THE INFLUENCE OF REINFORCEMENT BEAMS IN THE IRRADIATION EFFICIENCY OF SIMPLY SUPPORTED PLATES

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Abstract: Studies on radiation efficiency of structural components have been undertaken with the objective of determining sound power radiation. Offshore platform structures are mainly made of beam-reinforced plates. It is known that the reinforcing beams affect the radiation efficiency of plates, which is increased since sound cancellations effects are interrupted in the vicinities of the beams. To calculate the irradiation efficiency it is necessary the knowledge of the sound pressure field on the plate. This is obtained from the Rayleigh Integral. The integral is solved by Fourier Transforms, applied to the velocity distribution of the plate. The pressure is obtained by applying the inverse transform to the product of the transformed velocity with the transformed Green function. The use of Fourier Transform is very advantageous, since FFT is used for the calculations. This work presents results of irradiation efficiency of plane plates, considering the effects of the flexibility (web and flange own modes) of the reinforcing beams. The velocity distribution is obtained by Finite Elements. Some plate configurations, with and without reinforcements are analyzed, and the results show an increase in the irradiation efficiency as more beams are attached to the plate.

Keywords*: Sound Irradiation, Beam-Reinforced Plates, Rayleigh Integral, Structural Acoustics, Irradiation Efficiency*

1. Introduction

The sound irradiation and irradiation efficiency of structural components has been studied for some time, since its complete understanding gives an insight to predict the sound pressure in the environment surrounding these components. This is a consequence of more severe legislations regarding the health of the population, from the hearing point of view, especially for those people who work in noisy environments.

Most machines generate vibration while operating. As a consequence, structural components from the machine, foundation and floor irradiate noise. This is commonly observed in offshore platforms, which is composed basically by plates, beams and beam-reinforced plates.

Studies about radiation from beam-reinforced plates treat the discontinuities, as been ideal situations of supports and clamps. The beam effect has been considered in many articles (Maidanik, 1962; Lin e Hayek, 1972; Mace, 1980; e Berry e Nicolas, 1994), but only the rigidity and inertia of the reinforcements. Irradiation efficiency predictions for plates reinforced by beams, as made today, have non-tolerable errors, since they do not consider the real effects of the beams in the velocity field of the plates.

The pressure filed generated by plane vibrating surfaces can be obtained from the Rayleigh Integral (Williams, 1999). This integral can be solved by direct numerical integration or by means of Fourier Transforms (Williams e Maynard, 1982; e Williams, 1983). The Rayleigh Integral relates the pressure in a certain observation point with the normal velocity of each area element of the surface. Integrating all elements, all over the surface, we have the total pressure irradiated by the surface, calculated at the observation point.

To solve the Rayleigh Integral using Fourier Transforms we have to apply the transform to the velocity distribution of the plate. The pressure is obtained applying the inverse transform to the product between the transform of the velocity and the transform of the Green function for the plate. The use of Fourier Transforms can be very advantageous when FFT algorithms are used to calculate the direct and inverse transforms.

The irradiation efficiency is given by (Williams, 1999):

$$\sigma_{rad} = \frac{W_{rad}}{\rho c \left\langle \overline{v}^2 \right\rangle} \tag{1}$$

where W_{rad} is the irradiated sound power of the surface, ρ is the density of the medium, c is the sound speed of the medium, S is the surface area, and $\left\langle \overline{v}^2 \right\rangle$ is the mean quadratic velocity, in time and space. Finite Elements will calculate the velocity distribution, since this method can incorporate the resonances of the beam in the velocity of the plate. This velocity will be used to calculate the sound power and the quadratic velocity, necessary to the efficiency. The sound pressure and sound power are calculated with FFT (Fast Fourier Transform) algorithms, (Press et al, 1992).

The choice for Finite Element Methods (FEM) for velocity distribution calculation is based on the fact that there is no analytical formulation that represents the real velocity of a beam reinforced plate. With FEM, one can obtain a velocity that considers the influence of the beams, since it is possible to model the beams in a way that the resonances, loads and displacements acting in the beam are accounted for in the velocity distribution of the plate. The software used was Ansys 5.4. Analyses where made until frequencies higher than the coincidence frequency, as the irradiation efficiency tends to be constant (equals one) for frequencies higher than the coincidence.

The objective of this work is to calculate, with a better precision, the irradiation efficiency of beam reinforced, simply supported plates on all sides, considering the own modes of the web and flange of the beams, using FFT algorithms to solve the Rayleigh Integral. The irradiation efficiency of several plate/beams configurations is investigated, and the results show that the efficiency increases with the presence of the beams.

