

APPLICATION OF THE DISCRETE ORDINATE METHOD TO RADIATIVE HEAT TRANSFER TO TWO-DIMENSIONAL ENCLOSURES WITH DIFFUSELY EMITTING AND REFLECTING BOUNDARY WALLS

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Abstract. *Radiative transfer is the dominant mode of heat transfer in many engineering problems, including combustion chambers, greenhouses, rocket plume sensing among others. A complete genuinely multi-dimensional discretization in two-dimensional discrete ordinates method is formulated to solve radiative heat transfer in a rectangular enclosure composed of diffusely emitting and reflecting boundaries and containing homogeneous media that absorbs, emits and scatters. The major objetive of this study is to use an efficient method capable of eliminating the ray effect in complex 2D situations. One new genuinely multidimensional differencing scheme is used to solve a radiative transfer equation with* S_4, S_6, S_8, T_6, T_7, T_8 *and* T_9 *angular quadrature scheme. Different cases are analized and the results are compared, when possible, with those obtained by other researchers.*

Keywords: Radiative transfer, Discrete ordinates method, Reflecting boundary

1. Introduction

Radiation heat transfer plays an important role in energy transfer in many scientific and engineering applications specially in applications involving high temperature processes where radiation is the dominant mechanism of energy transfer as in the cases of combustion, nuclear fusion and similar applications. In such applications, it is generally required to solve a multidimensional radiation field using computational techniques. In many combustion applications the radiation heat transfer is the dominant way of energy transfer and can significantly affect the gas temperature and walls. Due to the fact that the reaction rates and density distribution are closely related to the local gas temperature, the influence of radiation heat transfer on the combustion dynamics is very strong. In practice radiative heat-transfer calculations are complex and consequently many approximate solutions were proposed. Including among others, methods based upon diffusion approximation, formulations of the equations of radiation hydrodynamics, the methods of P_n and simplified P_n , the method of discrete ordinates, the method of discrete transfer and the method of control volume.

In an optically dense medium, radiations travels only a short distance before being scattered or absorbed. For this situation it is possible to transform the integral equation for the radiative energy balance into a diffusion equation like that for heat conduction, Siegel and Howell (1992). The Rosseland diffusion equation for radiative energy transfer has the same form as the Fourier law of heat conduction, this allows solution of some radiation problems by heat conduction methods and it is using for simplified cases in same CFD codes, as Phoenics (2003) and CFX (2003)

The P_n approximation introduced initially by Krook (1955) and Cheng (1964) is simply taking the momentum of the radiative transport equation to obtain a system of equations free of the angular dependence. Each P_n approximation results in a system of n^2 equations and in the asymptotic limit, that is, for large n, the P_n approximation results in the solution of the transport equation, Ratzell and Howell (1983) applied and commented these models. Later Liu and Gelbard (1986) formulated a simplified P_n method or SP_n. Balsara (2001) reported that Morel(1993), and Larsen and Morel (1993) formulated and applied another simplified SP_n method but the SP_3 approximation does not retain the same level of angular dependence as the higher order of S_n approximations.

The method of discrete ordinates DOM proved to be a very attractive simplied method to handle and solve radiative transfer, and in Fiveland (1984, 1988), Truelove (1988), Balsara (2001) and Thurgood (1992, 1995) one can to find angular quadratures for the method. This method was originally formulated by Chandrasekhar, in Siegel and Howell (1992) and developed by Lathorp and Carlson (1966). Fiveland (1984, 1988) formulated an accurate method of discrete ordinates of the first order based upon the method of control volumes for two-dimensional and three- dimensional enclosures and present a general outline of the method. Viskanta and Menguc (1987) present a review of works in ordinates discrete and using the method in combustion problems, while Ramankutty and Crosbie (1997, 1998) present a more recent and extensive review of studies, which use this method and formulated the so-called modified discrete ordinates. Techniques based upon finite element methods were explored by Fiveland and Jesse (1994, 1995). Later and Sakami et all (1998), Sakami and Charette (2000) applied the modified discrete ordinates using triangular grids and finite elements methods. The techniques of total variation (TVD), presented by van Leer (1974) and Hearten (1983), were applied by Jesse and Fiveland (1997) to solve radiation problems. Based upon the techniques TVD, Fiveland and

