MULTIOBJECTIVE SOLUTIONS FOR PRACTICAL ENGINEERING TRUSSES SYSTEMS

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Abstract. *This paper address to the task of obtaining designs in which several objective functions are simultaneouosly considered. This is a very attractive issue in practical engineering design. However, due to the commonly conflicting nature of the objectives involved in such problems, trade offs or compromise solutions are obtained in the multiobjective optimization context. Here these solutions will be obtained using the Pareto concept. The traditional weighted sum (WS) method and a more robust technique named normal boundary intersection (NBI) will be investigated in detail. Other existing techiniques are briefly mentioned. Some examples are provided showing the solutions when considering the two investigated strategies.*

Keywords. *Multiobjective, optimization, trusses*

1. Introduction

In practical engineering, several tasks must be satisfied together in order to obtain an optimal design solution. Traditionally, the task of find an optimum design for these kind of problems are tackled as a single objective optimization problem with several constraints such that the combination of objective and constraints involves all desired goals in the design. This approach has the drawback of limiting the choices available to the designer, making the optimization process a rather difficult task. Alternatively, multiobjective optimization (MO) techniques allow a designer to model a specific problem considering a more realistic behavior, which commonly involves the satisfaction of several targets simultaneously. This approach is therefore nearer to the technical reality than the conventional scalar optimization.

MO techniques started to be applied in the structural framework in the middle 1970's. Since then the use of such tools in engineering design grew substantially over the following decades as can be observed in some existing literature surveys (Stadler, 1984; Coello, 1999). Optimal solutions in multiobjective studies commonly refers to the possible form of modification of a design, which best satisfy the involved objectives simultaneously. In this way, trade-off or compromised solution should be investigated in which the response characteristics would improve when compared to the initial design. A widely used approach, which is adopted in the multicriteria optimization task, is based on the so-called Pareto optimal concept. Two different methods based on such concept are here considered. They are the WS and the NBI methods. The NBI procedure proves to be the best of finding trade-offs among the competing objectives.

The present work highlights the importance of multiobjective structural optimization and overviews the basic involved concepts and the most relevant work carried in this area. The application is address here for two truss design problems that are commonly used in literature. The objective functions involved can be related to stability conditions, stress conditions and free vibrations conditions. We also highlight the importance of investigation of alternatives procedures to calculate the objective functions and constraints for large truss systems such that a MO system results in a very computational effective tool.

2. Optimization procedure

In this work, optimal designs fulfilling several simultaneous tasks are obtained through the use of an automatically sizing optimization (SO) procedure. The integrated system incorporates several tools such as: Geometry and discretization definition, finite element (FE) analysis, sensitivity analysis and a module to link the whole procedure with MO strategies and mathematical programming algorithm. Here the FE module can perform statics, free vibration and linear buckling analysis. Direct and adjoint sensitivities calculations are implemented for all the analysis types mentioned above. The sequential quadratic programming algorithm (SQP) (Powell, 1978) is used as optimizer. The details of each of such aspect can be found elsewhere (Afonso, 1995; Macedo, 2002). Here we will explore the basic concepts used in MO and present some details related to the investigated strategies.

2.1. Mathematical definition

The MO problem may be written as

Minimize:

$$\mathbf{F(s)} = \{f_1, f_2, \ldots, f_{nobj}\}$$

$$\begin{array}{ll} \text{subject} & \\ h_k(\mathbf{s}) = 0 & k = 1, \ldots, ne \\ g_i(\mathbf{s}) \leq 0 & i = 1, \ldots, ni \\ s_{lj} \leq s_j \leq s_{uj} & j = 1, \ldots, ndv \end{array} \tag{1}$$

In which **s** is the design variable vector, **F**(**s**) is the set of objective functions, which is to be minimized, or maximized, *nobj* is the number of objective functions, g_i (**s**) is an inequality constraint, h_i(**s**) is an equality constraint and s_{lk}, s_{uk} are respectively the lower and upper bounds on a typical design variable.

3. Pareto optimum concept

In this section we present the main features of the Pareto optimality. The detailed discussions about this concept can be found elsewhere (Hwauang et all, 1980; Steuer, 1985; Eschenauer et all, 1990 and Hernández, 1994).

A point $\mathbf{s}^p$ in the feasible design is Pareto optimal if for every **s** either

a) $f_k(\mathbf{s}) \leq f_k(\mathbf{s}^P) \quad k = 1, \ldots, nobj$

Or there is at least one objective function *k* such that

b) $f_k(\mathbf{s}) < f_k(\mathbf{s}^P)$.