2. Basic Equations and Definitions

The sound pressure field generated by a vibrating surface can be obtained solving the wave equation, subjected to the surface boundary conditions. The wave equation and the boundary conditions can be combined in an integral equation, named Kirchhoff-Helmholtz Integral (Fahy, 1985). The equation for the acoustic pressure in a position $\vec{R}$ (Figure 1), resulting from the vibration of a surface S, is given by:

$$p(\vec{R}) = \frac{1}{4\pi}\int_S \left[p(\vec{R}_0)\frac{\partial}{\partial n}\left(\frac{e^{-ikR}}{R}\right) + i\omega\rho v_n(\vec{R}_0)\frac{e^{-ikR}}{R} \right] dS \tag{2}$$

where $p(\vec{R}_0)$ is the pressure on the surface in the position $\vec{R}_0$, v_n is the surface velocity component in the normal direction, R is the distance ($|\vec{R} - \vec{R}_0|$) between the points on the source and the medium and the integration is made all over the surface.

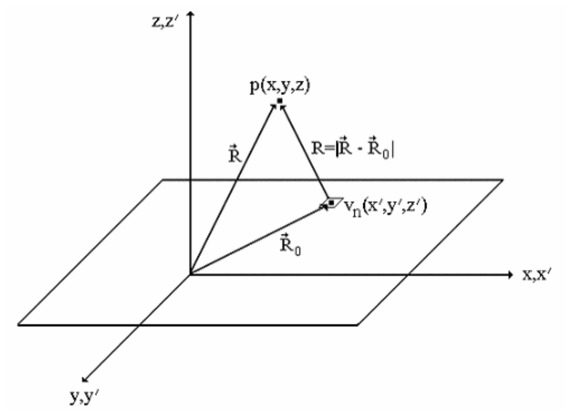

Figure 1: Geometric interpretation for the pressure.

When the vibrating surface is plane, Eq. (2) can be simplified to the Rayleigh Integral, given by (Fahy, 1985, Williams, 1999):

$$p(x,y,z) = \frac{j\omega\rho}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\dot{w}(x',y',z')\frac{e^{-ik|r-r'|}}{|r-r'|}\right]dx'dy' \tag{3}$$

where $|r - r'| = \left[(x-x')^2 + (y-y')^2 + z^2\right]^{1/2}$.

With the knowledge of the pressure field and the velocity distribution of the surface, the irradiation efficiency can be calculated. As already seen, the irradiation efficiency is calculated from:

$$\sigma_{rad} = \frac{W_{rad}}{\rho c S\langle \overline{v}^2 \rangle} \tag{4}$$

The irradiated sound power W_{rad}, is:

$$W_{rad} = \frac{1}{2}\int\int_S \mathrm{Re}\left[p(x,y,0)v^*(x,y,0)\right]dS \quad (5)$$

and the mean square velocity is:

$$\left\langle \bar{v}(x,y)^2 \right\rangle = \frac{1}{2S}\int_S v(x,y)^2\, dS \quad (6)$$

The sound radiation is a consequence from the cancellations that occur in the air movement over the plate, depending on the mode configuration (Beranek, 1988). Cancellation is a phenomenon where the air displaced outward by one section of the plate moves to occupy the space left by the motion of adjacent section, without being compressed and irradiating very little power. Finite plate’s modes have a well-known behavior. For frequencies below the coincidence, which is the frequency where the bending speed of the plate is equal to the sound speed of the medium, corner and edge modes are present. For frequencies higher than the coincidence, surface modes occur. Corner and edge modes are poor irradiators, while surface modes are good irradiators.

3. Irradiation Efficiency and FFT

The Rayleigh Integral, with time dependence $e^{-i\omega t}$, is given by (Williams e Maynard, 1982):

$$p(x,y,z) = -i\omega\rho\int_{-\infty}^{\infty}\int v(x',y')\frac{e^{ikR}}{2\pi R}dx'dy' \quad (7)$$

where $R^2 = (x-x')^2 + (y-y')^2 + z^2$ and the integration is made over an infinite surface (z = 0). Eq. (7) can be rewritten in a condensed form defining a kernel h:

$$h(x,y,d) = -i\omega\rho g(x,y,d) = -\frac{i\omega\rho}{2\pi}\frac{e^{ik\left(x^2+y^2+d^2\right)^{1/2}}}{\left(x^2+y^2+d^2\right)^{1/2}} \quad (8)$$

where g(x,y,d) is the Green function for the plate. It is possible to visualize Eq. (7) as being a convolution between v(x,y) and h(x,y,d), making z = d in p(x,y,z). The pressure, then, is:

$$p(x,y,d) = v(x,y)\circ h(x,y,d) \quad (9)$$

Taking the Fourier transform of Eq 9:

$$F[p(x,y,d)] = F[v(x,y)\circ(x,y,d)] \quad (10)$$

The Convolution Theorem (Bendat e Piersol, 1986) states that the Fourier Transform of the convolution of two functions is equal the product between the Fourier Transforms of the two functions. With that in mind, Eq. (10) can be changed to:

$$F[p(x,y,d)] = F[v(x,y)]F[h(x,y,d)] \quad (11)$$

Taking now the Inverse Fourier Transform of Eq. (11), we have:

$$p(x,y,d) = F^{-1}\left[\hat{V}(k_x,k_y)\hat{H}(k_x,k_y,d)\right] \quad (12)$$

where F^{-1} is the inverse Fourier transform and $\hat{V}(k_x,k_y)$ and $\hat{H}(k_x,k_y,d)$ are the Fourier transforms of v (velocity) and h (kernel), respectively.

