SPACE TRAJECTORIES IN THE BI-CIRCULAR RESCTRICTED FOUR-BODY PROBLEM

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Abstract. *The objective of this paper is to study possible trajectories for a spacecraft that is traveling governed by the dynamics given by the bi-circular restricted model of four bodies. The system considered is the Sun-Earth-Moon-spacecraft. This type of trajectories has many potential applications, such as: i) To use them as transfer orbits to transfer a spacecraft between the Earth and a Halo orbit in the Sun-Earth system; ii) To obtain trajectories that can transfer a spacecraft from a parking orbit around the Earth to the Moon, using the concept of gravitational capture. For gravitational capture it is understood a phenomenon where a massless particle changes its two-body energy around one of the primaries from positive to negative. This capture is always temporary and, after some time, the two-body energy changes back to positive and the massless spacecraft leaves the neighborhood of the primary. The importance of this temporary capture is that it can be used to decrease the fuel expenditure for a mission going from one of the primaries to the other, like an Earth-Moon mission. The results will show the basic information to characterize those orbits, in particular the ones that can satisfy the second application shown above.*

Keywords*: Astrodynamics, gravitational capture, four body problem.*

1. Introduction

In the restricted bicircular four body problem (M_1, M_2, M_3 and M_4) we consider the following system: i) the Earth and the Moon are the primaries, with both in circular orbits around the common center of mass; ii) the Sun is the third body, assumed in circular orbit around the center of mass of the Earth-Moon system and its orbit is coplanar with the orbit of the Moon; iii) we will study the motion of a particle of infinitesimal mass that has its motion governed by the gravitation of the three bodies previously described.

In Fig. 1 (ξ, η, ζ) and (x, y, z) denotes the position in an inertial and in a primaries-fixed rotating frame, respectively. In both frames, the barycenter of the Earth-Moon system is taken as the origin. The mass of the Earth and the Moon is μ_1 = 0.9878493317 and μ_2 = 0.0121506683 and their positions are denoted by ($-\mu_2$, 0, 0) and (μ_1, 0, 0), respectively, in primaries-fixed rotating frame.

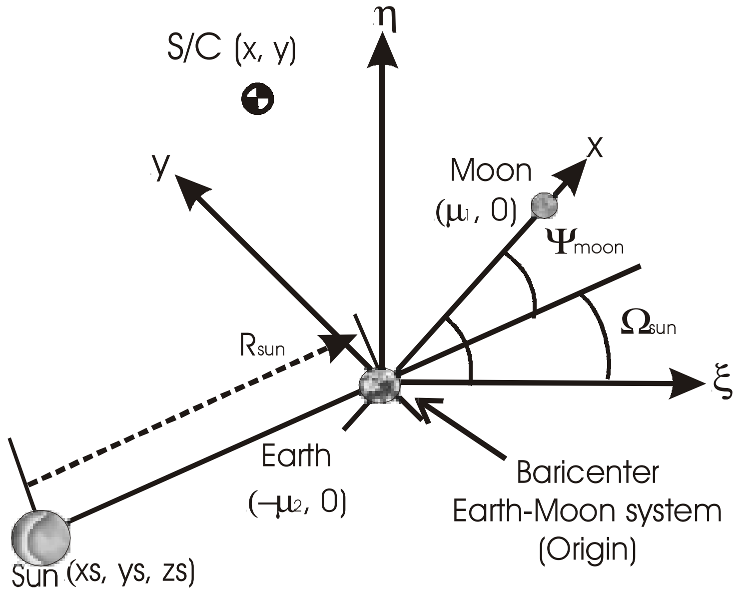

Figure 1 – Restricted four-body model (Cartesian Coordinate).