Jesse (1997) formulated a second order interpolation CLAM scheme while Howell et al (1999) used accurate formulation of the first order in combustion problems. Balsara (2001) formulated a new second order scheme with multidimensional interpolation and to solve the nonlinear system of equations obtained, using multigrid techniques in conjuntion with a Newton-Krylov method .

In the present study, the method of discrete ordinates based upon the method of control volume, the CLAM scheme and the multidimensional interpolation GM scheme are used to handle the problem of radiation within two-dimensional enclosure with diffusely emitting and reflecting walls. The predictions were validated with other results obtained by different approaches by other authors. The main objective is to use an efficient method capable of eliminating the ray effect in complex 2D situations and to use the developed code for other problems including combined conduction and convection and CFD codes.

2. Formulation

The radiative transport equation for an absorbing, emitting gray gas medium with isotropic scattering can be written as in Siegel (1992)

$$(\Omega.\nabla)\mathbf{I}(\mathbf{r},\Omega) = -(\kappa+\sigma)\mathbf{I}(\mathbf{r},\Omega) + \frac{\sigma}{4\pi}\int_{4\pi}\mathbf{I}(\mathbf{r},\Omega')\mathbf{d}\Omega' + \kappa\mathbf{I_b}(\mathbf{r}) \tag{1}$$

Where $\mathbf{I}(\mathbf{r},\Omega)$ is the radiation intensity in $\mathbf{r}$, and in the direction Ω; $\mathbf{I_b}(\mathbf{r})$, is the radiation intensity of the blackbody body in the position $\mathbf{r}$ and at the temperature of the medium; κ and σ are the medium gray absorption and scatter coefficients; and the integration is in incidents direction Ω'.

For diffusely reflecting surfaces the radiative boundary condition for equation (1) is given by

$$\mathbf{I}(\mathbf{r},\Omega) = \varepsilon\mathbf{I_b}(\mathbf{r}) + \frac{\rho}{\pi}\int_{\mathbf{n}.\Omega'<0}|\,\mathbf{n}.\Omega'|\,\mathbf{I}(\mathbf{r},\Omega')\mathbf{d}\Omega' \tag{2}$$

Where $\mathbf{r}$ belongs to the boundary surface Γ, and equation (2) applies for $\mathbf{n}.\Omega>0$. $\mathbf{I}(\mathbf{r},\Omega)$ is the intensity leaving the surface at the boundary condition position, ε is the surface emissivity, ρ is the surface reflectivity and $\mathbf{n}$ is the unit vector normal to the boundary surface.

In the method of discrete ordinates the equation of radiation transport is substituted by a set of M discrete equations for a finite number of directions Ω_m , and each integral is substituted by a quadrature series,

$$(\Omega_\mathbf{m}.\nabla)\mathbf{I}(\mathbf{r},\Omega_\mathbf{m}) = -\beta\mathbf{I}(\mathbf{r},\Omega_\mathbf{m}) + \frac{\sigma}{4\pi}\sum_{\mathbf{k}=1}^{\mathbf{M}}\mathbf{w_k}\mathbf{I}(\mathbf{r},\Omega_\mathbf{k}) + \kappa\mathbf{I_b}(\mathbf{r}) \tag{3}$$

Where $\mathbf{w}_k$ are the ordinates weight. This angular approximation transforms the original equation into a set of coupled differential equations, where $\beta = (\kappa+\sigma)$ is the extinction coefficient. Equation (3) can be simplified, Fiveland (1997) as follows:

$$\mathbf{S_m} = \frac{\sigma}{4\pi}\sum_{\mathbf{k}=1}^{\mathbf{M}}\mathbf{w_k}\mathbf{I}(\mathbf{r},\Omega_\mathbf{k}) \tag{4}$$