The above equations means that $\mathbf{s}^p$ is a Pareto optimal if there exists no vector **s** which would decrease some objecttive function value without causing a simultaneous increase in at least one objective function. In general the Pareto optimum does not find a unique solution, but rather a set of solutions called non-dominated solutions. These solutions can be used to construct a point-wise approximation to the Pareto curve or surface. There are several techniques to obtain the set of Pareto minima (Hwauang et all, 1980; Steuer, 1985; Eschenauer et all, 1990 and Hernández, 1994). In this subject, the challenging issues are related to obtain points in the concave regions of Pareto frontier (when this exist) and the ability to solve MO problems involving more than two objectives functions simultaneously. The WS method is the simplest and mostly used procedure. Other approaches have been presented in literature aiming to efficiently obtain the trade-offs solutions. Among them can be referred: the homotopy approach (Rakoswka and Haftka, 1991), the physical programming (Messac and Sundararaj, 2000) and the normal boundary intersection (NBI) technique (Das and Dennis, 1998). Currently, in literature, the later two strategies are pointed to have more success to obtain the Pareto curves. The NBI procedure and WS will be explored here.

3.1. WS method

This is the traditional approach considered in the MO framework. In this procedure, the original MO problem is converted into a single optimization problem through considerations of a substitute objective function which takes the following format:

$$F = \beta^T \frac{\mathbf{f}}{\mathbf{f}_0} = \sum_{k=1}^{nobj} \beta_k \frac{f_k}{f_{0k}} \tag{2}$$

in which f_{0k} is the *k* objective function evaluated in the initial design $\mathbf{s_o}$ and the elements β_k are the weighting coefficients. They represent the relative importance of each objective and are normalized according to:

$$\sum_{k=1}^{nobj} \beta_k = 1 \quad . \tag{3}$$

The Pareto optimum points are obtained solving many scalarizations of the MO problem for various β. This technique presents the drawback that an even spread of weights rarely produce an even spread of points on the Pareto curve/surface. In certain cases, the difficulty to find Pareto minima points increases and the designer cannot have an estimation of the shape of the trade-off curve/surface. Different order of polynomials can also be applied for the weighted factors as an alternative to improve the standard form of this scheme.

3.2. NBI method

This procedure is based on the so called Das parameterization of the Pareto curve and produce an even spread Pareto points distribution. This property turns out this method very adequate for obtaining the trade-off solutions among the various conflicting objectives. The NBI procedure is here briefly described. The details of such scheme can be found elsewhere (Das and Dennis, 1997 and Das and Dennis, 1998).

3.2.1. Basic definitions

The utopia point or shadow minima **F*** is defined as

$$\mathbf{F}^* = \{f_1^*, f_2^*, \ldots, f_{nobj}^*\} \tag{4}$$

in which each component f_i^* represents an individual local minima. Let $\mathbf{s}_i^*$ be the respective minimizer of $f_i(\mathbf{s})$. The pay *off* matrix is defined as

$$\Phi(i,j) = \left(\mathbf{F}_i^* - \mathbf{F}^*\right)_j, \; i = 1, \ldots, nobj; \quad j = 1, \ldots, nobj \tag{5}$$

in which

$$\mathbf{F}(\mathbf{s})_i^* = \{f_1(\mathbf{s}_i^*), f_2(\mathbf{s}_i^*), \ldots, f_{nobj}(\mathbf{s}_i^*)\} \quad . \tag{6}$$

Finally the set of points $\Re^{\mathrm{nobj}}$ that are convex combinations of Φ, i.e.

$$\left\{\Phi\beta : \beta \in \Re^{nobj}, \sum_{i=1}^{nobj} \beta_i = 1, \beta_i \geq 0\right\} \tag{7}$$

is referred to as the *Convex Hull of Individual Minima (CHIM).* In addition, it is defined $\Im$ to be the set of feasible objective vectors**.** The boundary of $\Im$ is denoted $\partial\Im$.

3.2.2 Generation of Pareto points

The efficient points in $\Im$, in most of the cases the Pareto points, are found by the intersection between the boundary $\partial\Im$ and the normal pointing towards emanating from and point in the CHIM. Denoting **n** the unit normal to the CHIM, the set of points defined by $\mathbf{p} = \Phi\beta + t\mathbf{n}$ lies on that normal. Then the intersection point between **p** and $\partial\Im$ closest to the origin is the solution of the following sub problem:

$$\max_{\mathbf{x},t} \; t \tag{8}$$

$$\text{subject to:} \begin{cases} \Phi\beta + t\hat{\mathbf{n}} = \mathbf{F} - \mathbf{F}^* \\ h(\mathbf{s}) = 0 \\ g(\mathbf{s}) \leq 0 \\ s_l \leq s \leq s_u \end{cases}$$

The sub problem above is called the NBI sub problem. The parameters β provides an alternate parameterization of the Pareto set, which is called Das parameterization (Das and Dennis, 1998). In this procedure an even spread of β is used. This leads to an evenly distribution in the points Φβ on the CHIM. The normals emanating from these evenly spacedpoints intersect the boundary of the set of attained vectors, containing the Pareto optimal points. This entire process forces the arcs joining two consecutive Pareto points to have equal projections on the CHIM, hence the points obtained are uniformly spread in this sense. The whole concept extends easily for more than two objectives.

