}{dS} + F \cdot sin(\phi_L + \alpha - \phi)\right\} \tag{11}$$

Non-Linear Elastic Material

If the material is non-linear elastic the neutral axis and the bending moment need to be numerically calculated by Eqs. (9a) and (9b), respectively. However, if the material is symmetric the neutral axis coincides with the centroid of the area (i.e., $\eta = 0$). The bending moment may be also written by an adjusted power series:

$$M = A\kappa + B\kappa^2 + C\kappa^3 + \cdots \tag{12}$$

Where $A(S), B(S), C(S), etc$ are interpolated coefficients. Considering a third order approximation for the bending moment power series expansion and substituting Eq. (12) in (3c) and using (5) yields:

$$\frac{d\kappa}{dS} = -\frac{1}{A + 2B\kappa + 3C\kappa^2}\left[\frac{dA}{dS}\kappa + \frac{dB}{dS}\kappa^2 + \frac{dC}{dS}\kappa^3 + F\ \mathrm{sen}(\phi_L + \alpha - \phi)\right] \tag{13}$$

2.5. Numerical solution

The two-point boundary value problem is set by the governing Eqs. (1a) - (1c) and (11) or (13) if the material is respectively assumed linear or non-linear elastic. The boundary conditions are given by Eq. (2).

An explicit analytical approach for this boundary value problem is not possible even for linear elastic materials (except for constant bending stiffness or constant curvature), and consequently it is necessary to pursue a numerical solution for this problem. The mathematical package Matlab® has been employed in the solutions. Bend stiffeners often present geometrical discontinuities, such as in the transitions cylindrical to conical shapes and stiffener-pipe, so the Rosembrock robust method, recommended for stiff functions, was adopted. A one parameter shooting method based on an incremental-iterative user supported scheme was employed for the solution.

3. Case Study

In this example, it is studied the structural behaviour of a 4 inch flexible pipe, 3.2 m long, protected by a 1.9 m bend stiffener as schematically shown in Fig. 2. The pipe bending stiffness and minimum bending radius are respectively 10 kNm^2 and 2.0 m. Furthermore the angles α and ϕ_L are respectively defined as zero and 45^o.

3.1. Linear symmetric material

The polyurethane Young Modulus at 10% strain is 45 MPa. Hence the boundary value problem is solved and results of curvature as a function of the arclength were compared with Boef and Out (1990), which showed good agreement.

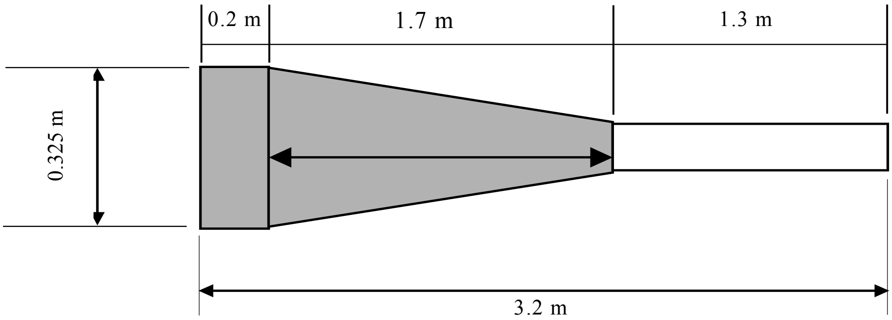

Figure 2. Schematic of the Bend Stiffener.

3.2. Non-Linear asymmetric material

When the material is non-linear elastic asymmetric, the solution of the boundary value problem is not so simple. In the following example a third order power series approximation for the bending moment distribution was employed. Furthermore the coefficients $A(s)$, $B(s)$ and $C(s)$ were also expanded by a power series of the type:

$$A(S) = a_0 + a_1 S + a_2 S^2 + a_3 S^3 + a_4 S^4 + \cdots \tag{14a}$$

$$B(S) = b_0 + b_1 S + b_2 S^2 + b_3 S^3 + b_4 S^4 + \cdots \tag{14b}$$

$$C(S) = c_0 + c_1 S + c_2 S^2 + c_3 S^3 + c_4 S^4 + \cdots \tag{14c}$$

If the Eq. (12) is solved for several sections an interpolation for the coefficients $A(S)$, $B(S)$ and $C(S)$ may be found and consequently the boundary value problem may be solved. A similar approach is carried out for the determination of the neutral axis, which is also a function of the curvature for different cross-sections (see Eq. (9a)).

The polyurethane stress-strain asymmetric curve obtained from Meniconi (1999), presented in Fig. 3, is employed in this case study. It is now necessary to calculate the neutral axis position and the bending moment for several cross sections as a function of the curvature. The coefficients $A(s)$, $B(s)$ and $C(s)$ are then calculated and the boundary value problem may be incremental-iteratively solved.

