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PARTICLE'S ORBITS AROUND THE EARTH ABOUT THE EFFECTS OF THE RADIATION PRESSURE AND OBLATENESS

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Abstract. The effect of the radiation pressure produces variations in eccentricity's particles with different oscillation periods for different particle's sizes. The long period variation presents a configuration that repeats with lower period to the orbital period of the Earth for smaller particles. When we included the planet's oblateness this oscillation period present changes. The combined effect of the radiation pressure and Earth's oblateness provokes variation of long period in the eccentricity increasing or reducing the period of the oscillation depending on the orbital area that this particle is.

Keywords: *radiation pressure, oblateness, long period variation, orbital region*

1. Introduction

Small particles that around of the Earth are subject to the effects of the gravitational and no disturbances. These disturbances provoke variations in the particle orbital evolution. If it onsiders the disturbance no gravitational of the radiation pressure this disturbance provokes variation in the particle eccentricity. Then, a particle initially in circular orbit would develop for an eccentric orbit offering a larger collision risk with a space equipment that it was orbit around of the Earth. Another disturbance is the gravitacional disturbance of Earth's oblateness that can affect the eccentricity of this particle when the radiation pressure is considered simultaneously. It is important to remind that the isolated effect of the Earth's oblateness would not provoke any alteration in the its eccentricity.

In this work we studied the dynamics of particles originary from space debris around the Earth subject to the no gravitational and gravitational disturbances of the radiation pressure force of the Earth's oblateness numerically. Our main goal is to study the evolution of its exccentricity to the circular orbits that it crosses. The initial considered datas were particles in circular orbits, particle density of 3.0 g/cm^3, particle sizes from 1 to 1,000 micrometers, that it corresponds approximately to the mass value in the range from 10^{-15} to 10^{-6} kg, and orbital radius in the range of 8,000 km to 44,000 km.

2. Dynamical System

Small particles, μm size particles, around the Earth are subjected to the effect of radiation pressure and Earth's oblateness. Theses disturbance provokes variations in the orbital evolution of these particles. The radiation pressure produces variations in its eccentricity, increasing the collision risk between these particles and a space equipment. Therefore, in this work we studied the evolution of the eccentricity of these particles across circular orbits of satellites around of the Earth (Fig.1). The considered initial conditions were particles in circular orbits, particle density of 3.0 g/cm^3, particle sizes from 1 to 1,000 micrometers and orbital radius in the range of 8,000 km to 44,000 km.

The radiation pressure and oblateness forces components, to circular case, in the planetocentric frame are given by:

$$F_x = -\frac{SA}{c} Q_{pr} \cos(n_{Sun} t) - GM_{\oplus} J_2 \frac{3}{2}\left(\frac{R_{\oplus}}{r}\right)^2 \frac{x_1}{r^3} \tag{1}$$

$$F_y = -\frac{SA}{c} Q_{pr} \sin(n_{Sun} t) - GM_{\oplus} J_2 \frac{3}{2}\left(\frac{R_{\oplus}}{r}\right)^2 \frac{x_2}{r^3} \tag{2}$$

where S is the solar flux., A is the particle cross section, c is the speed of light, Q_{pr} is the radiation pressure coefficient, n_{Sun} is the mean motion of the Sun around the Earth, G is the gravitational constant, $M_{\oplus}$, J_2 and $R_{\oplus}$ are mass, zonal coefficient and radius of the Earth, respectively, and $r = \sqrt{x_1^2 + x_2^2}$ is the particle's position vector. The first constant term of the above equations can be given by:

$$\frac{SA}{c} Q_{pr} = b\ F_{grSun} \tag{3}$$

where F_{grSun} is the Sun gravitational force and b is a parameter that depends on the partcles density and size (Burns et al., 1979).

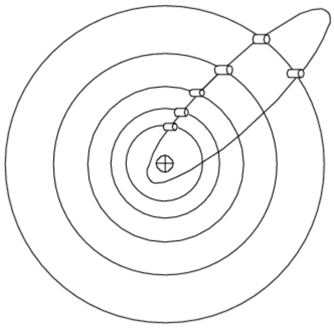

Figure 1 - Positions of hypothetical space vehicles in circular orbits around of the Earth and a particle crossing the orbits of these space vehicles.

3. Numeric Simulations

The numerical simulations of the particle's eccentricity around the Earth subject to combined effect of the radiation pressure and oblateness is presented in this section. Firstly, the particle being initially in circular geostationary orbit and after, in another orbital regions.