4. Examples

4.1. Ten-bar truss

To illustrate the capabilities of the procedure presented we will considered first the ten-bar structure shown in Fig. (1). The material properties are Elastic Modulus E = $2{,}0684.10^{11}$ N/m^2, material density ρ = $2{,}714.10^4$ N/m^3. The geometry data is L = 9.144m and area A_0 = 0,127m^2. The structure is submitted to a vertical load P = $4{,}4482.10^5$ N at nodes 4 and 6 as indicated in the figure. The cross-sectional area of each bar is considered as a design variable giving a total of ten variables. The lower and the upper limit imposed are 0.00254m and 0.254m, respectively.

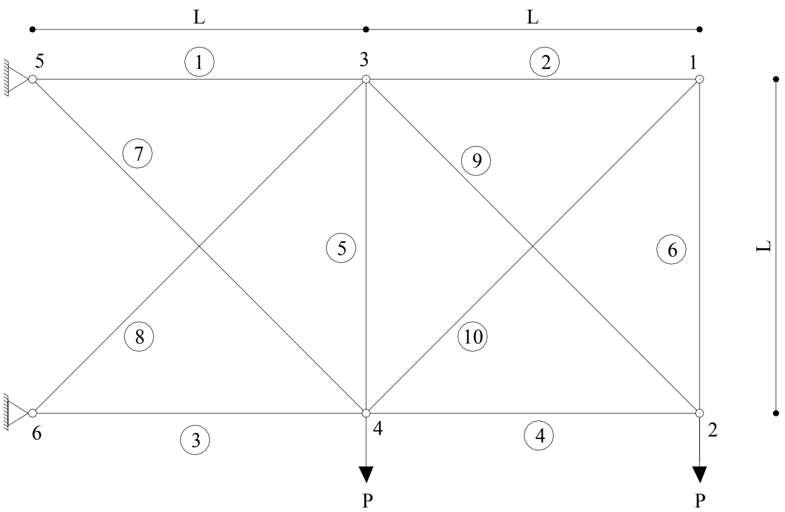

Figure 1. Ten bar truss example --- geometry and applied loads.

Apart of the limits on the design variables, two different sets of constrains are here considered:

constraint set 1: Stress on the ten bars. The allowable stress value is σ_{allow} = 1.7237. 10^8 N/m^2 (both tension and compression) to all bars except bar number nine in which σ_{allow} = 5.1711. 10^8 N/m^2 (tension and compression).

constraint set 2: Buckling Stress constraint in all ten bars is added to constraint set 1 described previously.

The objectives to be considered for optimization are: minimization of the total weight of the structure, maximization of the fundamental frequency and minimization of the vertical displacement at node 2 of the truss. The single optimization results found are the same reported in literature (Arora,1989). For MO solutions Both NBI and WS Schemes are considered. For both constraints set above described, the MO solutions are conduct in the following sequence:

(a) All three objectives combined together;
(b) displacement and weight minimization;
(c) frequency maximization and weight minimization and
(d) frequency maximization and displacement minimization.

The MO problem (a) is optimized for both constraints sets 1 and 2. For this case an uniform step size δ = 0.05 is considered to generate the parameters β from the NBI and the WS schemes. As a consequence 231 NBI and WS sub problems are solved. The NBI solution for this case, gives 231 and 223 distinct points on the Pareto surface when using constraints set 1 and constraint set 2 respectively. However, the number of distinct points found using WS is much less when compared to the NBI points generated. Fig. (2) plots all the Pareto points obtained using the NBI scheme under the different sets of constraints.

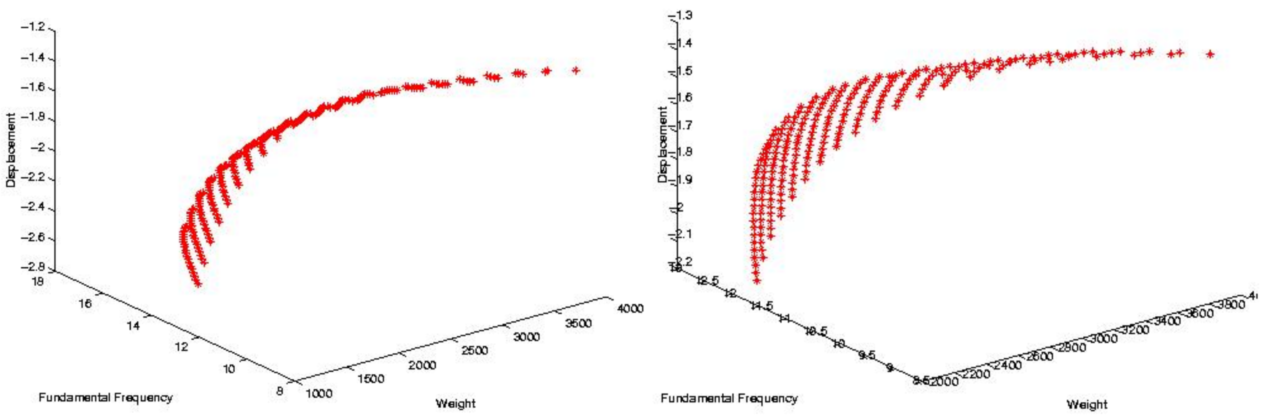

Figure 2. Ten bar truss example NBI Pareto surface for MO problem (a): (a) constraint set 1 and (b) constraint set 2.