The algorithm to calculate p(x,y,d), knowing v(x,y) and using Fast Fourier Transforms is implemented this way:

(1) calculate the Discrete Fourier Transform (DFT) of v(x,y), using FFT, and call it $\hat{V}_D$.

(2) calculate the analytical form of $\hat{H}(k_x, k_y, d)$.

(3) multiply the results from (1) and (2) and calculate the Inverse Discrete Fourier Transform of the result.

Symbolically speaking, we have:

$$p_D(x, y, d) = D^{-1}\left[\hat{V}_D(k_x, k_y)\hat{H}(k_x, k_y, d)\right] \quad (13)$$

where the subscript D refers to calculations via DFT and D^{-1} represents the inverse DFT.

Equations 4 to 6, along with the FFT technique, were implemented in a executable program (Fiates, 2003). The program uses a FFT algorithm based in the Cooley approach (Cooley e Tuckey, 1960), where the function to be transformed must have 2^n points. As we are interested in finite plated and due to stationary waves in this kind of plates, the most part of vibratory energy is irradiated in these frequencies. So, the program calculates the efficiency only for the resonance frequencies (modes) of the plates.

The program has as entry data the velocity field for each mode, and all calculations are repeated for each mode, in a frequency loop. The resonance frequencies values are obtained from the finite element analysis and are also an entry data for the program. The program calculates, then, the mean square velocity, (Eq. 6), which is stored for later use, and the velocity is also stored for the calculation of the sound power (Eq.5). But most of the calculations are made to obtain the sound pressure (Eq. 12).

The velocity matrix is patched with zeros to simulate a baffle condition and to improve the FFT accuracy. This matrix is transformed and the transformed Green function is calculated, on the same points of the discretizated plate. These results are multiplied and the inverse transform is applied, resulting in the sound pressure. The zeros are taken off and the sound pressure in the plate is obtained. With the pressure and the velocity, the sound power is, then, calculated. The efficiency is determined from the power recently calculated and the mean square velocity calculated in the beginning of the loop. The calculation time for a typical plate was around five minutes, while solving the Rayleigh Integral by direct numeric integration took about one hour.

The Finite Element Method is used in part of this work to obtain the plate velocity, with or without reinforcements. As the efficiency will be calculated for each resonance frequency, it is necessary to know the velocity for each correspondent mode. Ansys offers an easy way to accomplish that, performing only a modal analysis of the structure. The shell element used was Shell63, an element with bending and membrane capabilities, four nodes and six degrees of freedom (three displacements and three rotations) per node. The analyses were made until frequencies above the coincidence and the number of elements obeyed the rule of a minimum of 12 elements per wavelength.

4. Results

The efficiency was calculated for a series of plate/beam setups, from a simple one with no reinforcements to one with two reinforcements. For all cases the plate was the same, of steel, with dimensions L_x = 1,0 m and L_y = 0,8 m; thickness h = 7 mm; density ρ_s = 7860 kg/m^3 , loss factor η = 0,005 and Young modulus E = 210 G Pa. The reinforcements were T inverted beams, with thickness h_T = 7 mm; web h_a = 10 cm and flange b_f = 5 cm, as seen in the Fig. (2).

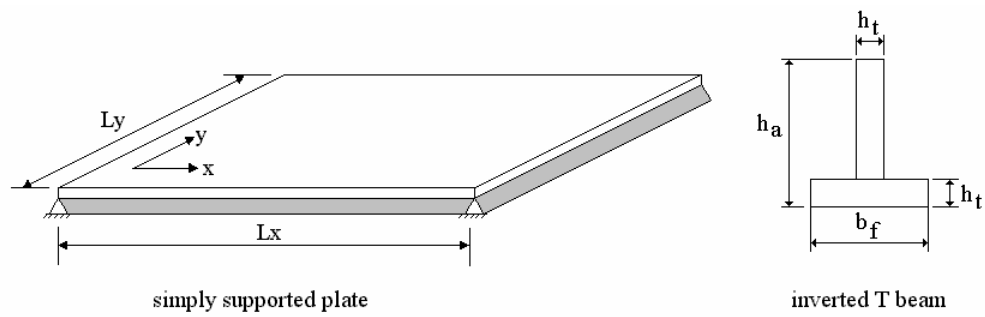

Figure 2: Scheme of the plate and beam used in the efficiency analysis.

The following convention was used to nominate the plate configurations:

Plate I – simply supported plate, no reinforcements.
Plate II – simply supported plate, with a central support in the y direction.
Plate III – simply supported plate, with a central beam in the y direction.
Plate IV – simply supported plate, with two crossed beams, in the center of the plate.
Plate V – simply supported plate, with two parallel beams, in the y direction.