2. Matematical Model

To obtain the equations of motion we will use the lagrangian formulation, in the same way used in Yamakawa, 1992. Then, defining the lagrangian of the system:

$$L = T - U \tag{1}$$

Where T is kinetic energy and U is potential energy as shown below. U takes the form of the effective potential energy, where the effect that Sun rotating around the barycenter of the Earth-Moon system (origin) is taken into account:

$$T = \frac{1}{2}\left(\left(\frac{d\xi}{dt}\right)^2 + \left(\frac{d\eta}{dt}\right)^2 + \left(\frac{d\zeta}{dt}\right)^2\right) = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right) + \left(x\frac{dy}{dt} - \frac{dx}{dt}y\right) + \frac{1}{2}\left(x^2 + y^2\right) \tag{2}$$

$$U = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \mu_{sun}\left[\frac{1}{r_{sun}} - \frac{xx_s + yy_s}{R_{sun}^{\;3}}\right] \tag{3}$$

Where:
r_1, r_2, and r_{sund} are the distances of the spacecraft with respect to the Earth, Moon and Sun, respectively;
R_{sun}= 389.1723985 is the distance between the Sun and the origin;
(x_s,y_s,z_s) corresponds to Sun´s position with respect to the origin in Earth-Moon fixed rotating frame;
μ_{sun} is the gravity constant of the Sun = 328900.48 under normalization.
r_1, r_2 and r_{sun} are expressed as follows:

$$r_1^{\;2} = (x + \mu_2)^2 + y^2 + z^2 \tag{4}$$

$$r_2^{\;2} = (x - \mu_1)^2 + y^2 + z^2 \tag{5}$$

$$r_{sun}^{\;\;2} = (x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2 = (x + R_{sun}\cos\psi_{moon})^2 + (y - R_{sun}\sin\psi_{moon})^2 + z^2 \tag{6}$$

Where ψ_{moon} corresponds to the angle that specifies the position of the Moon in the Sun-Earth fixed rotating frame (see Fig. 1). ψ_{moon} is described as follows;

$$\Psi_{moon} = \alpha + (\omega_{E-M} - \Omega_{sun})t \tag{7}$$

$$\Omega_{sun} = \alpha + \omega_{E-M} t \tag{8}$$

Where:

α = initial Sun phase angle expressed by (anti-Sun-direction)-(origin)-(Moon) which is equivalent to the initial Moon´s positional phase in sun-earth fixed rotating frame;

ω_{E-M} = angular velocity of the Earth and Moon around their origin (=1.00);

Ω_{sun}= angular velocity of the Sun around the origin (=0.07480133).

Using the Lagrangian formulation, the equations of motion are expressed as follows:

$$\begin{aligned}
&\frac{d^2x}{dt^2} - 2\frac{dy}{dt} - x - \frac{\mu_{sun}}{R_{sun}^{\,2}}\cos\psi_{moon} = -\frac{\delta U_n}{\delta x} \\
&\frac{d^2y}{dt^2} + 2\frac{dx}{dt} - y + \frac{\mu_{sun}}{R_{sun}^{\,2}}\sin\psi_{moon} = -\frac{\delta U_n}{\delta y} \\
&\frac{d^2z}{dt^2} = -\frac{\delta U_n}{\delta z}
\end{aligned} \tag{9}$$

Where:

$$U_n = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{\mu_{sun}}{r_{sun}} \tag{10}$$

and:

$$\begin{aligned}
&\frac{\delta U_n}{\delta x} = \frac{\mu_1}{r_1^{\,3}}(x+\mu_2) + \frac{\mu_2}{r_2^{\,3}}(x-\mu_1) + \frac{\mu_{sun}}{r_{sun}^{\,3}}(x + R_{sun}\cos\psi_{moon}) \\
&\frac{\delta U_n}{\delta y} = \frac{\mu_1}{r_1^{\,3}}(y) + \frac{\mu_2}{r_2^{\,3}}(y) + \frac{\mu_{sun}}{r_{sun}^{\,3}}(y - R_{sun}\cos\psi_{moon}) \\
&\frac{\delta U_n}{\delta z} = \frac{\mu_1}{r_1^{\,3}}(z) + \frac{\mu_2}{r_2^{\,3}}(z) + \frac{\mu_{sun}}{r_{sun}^{\,3}}(z)
\end{aligned} \tag{11}$$

3. Gravitational Capture

The term gravitational capture means that the spacecraft approaches one of the primaries from a long distance and stays close to that primary with negative two-body energy. This changes of sign of energy is obtained thanks to the gravititional effects of the third and fourth body involved in the dynamics.