MO problems (b) to (d) were solved for imposed constraints set 2. The MO solutions were conducted for 21 uniformly distributed parameters β. Figures (3) to (5) present the Pareto curves obtained for these problems. In each of these problems 21 NBI sub problems are solved with success. As a consequence, 21 distinct Pareto points are obtained. Moreover, it is observed that in all situations, the NBI solution gives a very smooth trade-off curve between the investigated objectives. The above-mentioned figures also show the comparative advantage obtained by the NBI uniformly spread points over the WS non-uniformly distributed points.

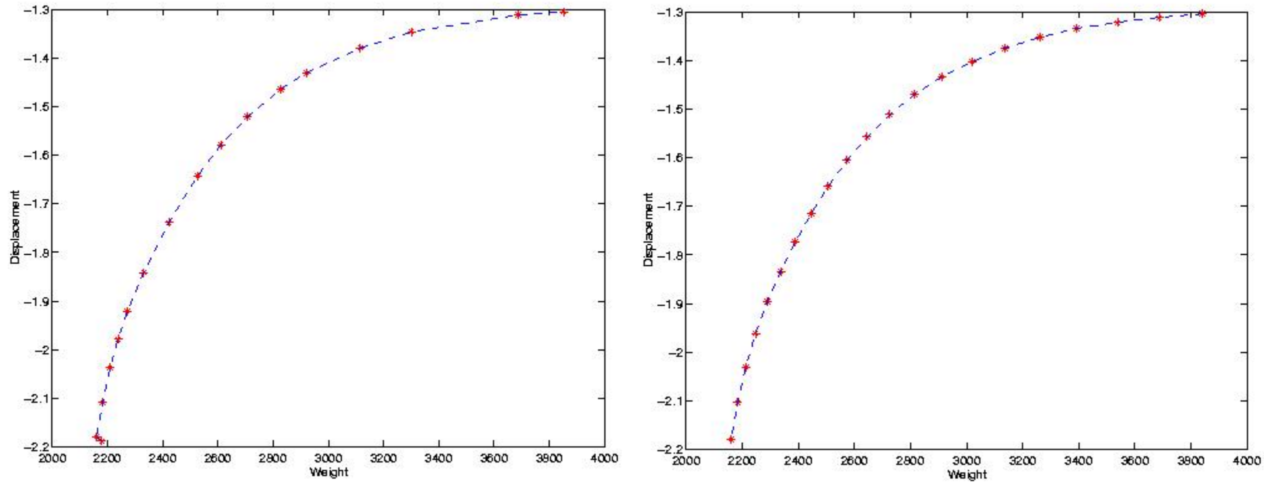

Figure 3. Ten bar truss example---Pareto surface for MO problem (b): (a) WS method and (b) NBI method.

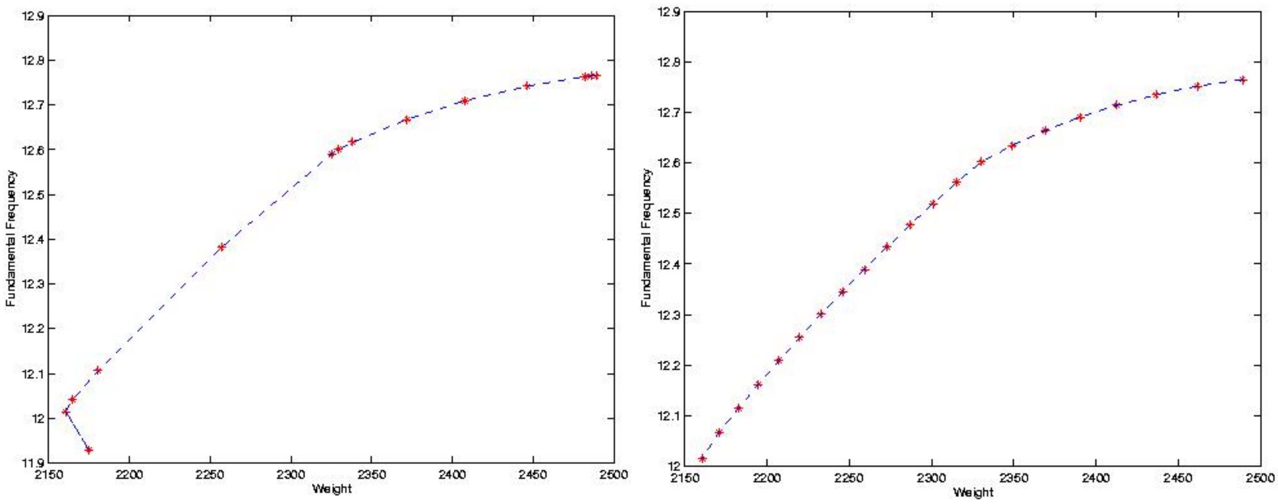

Figure 4. Ten bar truss example---Pareto surface for MO problem (c): (a) WS method and (b) NBI method.