To verify the accuracy of the developed program (using FFT), the efficiency for a simply supported (ss) plate was compared with a simplification suggested by Ver (Beranek, 1988). Figure (3) shows a good agreement between the two curves, indicating that the results from the program are valid. It can be seen that the efficiency oscillates around the simplification. Figure (4) shows the efficiency plotted in 1/3 octave bands. These results are the mean value of the efficiency, that is, for each band, the modes that fitted between lower and upper limits of the band were accounted for and the mean efficiency value was stipulated for that band. Also in Fig. (4) we can see that program gives reliable results.

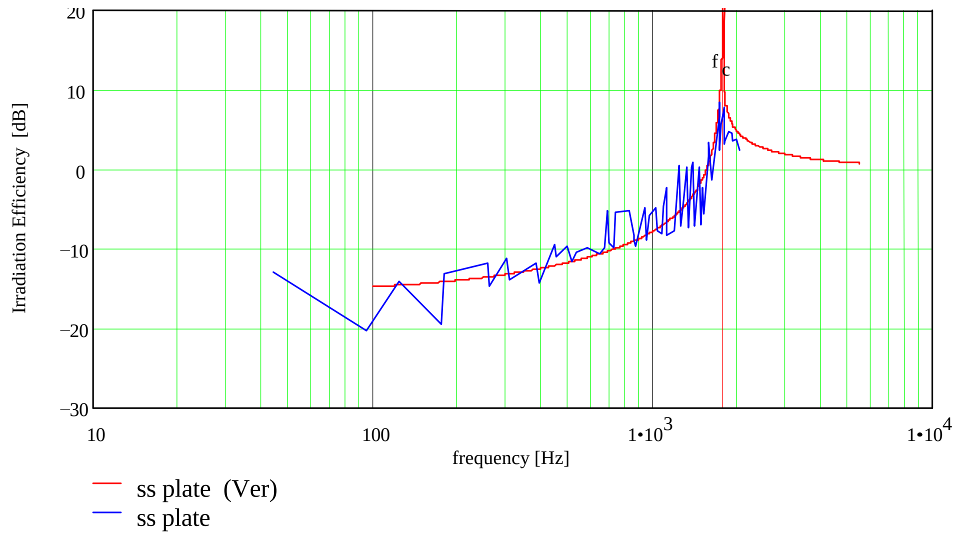

Figure 3: Irradiation efficiency calculated by the program and the simplification suggested by Ver, for the resonance frequencies, Plate I.

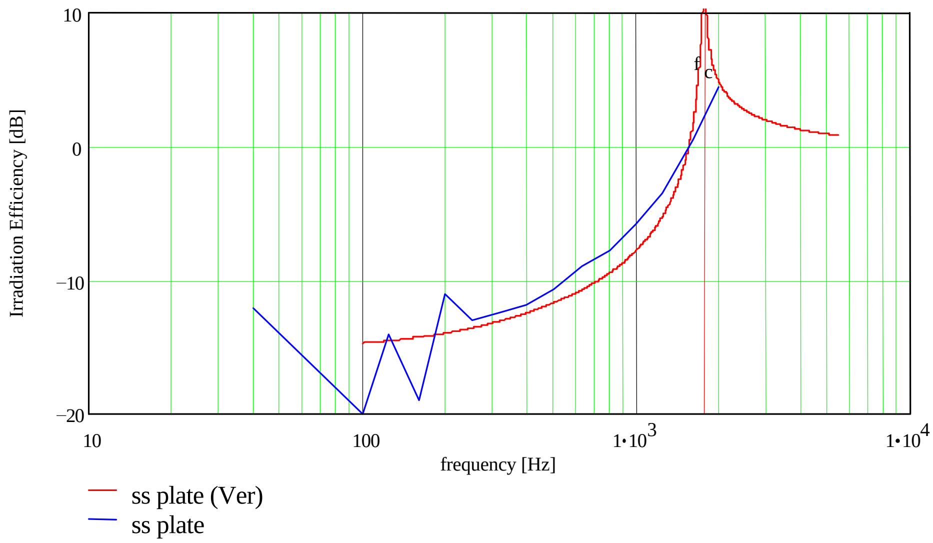

Figure 4: Irradiation efficiency calculated by the program and the simplification suggested by Ver, for 1/3 octave frequency bands, Plate I.

To verify the influence of the beam modes in the efficiency, a preliminary analysis was made studying a simply supported plate with a beam attached to its center. The beam dimensions were changed and two types of elements were used to model the beam: a shell element and a beam element. Three beam configurations were studied and the dimensions can be seen in Table (1).

Table 1: Beam dimensions.