3.1. Geostationary particles – 42,164 km

Geostationary orbit is defined as being that it presents the orbital period around of the Earth equal to the rotation orbital period of the Earth (period ~1 day). The radiation pressure effect provokes variation in the particle eccentricity. In this orbital region there are the communication satellites. Then, the knowledge of the particle dynamics in this orbital region is necessary. In the Fig. (2 a-f) the eccentricity variations are presented for different particle sizes in the range from 1 to 100 μm. The importance of the eccentricity variation study is that the eccentricity is related to the orbital radius. Then, if the eccentricity suffers great variations the orbital radius will also suffer. Therefore, the width variation of the orbital radius can turn a dangerous region for the space equipments. In other words, these variation can provoke damages to these space equipments. The curve in red represents the curve with the effect of the radiation pressure (PR) and the curve in blue the combined effect of the oblateness and of the radiation pressure (J_2 + PR). The line in green delimits the region where upper of this the predominant disturbance is the drag atmospheric. This disturbance still was not considered until the moment. Then, the results above this line should be despised a priori. The integration period, in these figures, was of three orbital periods of the Earth. If it considers only the results until the line in green in these figures it can be observed that the effect of the combined oblateness to the radiation pressure for a same particle size doesn't provoke difference in the maximum value of the eccentricity when it is compared to the results where only the radiation pressure was considered. The eccentricity maximum value practically is the same in the two situations. The eccentricity variation for particles $\leq$ 5 μm (Fig. 3a-b), considering the same particle size, capable to put it in the region where the predominant disturbance is the drag atmospheric, continues occurring approximately with the same period for the two cases (with and without oblateness). In other words, the effect of long period doesn't present relevant differences among the found results only considering the radiation pressure and the combined effect. For particles > 5 μm, Fig. (3c-f), it is noticed that this variation of long period decreased in all of the cases considered for particle sizes particles separately.

3.2. Particles in other orbital regions

As it can be observed in the previous section the eccentricity presents variation of long period for particle sizes > 5 μm in geostationary orbit and combined effect of PR and J_2 for particle sizes $\leq$ 5 μm.