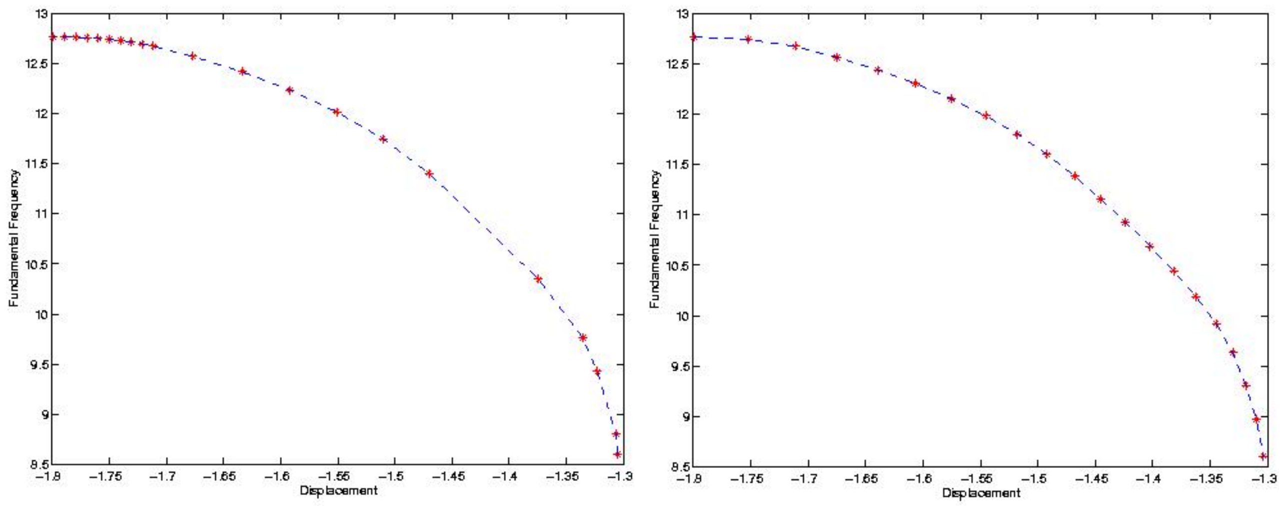

Figure 5. Ten bar truss example---Pareto surface for MO problem (d): (a) WS method and (b) NBI method.

4.1 Two hundred-bar truss

As a more realistic problem we will consider the structure indicated in Fig. 6 (Al-Khamis, 1996). The structure is subjected to loads $P_h = 4{,}4482.10^3$ N acting in positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71 and loads $P_v = 4{,}4482.10^4$ N acting in negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 24, 28, 71, 72, 73, 74 and 75. The material properties considered are: Elastic Modulus E = $2{,}0684.10^{11}$ N/m^2, material density ρ=7,838. 10^3 N/m^3. The design variables are taken to be the cross sectional areas of the two hundred member, with a lower limit of $3{,}226.10^{-3}$ m and an upper limit of $2{,}258.10^{-2}$ m and the initial values of $6{,}452.10^{-3}$ m. A total of 29 design variables are specified and the link relations to the 200 elements of the truss are indicated in Tab. (1). Stress constraints on the members are specified resulting a total of 400 inequality constraints for this problem. The allowable stress value is $\sigma_{allow} = 6{,}895.10^7$ N/m^2 (both tension and compression). In this particular example three objectives will be considered: minimize total volume, minimize compliance and maximize the fundamental frequency. The MO solutions are conduct in the following sequence:

(a) All three objectives combined together;
(b) compliance and volume minimization;
(c) frequency maximization and volume minimization and
(d) frequency maximization and compliance minimization.