Configuration	h_a [cm]	b_f [cm]	h_t [cm]
1	5	2,5	0,7
2	10	5	0,7
3	20	10	0,7

The beam element used was Beam44, an axial element with two nodes and bending, torsion and traction/compression capabilities. It has six degrees of freedom per node: displacements and rotations in the x, y e z directions. This element has an offset feature, where the gravity center can be dislocated in relation to the plate nodes, and the beam is situated below the plate, as seen in Fig. (5)

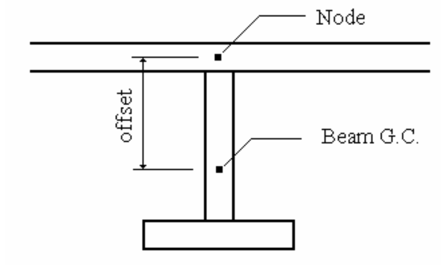

Figure 5: Offset for beam element.

The first comparison is made with a relatively small beam. Fig. (6) shows the efficiency for this configuration. It can be seen that there are no great differences between the two results, implying that for small beams the beam element can be used. Increasing the size of the beam, as in configuration 2, the difference between the efficiency results is more noticeable. The shell element shows more modes, which are not present by modeling the beam with beam elements. This is showed in Fig. (7).

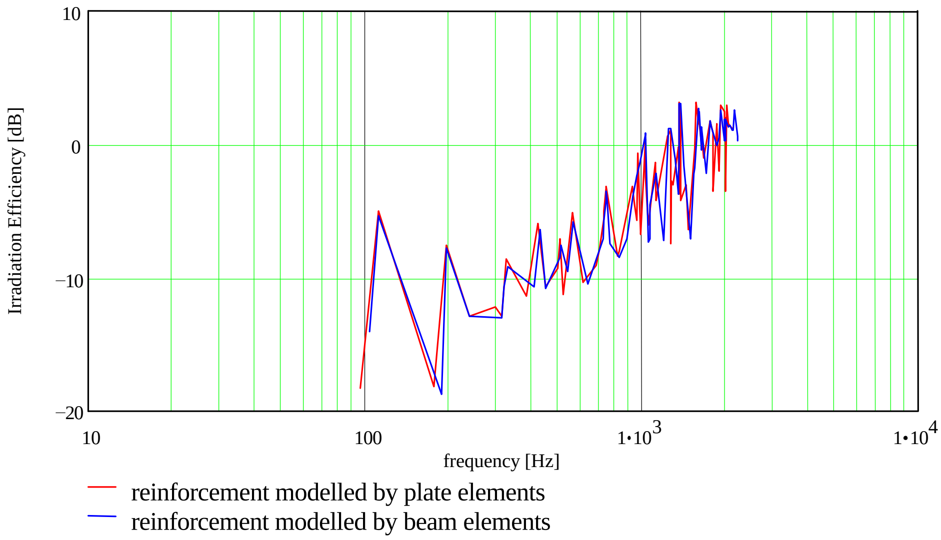

Figure 6: Irradiation efficiency for an ss plate with a central inverted T beam (configuration 1).

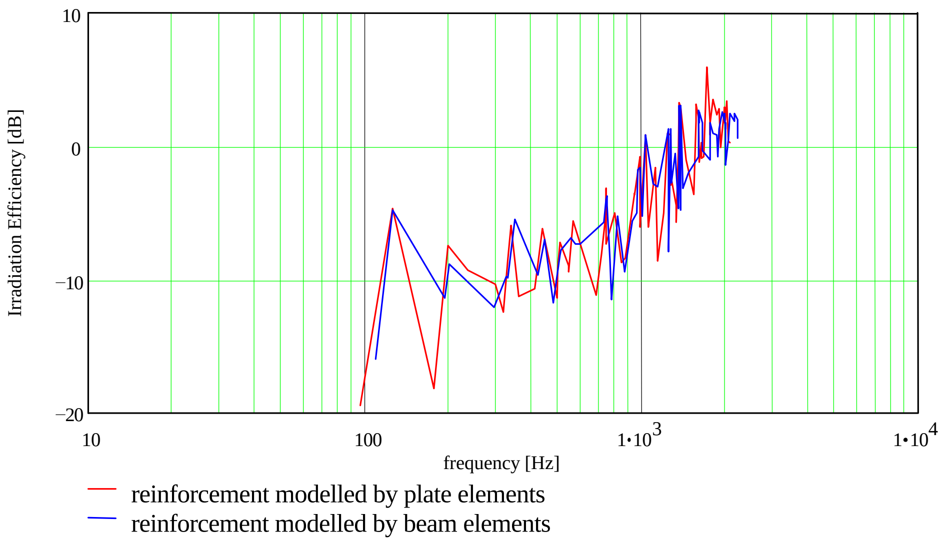

Figure 7: Irradiation efficiency for an ss plate with a central inverted T beam (configuration 2).

For a bigger beam, as in configuration 3, the efficiency results are very different. Figure (8) shows a constant tendency in the efficiency calculated with beam elements, especially in lower frequencies. The shell element seems to represent the real influence of the beam on the velocity of the plate, by considering the own modes of the beam, justifying its use.