In the resctricted three-body problem this capture is always temporary and the two-body energy changes back to positive after some time. Then, to consider that a trajectory ends in a gravitational capture we define a parameter C3 (twice the two-body energy with respect to the Moon) as:

$$C3 = V^2 - \frac{2\mu}{r} \tag{12}$$

Where: V and r are the velocity and the distance of the spacecraft with respect to the Moon.

Then, the variation of C3 is verified all the time during the trajectory. If the value of this parameter changes the sign from negative (closed trajectory) to positive (open trajectory) it is considered that the trajectory escaped, so, in the oposite sense of time, there is a gravitational capture. More detail can be found in Belbruno (1987), Vieira-Neto e Prado (1998). Yamakawa (1992), Vieira-Neto (1999).

4. Results

Using the numeric integrator Runge Kutta-4 and the Fortran Power Station 4.0 Compiler, graphs were generated that represent trajectories in the bicircular four bodies system with gravitational capture.

For the integration of the trajectories the initial conditions are calculated with a certain value of C3 with null relative radial velocity at the perilune distance. Then the value of C3 is monitored until the negative value turn into positive. A trajectory is considered a gravitational capture when the change of the sign occurs in a time smaller than 50 days.

The initial parameters of the trajectories are: R_p (perilune distance) = 1838 km, V is the velocity relative to the Moon, α is the angle between the perilune and the line Earth-Moon, measured, in the counterclockwise sense, starting from the opposite side of the Earth.

Figs. 3-5 shows trajectories in the Sun-Earth-Moon-spacecraft system with several angles of captures for diferents values of C3.

Fig. 3 show trajectories starting with C3 = 0.0 (parabolic orbits with respcet to the Moon with diferent values of α, as show in the figure. Fig. 4 shows equivalent trajectories, with initial values of C3 = - 0.1, and Fig. 5 show the results for C3 = - 0.2.

The results show the existence of trajectories of gravitational capture for most of the initial conditions simulated. The horizontal axis represents the x-axis of the rotating frame, that is in the Earth-Moon line and the vertical axis is axis perpendicular to the Earth-Moon line passing by the center of the mass of the system.

The center of the Moon is at the point x = 0.9878493317, y = 0. In the negative sense of time all the trajectories start close to the Moon at 1838 km from the center of the Moon, or 100 km above its surface, and then it goes to infinity. It means that, in the positive sense of time, the particle comes from infinity and then gets close to the Moon after some time has passed.

The trajectories are captured faster when the value of C3 is closer to zero. It is visible that when C3 goes from 0.0 to -0.1 most of the trajectories have some loops around the Moon before escaping/been captured. The number and duration of the loops increase when C3 goes from –0.1 to –0.2.

For C3 = 0.0 and C3 = 0.1 we obtained trajectories for all values of the angle α. For C3=-0.2, it was not possible to obtain solution for the case α = 180º. This result is expected and it also occurs in the restricted Three-Body problem (Yamakawa, 1992; Viera-Neto, 1999). In the situations close to the limit of minimum C3 (close to – 0.22) only some initial value of α allows the trajectoriy to exist.

5. Conclusions

The bi-circular resctrited four-body problem was implemented and tested with good results. Trajectories that ended in gravitational capture were found under this model, approaching the Moon from many different directions. Different values for the savings obtained from the gravitational capture were used. Those simulations showed very well the potential of this type of maneuver under the mathematical model tested.