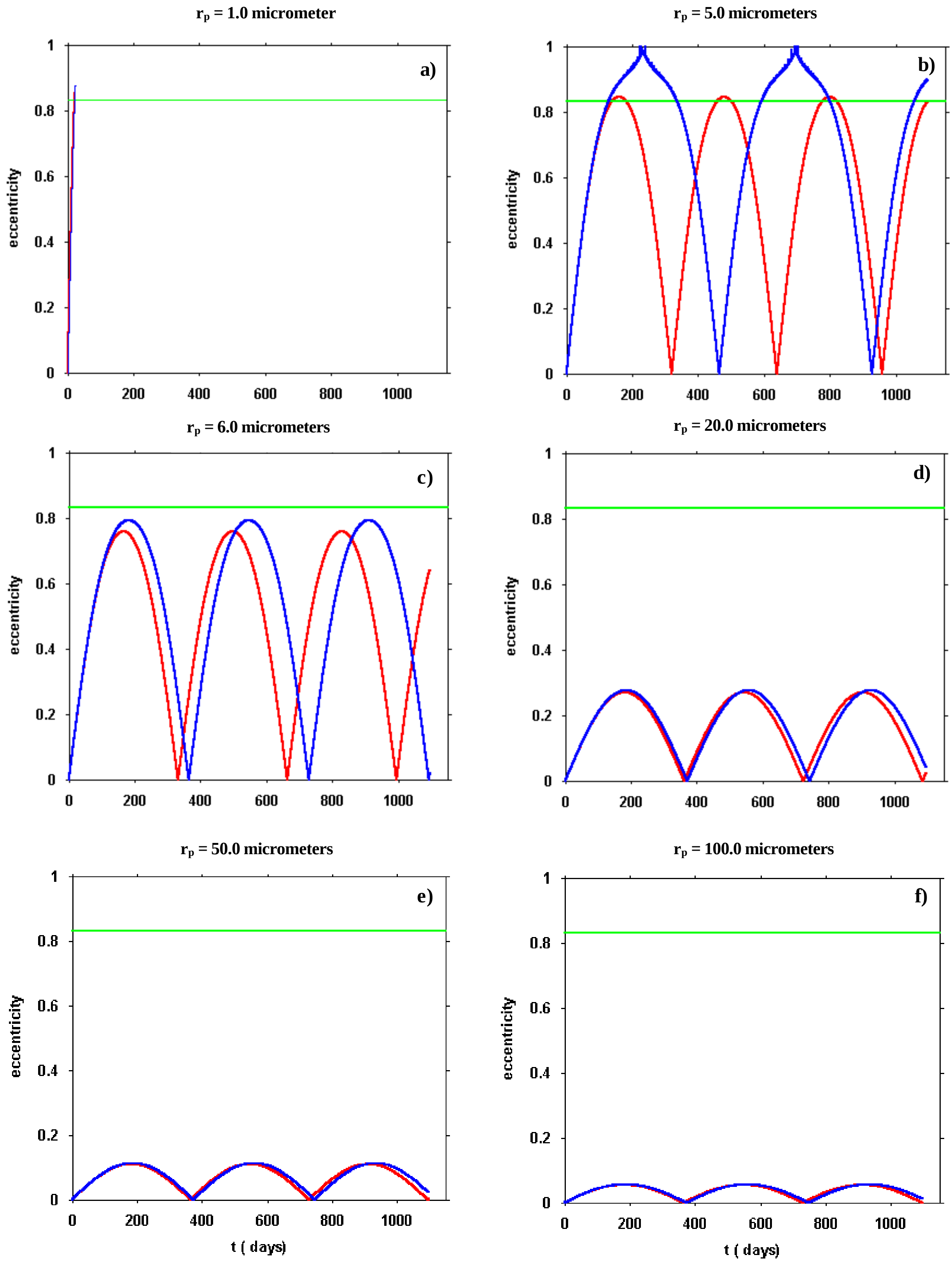

Figure 2a-f – Eccentricity variation for particles from 1 to 100 μm initially in geostationary orbit. The curve in red represents the effect of the radiation pressure (PR) and the curve in blue the combined effect of the radiation pressure and of the oblateness (PR + J_2). The line in green represents the limit where above this the predominant disturbance is the drag atmospheric.

This process performs as a natural mechanism of removal of these particles for this initial orbital region. Therefore, a study of these variations in function of different orbital region is relevant to complement these analyses. In this subsection the maximum eccentricities are presented for a particle of 20 μm in different initial orbital radius.

In the Fig. (3a-f) and (4a-f) the eccentricity variations are presented for particle of 20 μm in different orbital regions in the range between 44,000 and 8,000 km and they are subjected to the simultaneous disturbance of the radiation pressure and of the oblateness. In these figures the curve in red represents the curve with the effect of the radiation pressure (PR), the curve in blue the combined effect of the oblateness with the radiation pressure (PR + J_2), the curve in green with the effect of the Earth's oblateness and the line in yellow delimits the region in that the predominant effect is the atmospheric drag. This region is defined as being the region above the line yellow. In the Fig. (3a-b) it is observed that understood between 44,000 and 32,000 km the eccentricity maximum value when it considers orbital region in this range, for a same orbital region, it presents close values considering the case with and without the disturbance of the oblateness. These values are understood for orbital region of 44,000 km between $e_{max} \sim 0.28$, considering the disturbance of the radiation pressure, to $e_{max} \sim 0.281$, the combined effect of PR and J_2. In these figures it is possible to observe that the initial configuration doesn't repeat separately for the same period considering the radiation pressure and the simultaneous disturbance of the Earth's oblateness and of the radiation pressure. For the case in that the radiation pressure and the oblateness are considered together it is noticed that the period increased. Then, the simultaneous disturbance of the Earth's oblateness and of the radiation pressure provokes an effect of long period in the particle eccentricity variation that can be orbit around of the Earth.

Considering the understood orbital region between 28,000 km to 17,530 km, Fig. (3c-e), it is observed that , besides the variation of long period, they presents maximum values of different eccentricities only considering a same orbital region for the case of the disturbance of the radiation pressure and the case of the simultaneous disturbance of the radiation pressure and Earth's oblateness. For instance, for the orbital region of 17,530 km, Fig. (3e), it is observed that the maximum eccentricity considering only the radiation pressure is inferior to 0.20, while for the case of the combined effect of PR and J_2 it is very superior to this value.

For the orbital regions between 17,500 to 15,000 km, Figs. (3f and 4a-c), it is observed that the particle eccentricity variation is enough to put it in the region where the predominant effect is the atmospheric drag (no considering in this work stage). Then, for these orbital regions and particle sizes of 20 μm the combined effect of PR and J_2 can be considered a natural mechanism of removal.