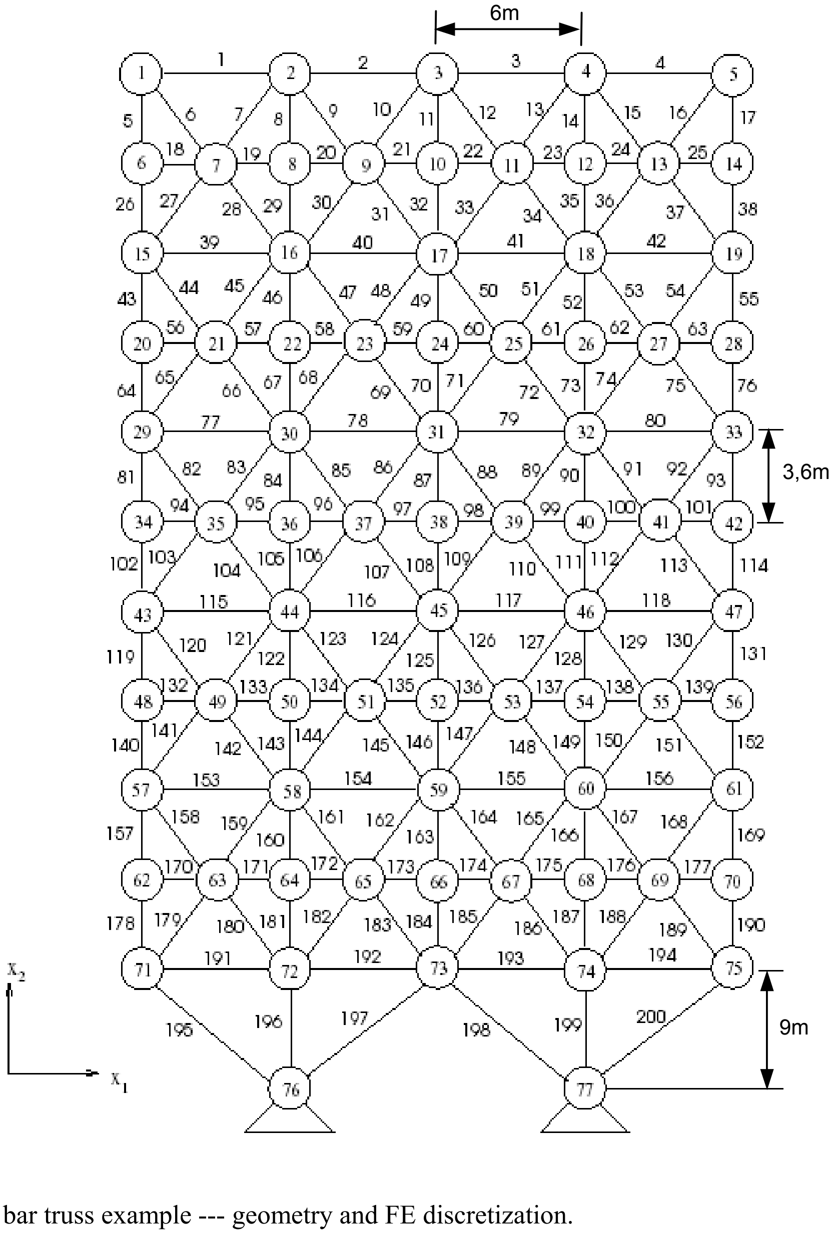

Figure 6. Two-hundred bar truss example --- geometry and FE discretization.

Table 1. Design variables link relations.

dv	Element number	dv	Element number
1	1,2,3,4	16	82,83,85,86,88,89,91,92,103,104,106,107,109,110, 112,113
2	5,8,11,14,17	17	115,116,117,118
3	19,20,21,22,23,24	18	119,122,125,128,131
4	18,25,56,63,94,101,132,139,170,177	19	133,134,135,136,137,138
5	26,29,32,35,38	20	140,143,146,149,152
6	6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37	21	120,121,123,124,126,127,129,130,141,142,144,145, 147,148,150,151
7	39,40,41,42	22	153,154,155,156
8	43,46,49,52,55	23	157,160,163,166,169
9	57,58,59,60,61,62	24	171,172,173,174,175,176
10	64,67,70,73,76	25	178,181,184,187,190
11	44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75	26	158,159,161,162,164,165,167,168,179,180,182,183, 185,186,188,189
12	77,78,79,80	27	191,192,193,194
13	81,84,87,90,93	28	195,197,198,200
14	95,96,97,98,99,100	29	196,199
15	102,105,108,111,114	---	---

For the three objective case, (MO problem (a)), an uniform step size $\delta = 0.25$ is considered to generate the parameters β from the NBI and the WS schemes. This consideration generates 15 sub problems to be solved. The results using WS and NBI methods are shown in Fig.7(a) and Fig.7(b) respectively. As expected, an even and nice distribution of points is found for the NBI scheme in contradiction to the WS scheme. For the later we can observe clustering of points instead of uniform distribution of points.