6. Acknowledgment

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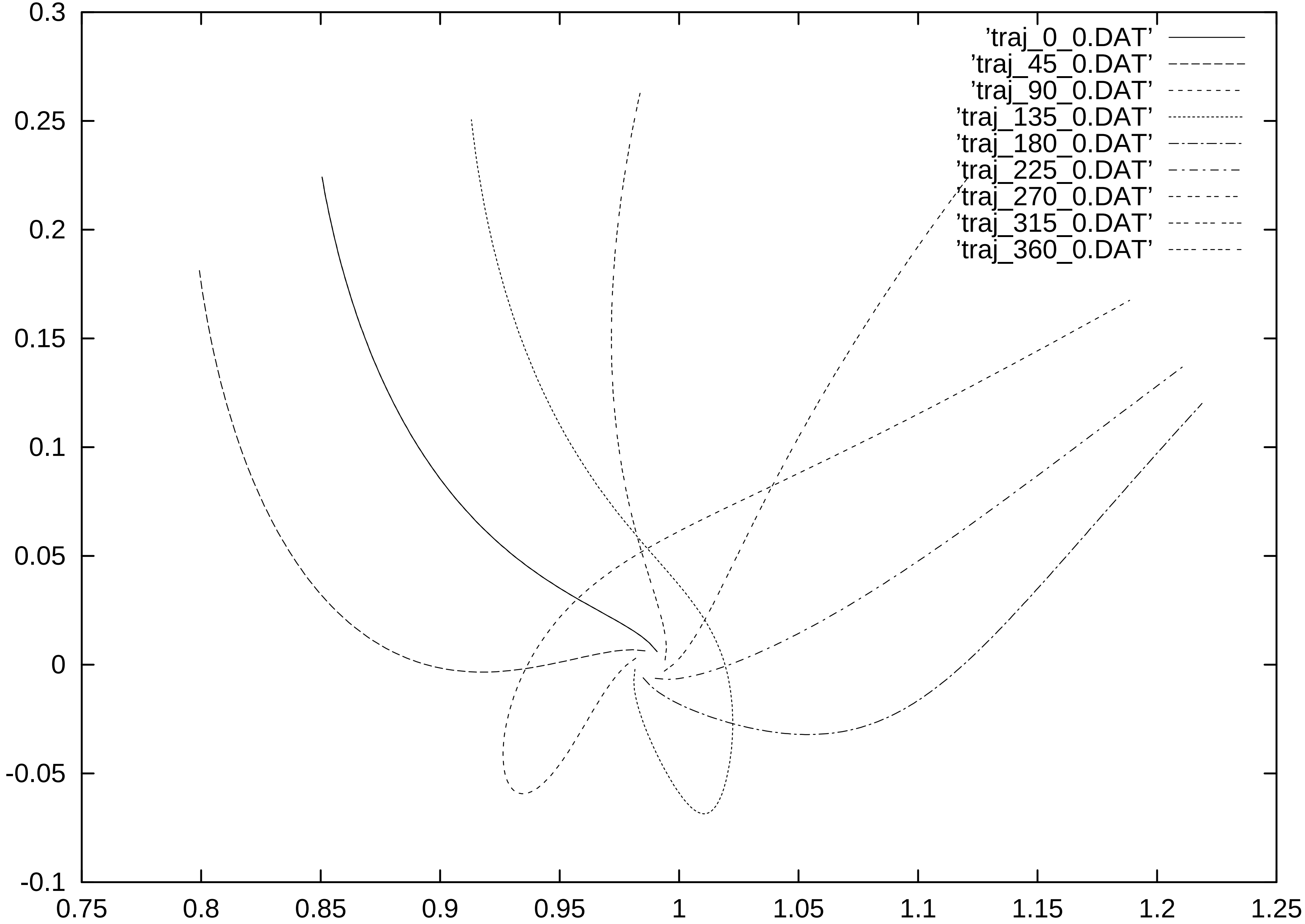

Fig. 3 – Trajectories with C3 = 0.0.