In the Fig. (4d-f), orbital region understood between 12,000 km to 8,000 km, it is noticed that the combined effect of PR and J_2 reduce the maximum value of the eccentricity so that the acquired maximum eccentricity is not enough to put the particle in the region where the predominant disturbance is the atmospheric drag. Besides, the period of the long period variation of the eccentricity decreases. Then, particles in these orbital regions present a contrary behavior for the evolution of the eccentricity in comparison with the other orbital regions here considered.

For best to visualize the behavior of the maximum eccentricity for a particle of 20 μm subjects the effect of the radiation pressure and the combined effect of PR and J_2 in different altitudes was done the Fig. (5). In this figure the points in black represent the behavior of the particle eccentricity just subjects the radiation pressure and the points in red the simultaneous effect of the radiation pressure and of the Earth's oblateness. The case in that only the radiation pressure was considered presents an adjustment of polynomial curve of second degree for the maximum eccentricity. This curve adjustment is given by the equation $e_{máx} = -5.16512 \times 10^{-11} r^2 + 6.90934 \times 10^{-6} r + 6.878 \times 10^{-2}$. The case in that the oblateness and the radiation pressure disturbances were considered it is noticed a growing behavior until the orbital region of 17,500 km. The value of the maximum eccentricity for this growing behavior is inferior 0.8. After this orbital region the behavior of the eccentricity is decreasing.

The eccentricity variation for a particle of 1,000 μm subjects the combined effect of PR and J_2 in three different orbital regions being them 8,000, 25,000 and 42,164 km is presented in the Fig. (6). It observes that larger particles in the low orbit Earth region, altitude of approximately 8,000 km (curves in blue), the radiation pressure and the oblateness practically don't provoke eccentricity variation in comparison the other orbital regions. For the geostationary (curves in red) and intermediate (curves in green) orbit regions of the Earth it is observed that a variation exists in the period of the long period variation of the eccentricity. For the orbital region of 42,164 km the particle presents this period approximately equal to the orbital period of the Earth around of the Sun while for the orbital region of 20,000 km this period is larger than the orbital period of the Earth.

The variation width of the radial distance for a particle of 20 μm subjects the combined effect of PR and J_2 for the orbital regions in that the eccentricity variation is enough to put the particle where the predominant disturbance is the atmospheric drag is presented in the Fig. (7a-b). This region is in the range between 15,000 and 17,500 km. In this figure the curve in green represents the variation width of the radial distance for the case in that only the radiation pressure was considered, the curve in red to the combined effect of PR and J_2 and the line in blue delimits the radial distance in that below this value the predominant disturbance is the atmospheric drag. Comparing these figures it observes that the necessary time for the particle to enter in the region where the predominant disturbance is the atmospheric drag is inferior for the orbital region of 15,000 km. In other words, its oscillation width of the orbital radius is inferior when compared orbital region of 17,500 km.

As it was exposed the combined effect of PR and J_2 here provokes long period variation in the eccentricity (Krivov, 1996, Hamilton and Krivov, 1996, Ishimoto, 1996, Juhász and Horányi, 1996) and, for consequence, in the orbital radius. However, it is had that the semi major axis can be related with these two greatness. However, the behavior of the semi larger axis for different orbital regions are constant in function of the time for a same orbital region. This result is in agreement with introduced them by Krivov et al. (1996) in that $\dot{a} = 0$. The discontinuity presented in the eccentricity variation for the case of the orbital region of 17,500 km to 15,000 km is responsible for the semi major axis variation for this same orbital region. Krivov et al. (1996) and Hamilton and Krivov (1996) studied the discontinuity in function of the particle size for the case of the ejected particles of Mars. Then, it is presupposed that particles around of the Earth presented the dependence with the particle size and the orbital region where this is.

4. Discussion of the results

For geostationary particles it was observed that the effect of the combined oblateness to the radiation pressure doesn't provoke difference in the maximum value of the eccentricity when it is compared to the results where only the radiation pressure was considered. The effect practically is the same in the two situations. The effect of long period, for particles ≤ 5 μm didn't also present relevant differences among the found results only considering the radiation pressure and the simultaneous the oblateness and the radiation pressure effects. In other words, the particle eccentricity variation capable to put it in the region where the predominant disturbance is the atmospheric drag continues occuring approximately with the same period of numeric integration for the two cases (without and with oblateness).