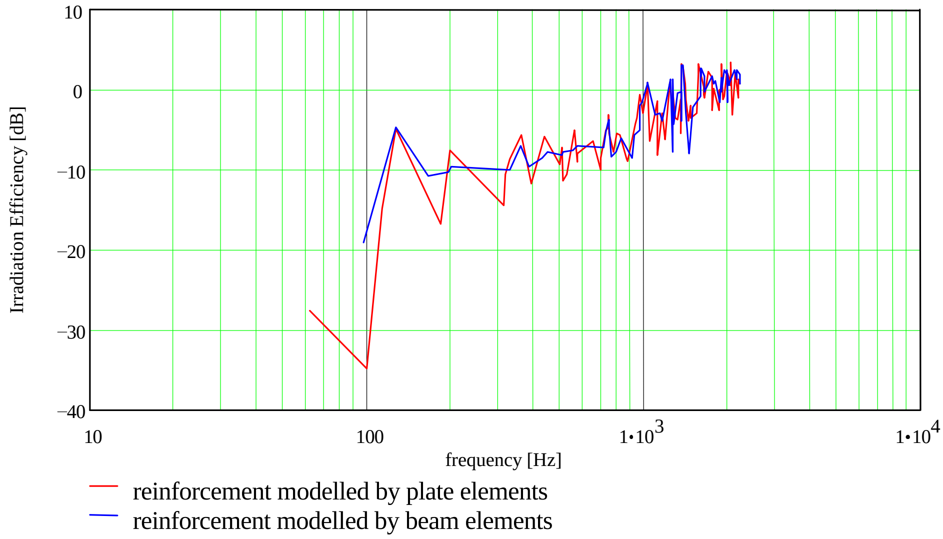

Figure 8: Irradiation efficiency for an ss plate with a central inverted T beam (configuration 3).

Five types of plate/beam configurations were analyzed. The first case was the simply supported plate with no reinforcements, which will serve as a reference. The next one was a plate with a central support, followed by a plate with a central beam. Next, a plate with two beams, crossed and in parallel, was analyzed. A 7 mm thick steel plate has a coincidence frequency of 1817 Hz. All analyses were made to around 2000 Hz, with the extraction of 65 to 80 modes. The meshes contained about 4000 to 5000 elements and the computation time was around half an hour, in a computer Athlon 650 MHz, 500 Mbytes ram memory and hard drive of 26 Gbytes, Win2000 operational system.

The first comparison is between the simply supported plate and plate with a central support. As expected, the support increases the number of non-cancelled areas in the plate, and the irradiation efficiency increases, in all spectra. Fig. 9 shows these results.

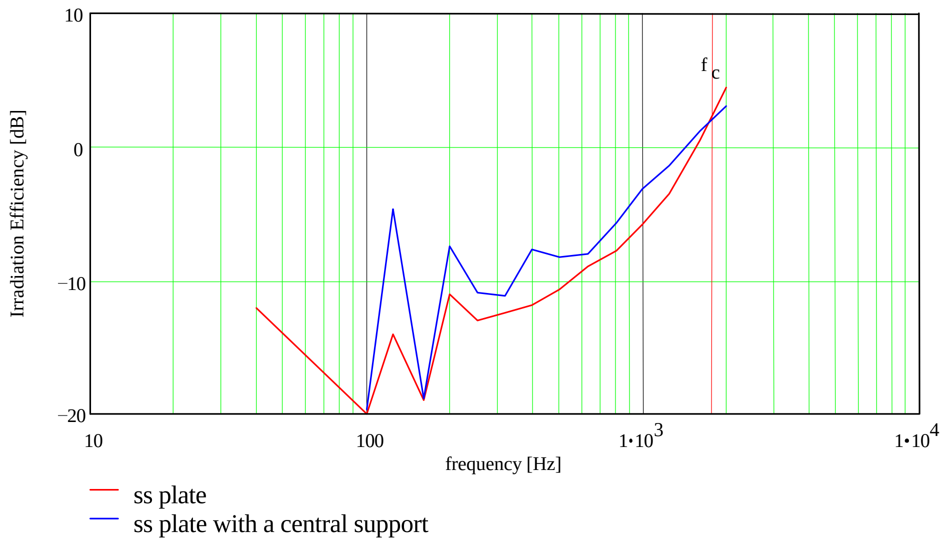

Figure 9: Comparison between the efficiency for a simply supported plate (Plate I) and a plate with a central support (Plate II), in 1/3 octave frequency bands.

The attachment of a central beam in the plate has an expected effect of increasing the efficiency, as happened with the support. Figure (10) shows that the irradiation efficiency has indeed increased. The values depend on the mode distribution of the plate.