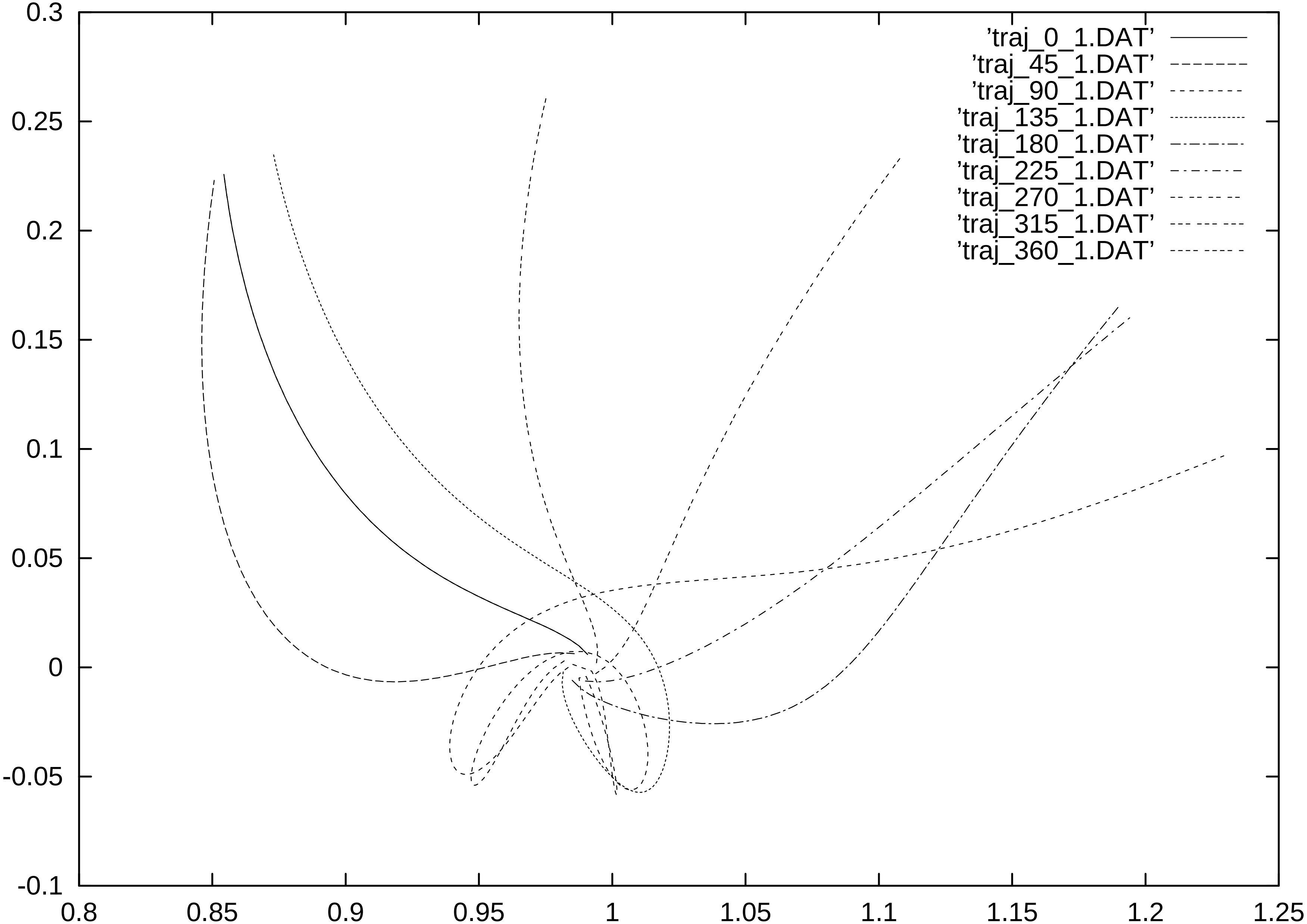

Fig. 4 – Trajectories with C3 = -0.1.