For the other orbital regions it was noticed that the effect of the simultaneous radiation pressure and oblateness disturbances for orbital region from 44,000 km to 32,000 km doesn't present relevant differences in its maximum eccentricity values when compared with the results in that the considered disturbance was only the effect of the radiation pressure. For inferior orbital region to 32,000 km until approximately 17,530 km the maximum eccentricity value of the particle increases quantitatively when it compares the results considering only the radiation pressure and simultaneously the oblateness and the radiation pressure. For the region between 17,500 km and 15,000 km the eccentricity increases enough to put the particle in the region where the predominant disturbance is atmospheric drag. It can be observed that this value was inferior the $e_{máx} = 0.8$. For the region between 12,000 km to 8,000 km the maximum eccentricity value decreases as the altitude of the particle decreases. For the region of 12,000 km it is had $e \sim 0.145$ and for 8,000 km it is had $e \sim 0.015$. Particles in the orbital region between 44,000 km to 17,530 km increase the period of the eccentricity long period disturbance . Particles in the orbital region between 12,000 km to 8,000 km reduce this eccentricity long period disturbance. The region between 17,500 km to 15,000 km is the transition region where above it particles present a type of behavior and below it particles present a contrary behavior due to simultaneous the radiation pressure and the oblateness disturbance.

The isolated effect of the oblateness doesn't provoke significant variation in the initial value of the eccentricity ($e \sim 0$). This result is in agreement with the results found in the literature in that $de/dt = 0$ (equation of Lagrange).

The case in that only the radiation pressure was considered presents an adjustment of polynomial curve of second degree for the maximum eccentricity. This curve adjustment is given by the equation $e_{máx} = -5.16512 \times 10^{-11} r^2 + 6.90934 \times 10^{-6} r + 6.878 \times 10^{-2}$. The case in that the oblateness and the radiation pressure disturbances were considered it is noticed a growing behavior until the orbital region of 17,500 km. The value of the maximum eccentricity for this growing behavior is inferior 0.8. After this orbital region the eccentricity behavior is decreasing.

Larger particles in the low orbit region of the Earth, altitude of approximately 8,000 km, the rediation pressure of the radiation and the oblateness don't provoke eccentricity variation and in the period of the eccentricity long period variation. For the region of geostationary and intermediate orbit of the Earth is observed that a variation exists in this period. For the orbital region of 42,164 km the particle presents this inferior period to the orbital period of the Earth around the Sun while for the orbital region of 20,000 km this period is larger than the orbital period of the Earth.

The orbital region between 15,000 km to 17,500 km, for a particle of 20 μm, the simultaneous effect of the radiation pressure and of the oblateness disturbance was shown larger for larger orbital region. However, the oscillation width of the orbital radius, being larger for the region of 17,500km, needs a larger time to drive the particle to enter in the atmospheric drag region.