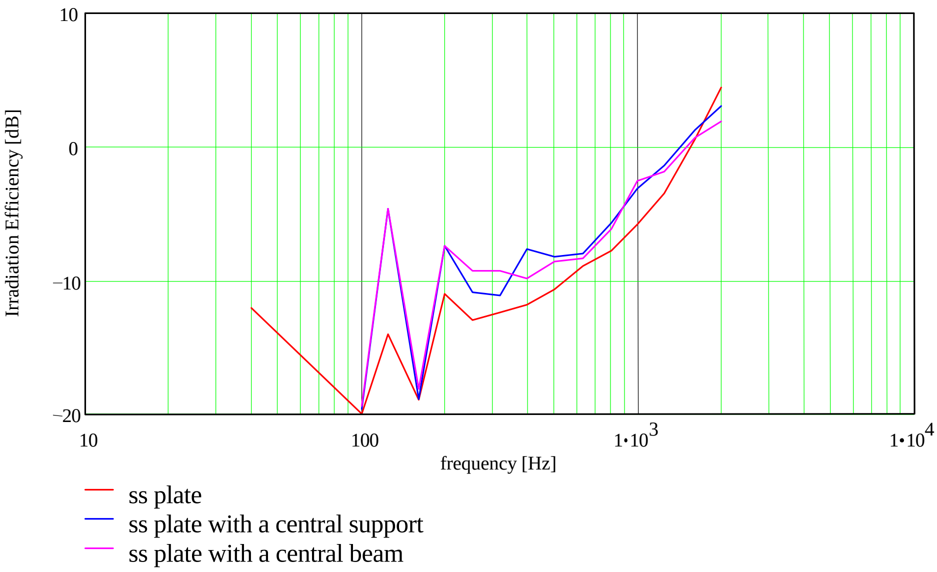

Figure 10: Comparison between the efficiency for a simply supported plate (Plate I), a plate with a central support (Plate II), and a plate with a central beam (Plate III), in 1/3 octave frequency bands.

In the sequence, a plate with two crossed beams was analyzed. The beams are disposed in the center of each side of the plate. These beams modify the velocity field, creating more non-cancelled areas than the previous configurations, increasing even more the efficiency. These can be concluded from Fig. (11). There is a difference between the resonance frequencies, and less efficient modes, became more efficient with the inclusion of the crossed beams.

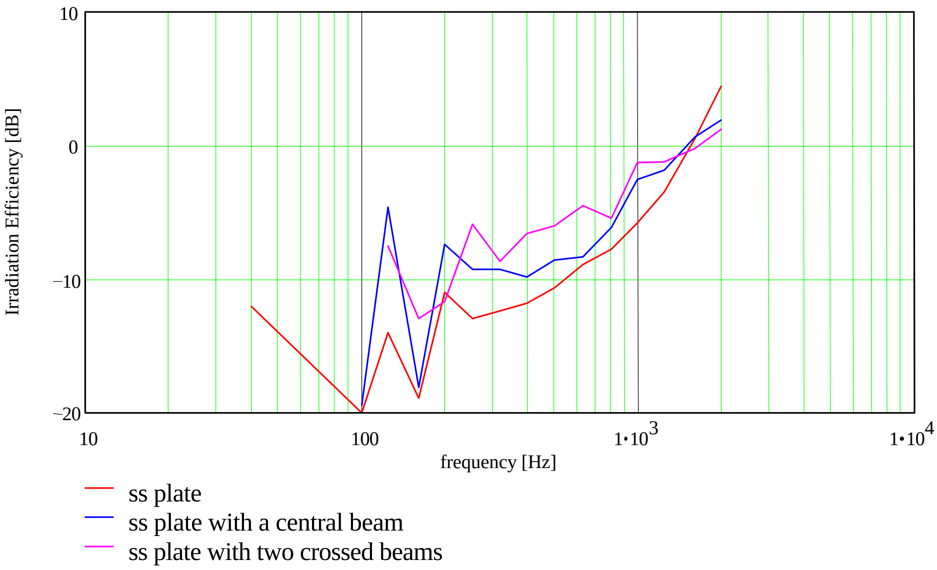

Figure 11: Comparison between the efficiency for a simply supported plate (Plate I), a plate with a central beam (Plate III), and a plate with two crossed beams (Plate IV), in 1/3 octave frequency bands.

The next step was to change the direction of the two beams. Now they are parallel to the shorter side of the plate. The beams, equally spaced, increase the non-cancellation areas of the plate, especially around the beams, and the consequence is an increase in the efficiency, compared to the case with one reinforcement. This can be seen in Fig. (12).