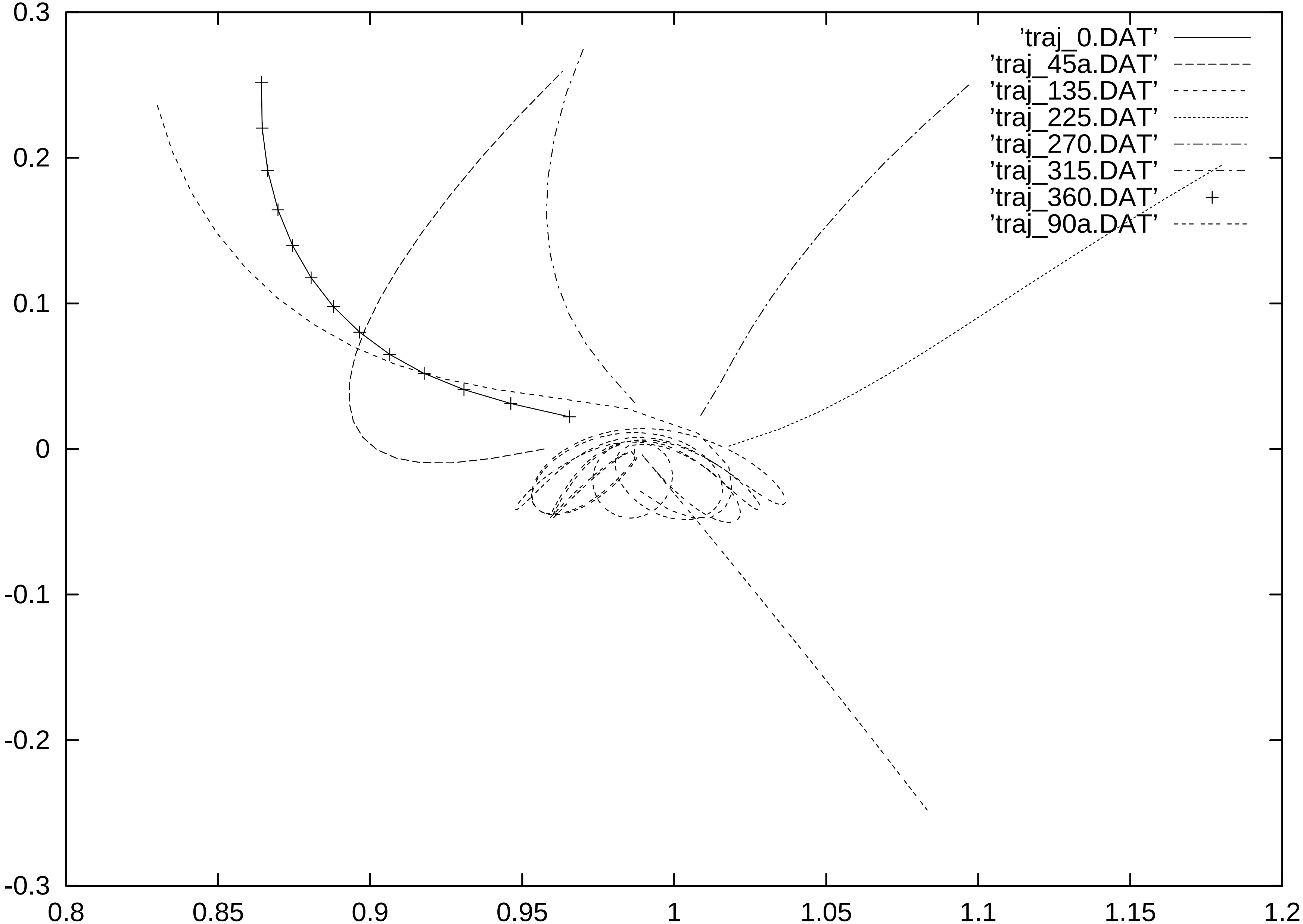

Fig. 5 – Trajectories with C3 = -0.2.