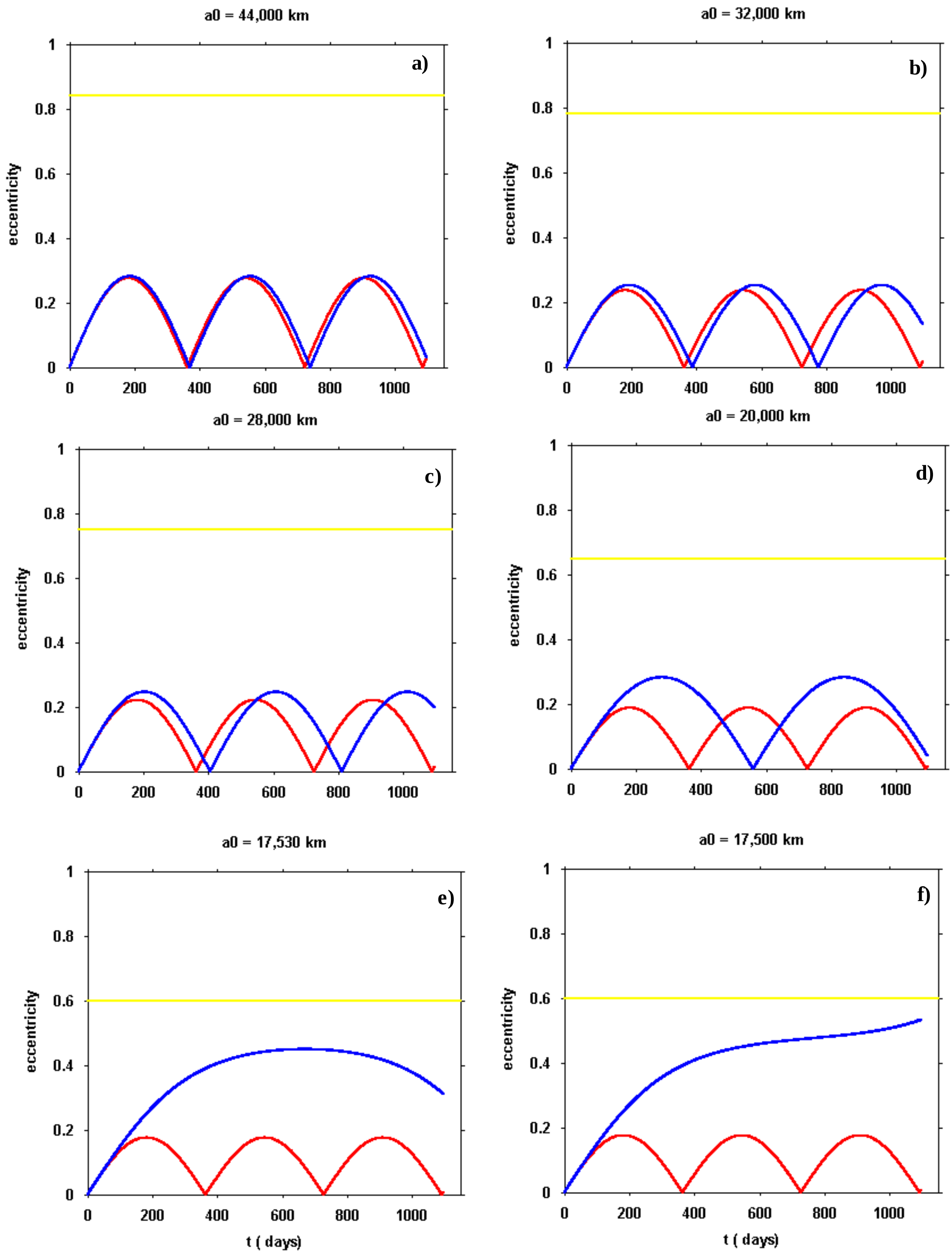

Figure 3a-f - Eccentricity behavior for particle of 20 μm in different orbital regions between 44,000 to 17,500 km subjected to combined effect of PR and J_2. The curve in red represents the curve with the effect of the radiation pressure (PR), the curve in blue the combined effect of the radiation pressure and of the Earth's oblateness (PR + J_2) and the line in yellow delimits the region in that the predominant disturbance is the atmospheric drag. This region is defined as the region above the line yellow.