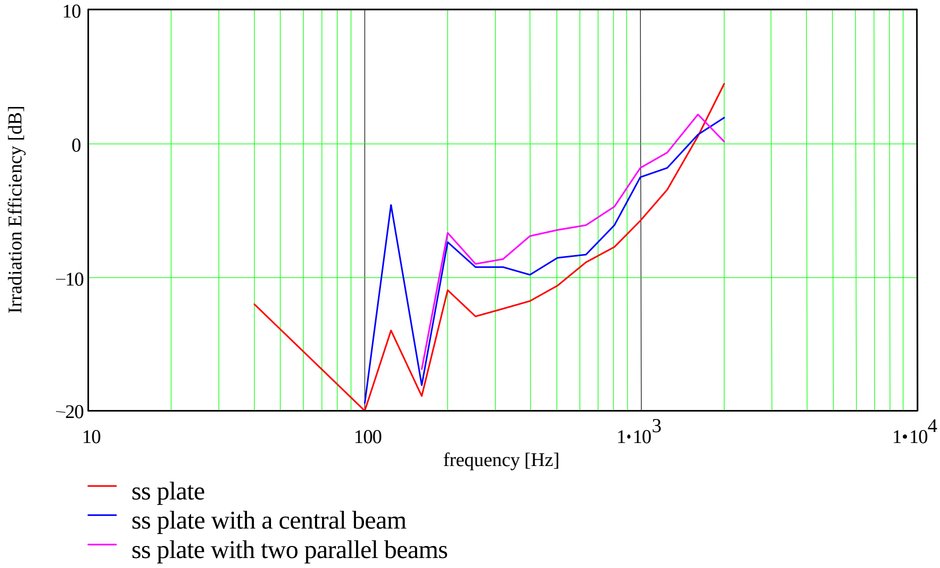

Figure 12: Comparison between the efficiency for a simply supported plate (Plate I), a plate with a central beam (Plate III), and a plate with two parallel beams (Plate V), in 1/3 octave frequency bands.

Figure (13) shows the results for the configurations with two reinforcements, crossed and in parallel. The plate with the crossed beams has higher irradiation efficiency than the plate with parallel beams. This happens because the crossed beams highly modify the mode distribution of the plate, creating more non-cancelled areas

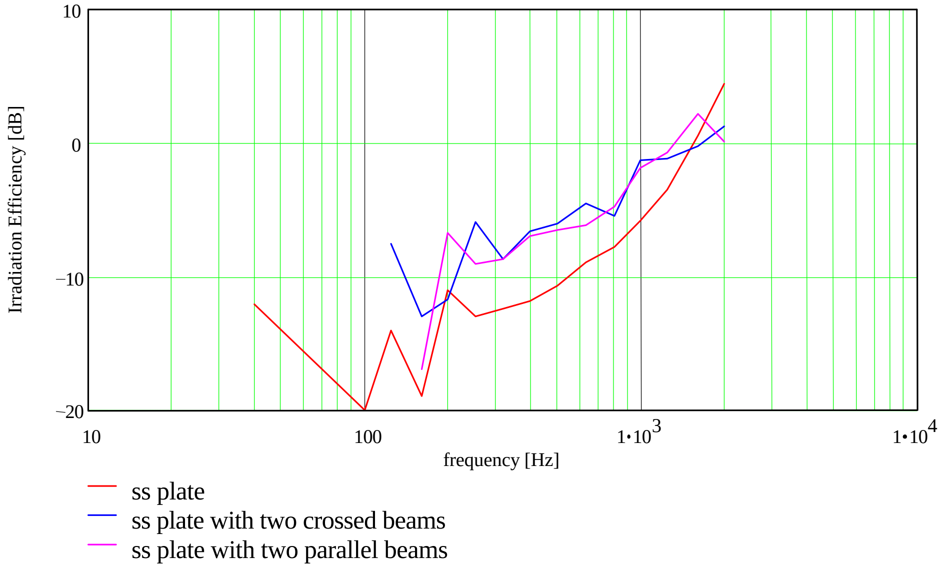

Figure 13: Comparison between the efficiency for a simply supported plate (Plate I), a plate with two crossed beams (Plate IV), and a plate with two parallel beams (Plate V), in 1/3 octave frequency bands.

5. Conclusions

A technique, based in the FFT, was implemented for the calculation of the irradiation efficiency of beam-reinforced plates. By modeling the beams with shell elements, the modes of the beam are accounted for in the velocity distribution of the plate and more precise results are obtained.

The FFT reduces significantly the computation time of Rayleigh Integral solution and the Finite Element Method is a necessary tool to obtain the efficiency of reinforced plates, since there are no analytical methods for calculating the plate's velocity.

The choice of the beam element plays an important role in the irradiation efficiency characterization of beam reinforced plates. As seen in the study, for typically offshore beams the chosen element must be a plate element, since this kind of element can represent the real effect of the beam (own modes) in the plate velocity distribution.

The type and distribution of the plates' modes affect the irradiation efficiency. The beam presence increases the efficiency, due to a higher number of non-cancelled areas on the plate, and this increase will be bigger as more disturbed the velocity field is by the beam. Some plate/beam configurations were studied, from a simple supported plate with no reinforcements to a plate with two reinforcements, crossed and in parallel. Every time a restriction was imposed in the movement of the plate, i.e., by attaching beams to the plate, the irradiation efficiency increases. Of the five configurations studied, the one with higher efficiency was that with two beams, crosswise, due to the higher number of non-cancelled areas in the plate.

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