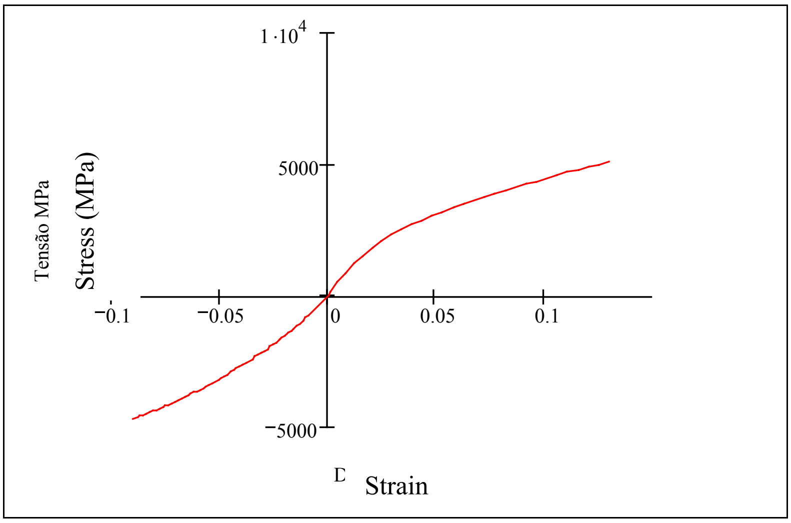

Figure 3. Stress-Strain Curve for a Polyurethane

The results $\kappa(S)$ for linear elastic and non-linear elastic asymmetric and applied forces F = 62.5, 125 and 250 kN can be seen in Fig. 4. Results evidence the significant potential influence of the material non-linearity on the stiffener behaviour.

It should be emphasized that the neutral axis eccentricity originates from the material asymmetry in tension – compression and this behaviour is not featured in the bend stiffener commercial tools analysis.

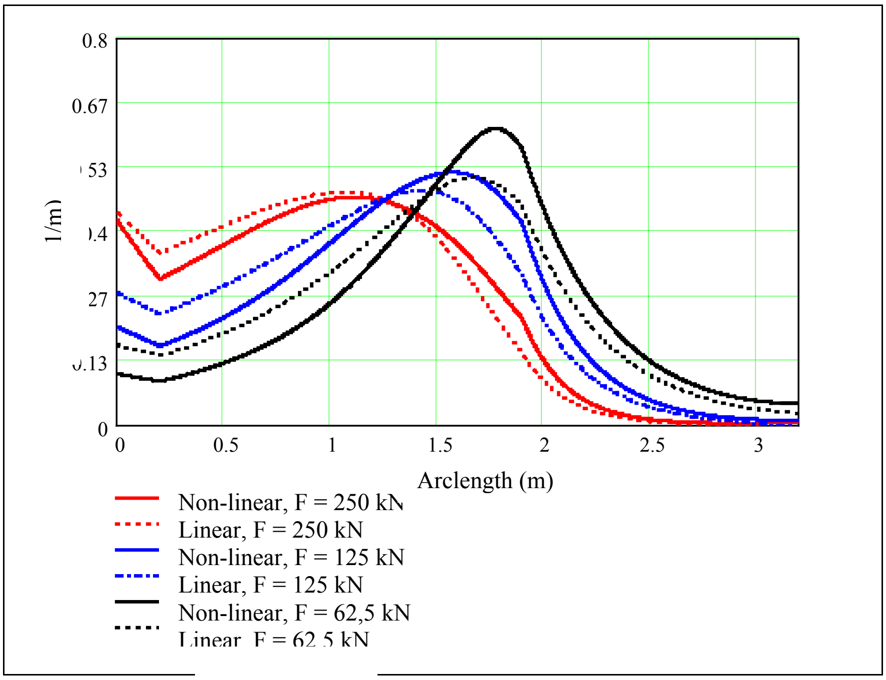

Figure 4. Curvature Distribution. Linear and Non-Linear Materials. Loads 62.5, 125 and 250 kN

4. Conclusions

A formulation and a solution for the geometrical and physical non-linear analysis of bend stiffeners are developed in this paper. Large displacements are accepted and the material is assumed non-linear elastic asymmetric. The governing equations are expressed by a system of four non-linear ordinary differential equations and the boundary conditions are specified at both ends. An incremental-iterative user supported scheme is developed and results show that a correct assessment of the constitutive representation of the polyurethane is important for an accurate design of the bend stiffener.

5. Acknowledgements

The authors acknowledge the support of the National Council for Scientific and Technological Development (CNPq) and Petrobras for this work.

6. References

Boef, W. J. C. and Out, J. M. M., 1990, “Analysis of a Flexible Riser Top Connection with Bend Restrictor”, Offshore Technology Conference, OTC 6436, Houston.

Custódio, A. B. and Vaz, M. A., 2002, “A Nonlinear Formulation for the Axisymmetric Response of Umbilical Cables and Flexible Pipes”, Applied Ocean Research, Vol. 24, No 1, pp. 21-29.

Féret, J. and Bournazel, C.H., 1987, “Calculation of Stresses and Slip in Structural Layers of Unbounded Flexible Pipes”, Journal of Offshore Mechanics and Artic Engineering (JOMAE), Vol. 109, pp.263-269.

Féret, J., Leroy, J.M. and Estrier, P., 1995, “Calculation of Stresses and Slips in Flexible Armour Layers with Layers Interaction”, Journal of Offshore Mechanics and Artic Engineering (JOMAE), Vol. V, pp.469-474.

Lane, M., McNamara, J. F., Gibson, R. and Tyrer A., 1995, “Bend Stiffeners for Flexible Risers”, Offshore Technology Conference, OTC 7730, Houston.

Meniconi, L. C. M., 1999, “Avaliação da Vida à Fadiga dos Enrijecedores dos Risers Flexíveis do FPSO Petrobras 33”, Technical Comunication No. 137/99 (in Portuguese), CENPES/SUPEP/DIPLOT/SEMEC.

Saevik, S., 1992, “On Stresses and Fatigue Flexible Pipes”, PhD Dissertation, University of Trondheim, Norway.

Witz, J. A. and Tan, Z., 1992, “On the Axial - Torsional Structural Behavior of Flexible Pipes, Umbilicals and Marine Cables”, Journal of Marine Structures, Vol.5, No 2 & 3.