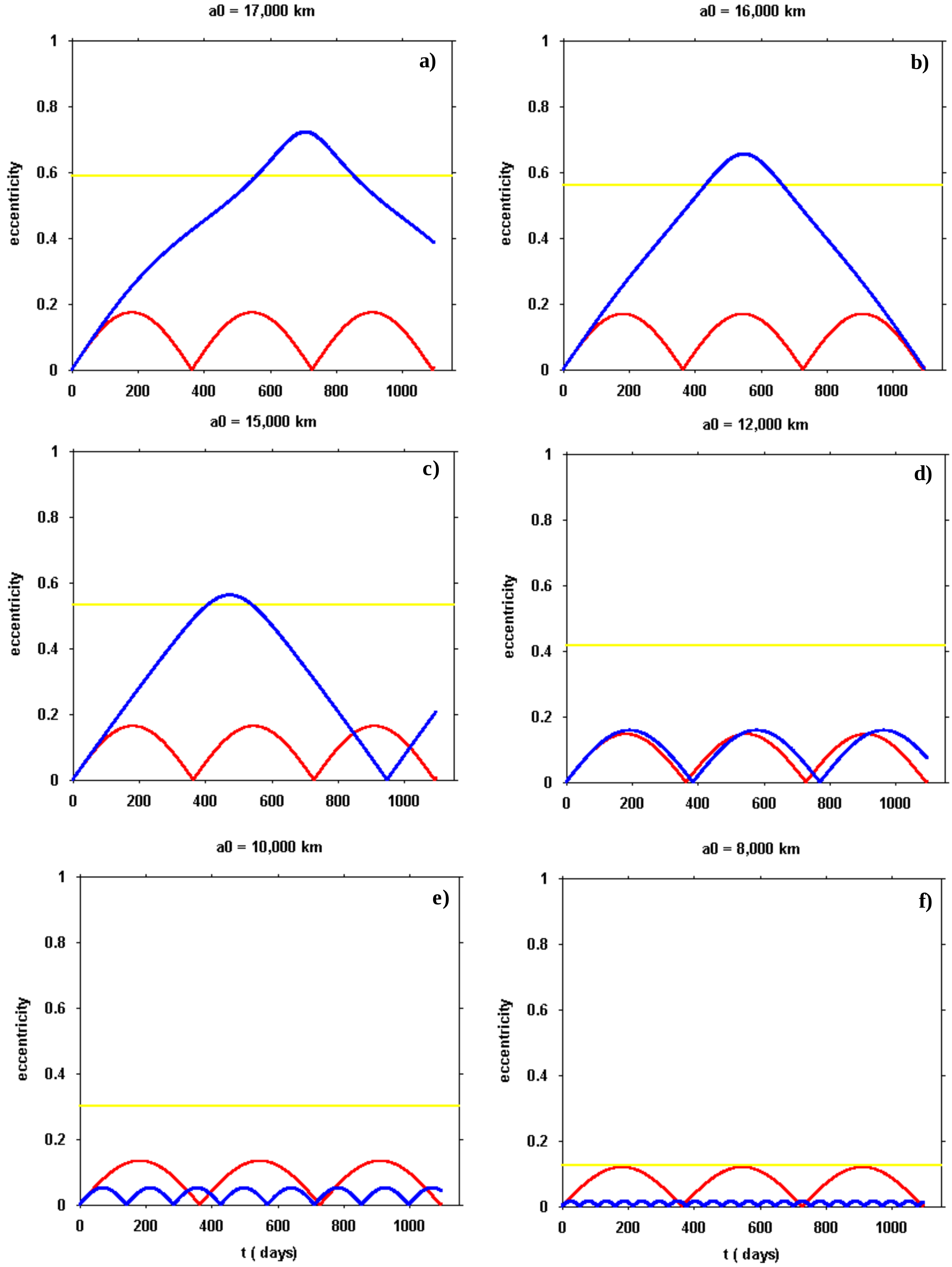

Figure 4a-f - Eccentricity behavior for particle of 20 μm for the orbital regions between 17,000 to 8,000 km. The curve in blue represents the combined effect of PR and J_2, in red, the effect of PR, in green, the effect of J_2 and, in yellow, the region where the predominant disturbance is the atmospheric drag.

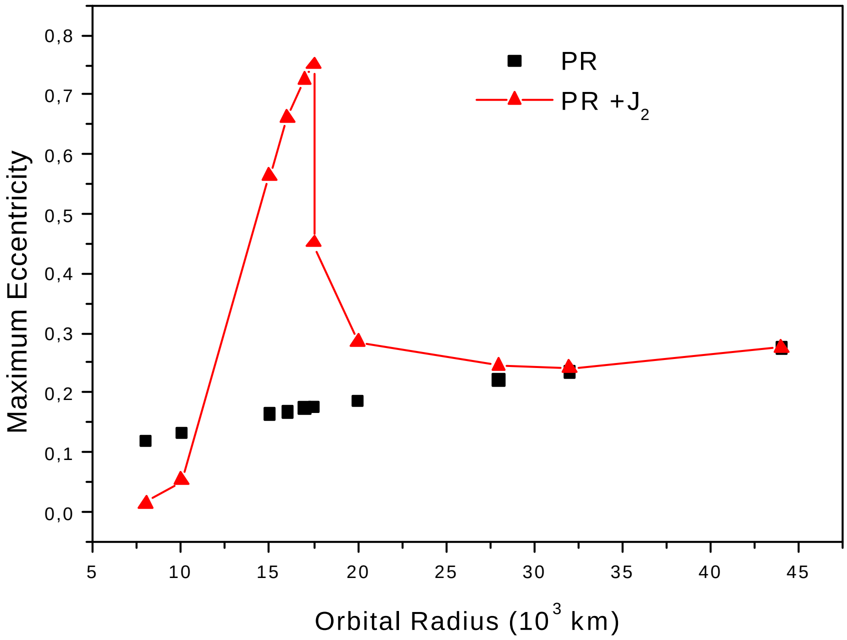

Figure 5 - Maximum eccentricity behavior for a particle of 20 µm subjects to the effect of the radiation pressure (black) and to the combined effect of PR and J_2 (red) in different orbital region. The case in that only the radiation pressure was considered presents an adjustment of polynomial curve of second degree for the maximum eccentricity given by the equation $e_{máx} = -5.16512 \times 10^{-11} r^2 + 6.90934 \times 10^{-6} r + 6.878 \times 10^{-2}$.