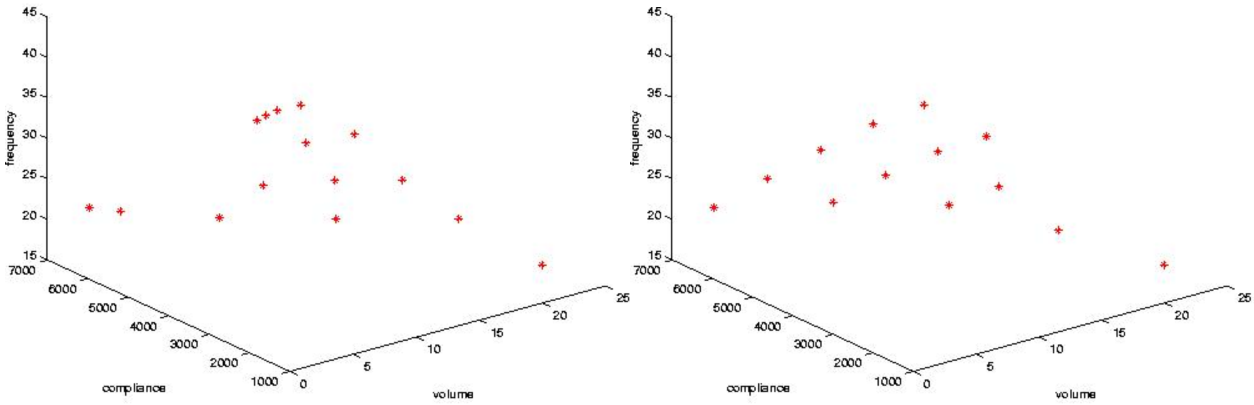

Figure 7.Two hundred-bar truss example---Pareto surface for MO problem (a): (a) WS method and (b) NBI method.

MO solutions for problems (b) to (d) were conducted for 21 uniformly distributed parameter settings. Figs. (8) to (10) present the results obtained for the schemes here considered. As can be observed, both WS and NBI methods manage to obtain Pareto curves with a sufficient number of points. However, the superiority of the NBI plots is easily observed. For the problems considering two objective functions, the combination: frequency maximization and compliance minimization was the most difficult task to solve due to the nonlinearities of the functions involved. Also there is one nonconvex region in the Pareto frontier as indicated in fig. 10(a). It is important to emphasize here that NBI technique is able to calculate such points in contradiction to the WS method, which assumes a convex combination between functions. In the remaining two objective functions combinations, such behavior was not observed, as the Pareto curves do not exhibit nonconvex parts.

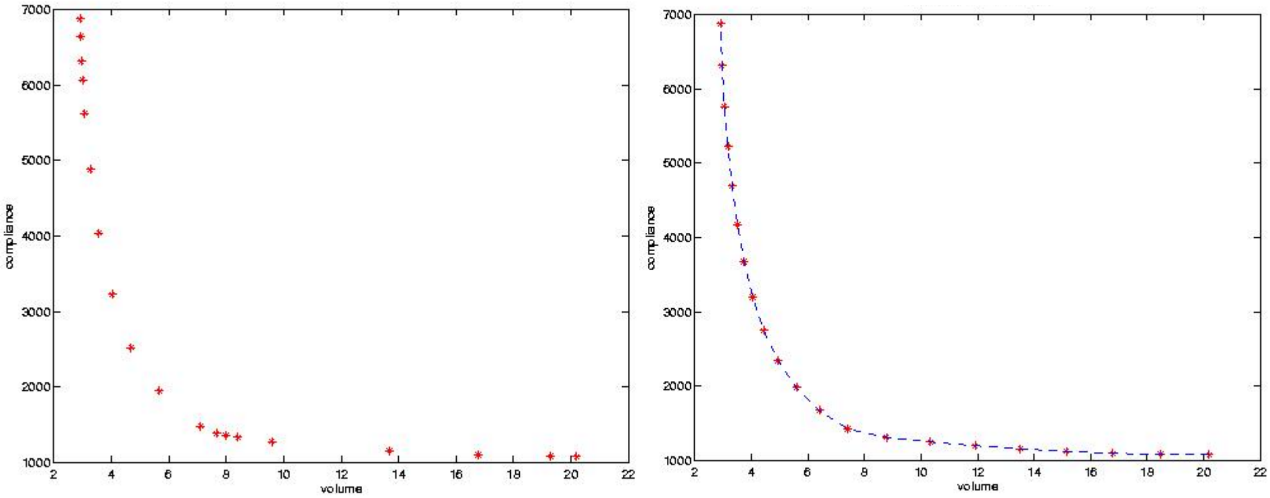

Figure 8. Two hundred-bar truss example---Pareto surface for MO problem (b): (a) WS method and (b) NBI method.