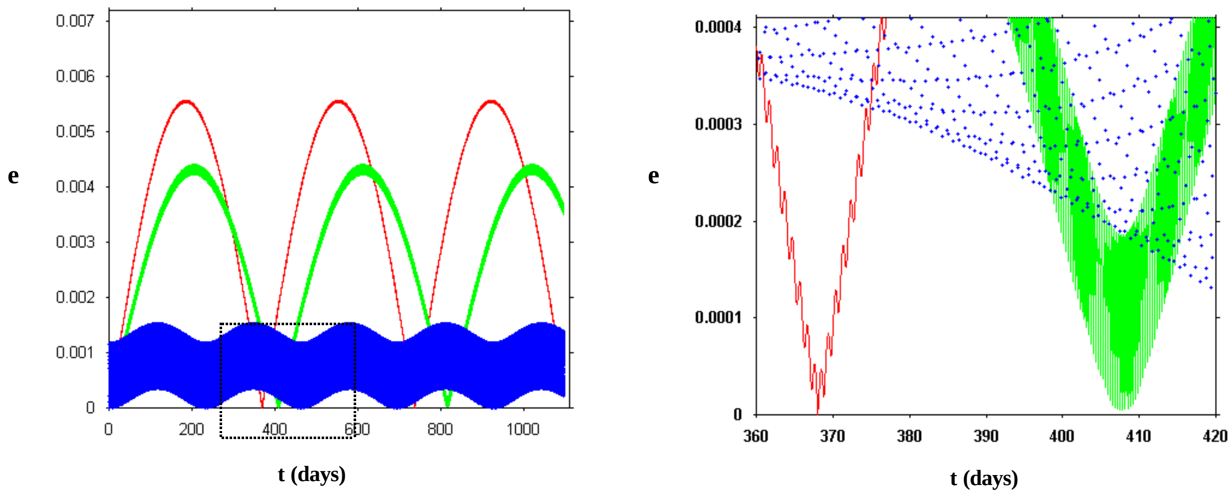

Figure 6 – Evolution of the eccentricity (e) for a particle of 1,000 µm subjects to combined effect of PR and J_2 in three different orbital regions, being them 8,000 (blue), 25,000 (green) e 42,164 km (red).

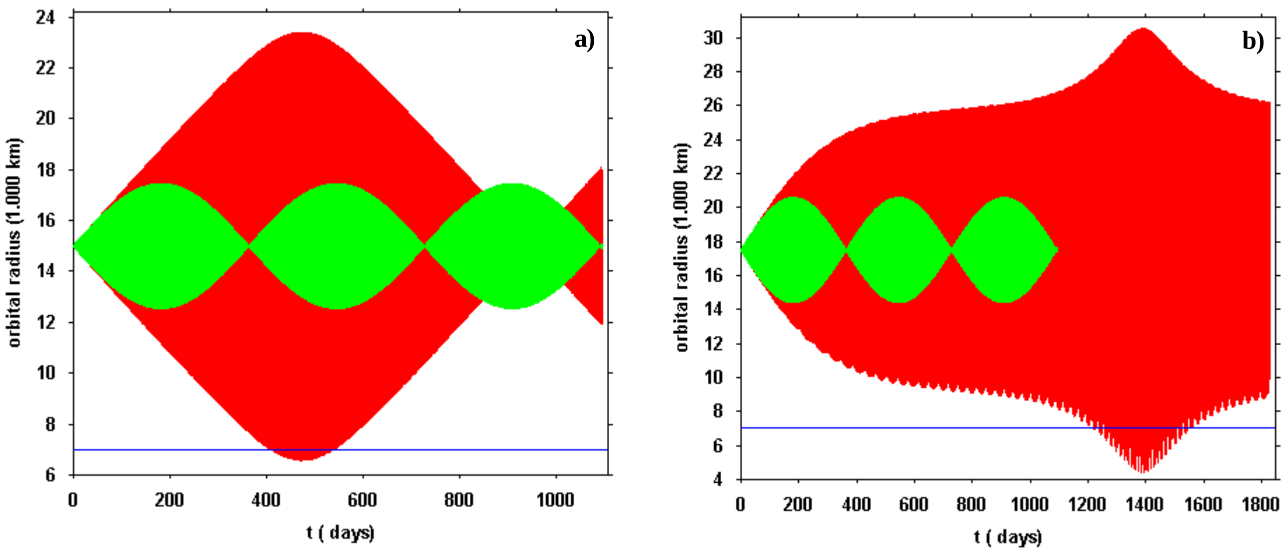

Figure 7a-b -Evolution of the orbital radius for a particle of 20µm subjects the radiation pressure disturbance (green) and the simultaneous disturbance of the radiation pressure (red) in two orbital regions: a) 15,000 km and b) 17,500 km. The blue line delimits the orbital region in that below this the predominant effect is atmospheric drag.

5. Acknowledgement

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