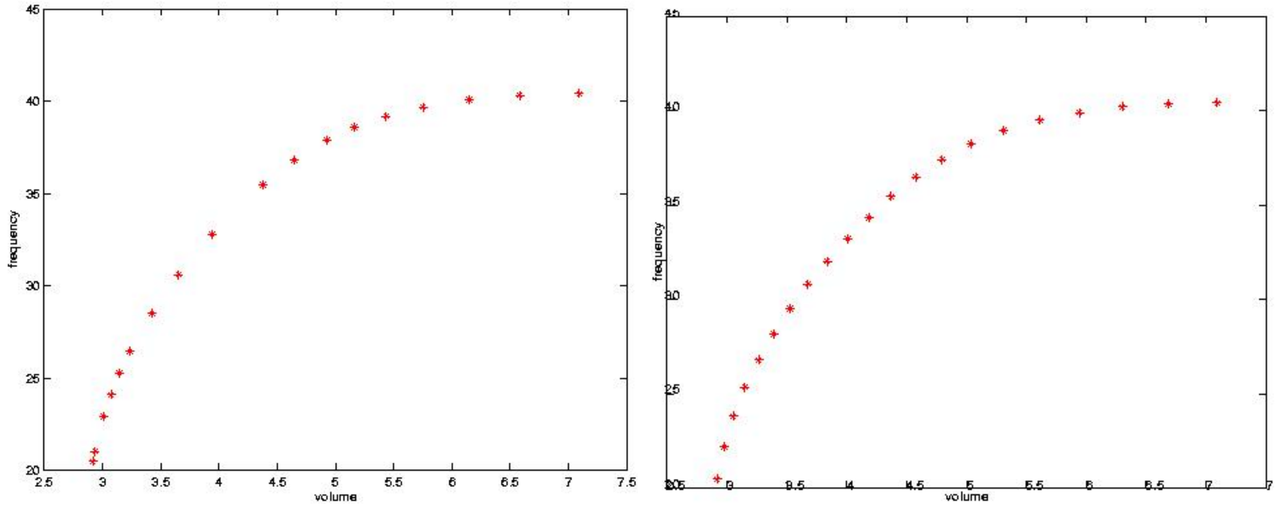

Figure 9. Two hundred-bar truss example---Pareto surface for MO problem (c): (a) WS method and (b) NBI method.

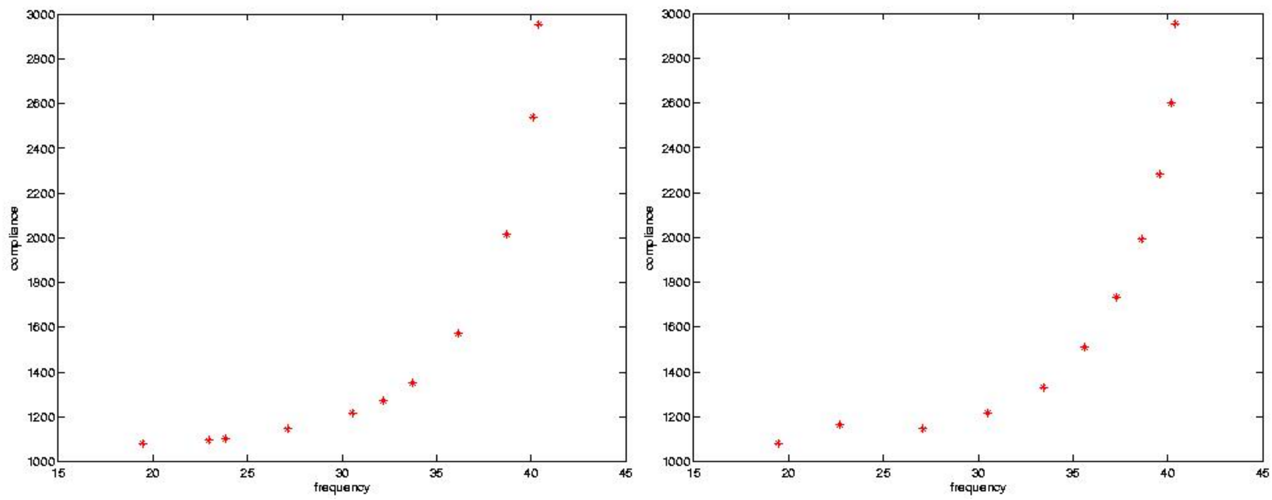

Figure 10. Two hundred-bar truss example---Pareto surface for MO problem (d): (a) WS method and (b) NBI method.

5. Conclusions

Multi criteria optimization was address here aiming to obtain a tool that can be used to obtain practical engineering designs, in which commonly several tasks need to be tackled simultaneously. Optimal Trusses designs were obtained under this framework. Some issues regarding to WS and NBI methods implemented here were discussed. The NBI procedure always generates a uniform spread of points representative of all parts of the Pareto frontier consequently, a better model of the trade-off curve/surface was obtained when using such scheme for MO solutions.

6. Further work

Real engineering problems commonly involves not only several tasks (objective functions) but also complex physics interactions and/or several design variables. To apply optimization techniques for such problems could be an issue as the computational cost required for multiple numerical simulations could be in certain cases prohibitive. One alternative to overcome such difficulties can be obtained by constructing some form of reliable and computationally effective approximations to the response solutions (or outputs) of the original (and costly) problem. In this context, it is currently under our investigation the reduced-basis output bound method (RBOBM) (Prud'homme et all, 2002). The purpose of such scheme is therefore to get high fidelity model information without the computational expense. The RBOBM is a Galerkin projection onto low order approximation spaces comprising solutions of the problem of interest at selected points in the parameter/design space. The numerical implementation of RBOBM consists of two stages: (1) the pre-processing or also called "off-line" stage in which the reduced basis and associated functions are computed at a prescribed set of points in parameter space; (2) the real time or "on-line" stage, in which the approximate output of interest (and their gradients) and corresponding rigorous error bounds are computed for any new parameter value of interest. The latter stage is very inexpensive, as it requires only the solution or evaluation of very small systems. Single and MO problems will be conducted in this framework.

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