Where $\mathbf{S}_m$ represents the scattering source term entering

After finalizing the angular discretization, the equations in discrete ordinates can be discretized with respect to space. Writing the discretized equations in discrete ordinates in the three dimensional space in the **m** direction we have

$$\mu_\mathbf{m}\frac{\mathbf{dI_m}}{\mathbf{dx}} + \xi_\mathbf{m}\frac{\mathbf{dI_m}}{\mathbf{dy}} + \eta_\mathbf{m}\frac{\mathbf{dI_m}}{\mathbf{dz}} = \kappa\mathbf{I_m} + \kappa\mathbf{I_b} + \sum_{\mathbf{k}=1}^{\mathbf{M}}\mathbf{w_k}\mathbf{I}(\Omega'_\mathbf{k}) \tag{5}$$

Where $\mu_\mathbf{m},\xi_\mathbf{m},\eta_\mathbf{m}$ are the directional cosines of $\Omega_\mathbf{m}$. The two-dimensional radiative transport equation in the **m** direction for an emitting absorbing and scattering medium is

$$\mu_\mathbf{m}\frac{\mathbf{dI_m}}{\mathbf{dx}} + \xi_\mathbf{m}\frac{\mathbf{dI_m}}{\mathbf{dy}} = \kappa\mathbf{I_m} + \kappa\mathbf{I_b} + \mathbf{S_m} \tag{6}$$

The reflection boundary condition in discrete ordinates can be written as

$$\mathbf{I_m} = \varepsilon \mathbf{I_b} + \frac{\rho}{\pi} \sum_{\substack{\mathbf{m'} \\ \mu'_m < 0}} \mathbf{w_{m'}} \, |\mu'_\mathbf{m}| \, \mathbf{I_{m'}} ; \quad \mu_\mathbf{m} > 0 \quad \text{in } x \in \Gamma \tag{7}$$

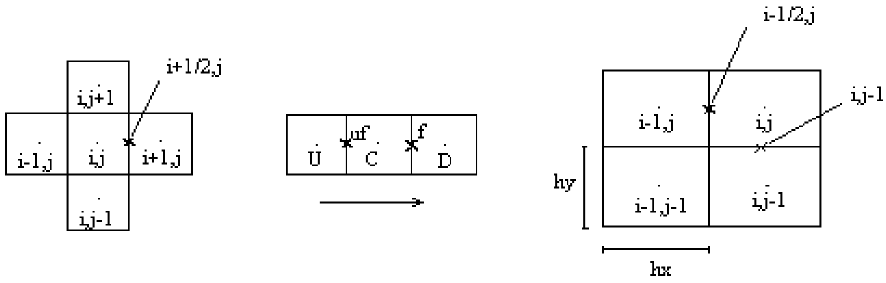
$$\mathbf{I_m} = \varepsilon \mathbf{I_b} + \frac{\rho}{\pi} \sum_{\substack{\mathbf{m'} \\ \xi'_m < 0}} \mathbf{w_{m'}} \, |\xi'_\mathbf{m}| \, \mathbf{I_{m'}} ; \quad \xi_\mathbf{m} > 0 \quad \text{for } y \in \Gamma \tag{8}$$

The equations in discrete ordinates are discretized in space using standard technique of finite volumes of equation (6). The discretization in finite volumes in discrete ordinates can be obtained by multiplying equation (6) by dx.dy and integrate over the control volume (i,j) as shown in figure (1.a)

$$\mu_\mathbf{m} \mathbf{A_x} (\mathbf{I}^\mathbf{m}_{\mathbf{i+\frac{1}{2},j}} - \mathbf{I}^\mathbf{m}_{\mathbf{i-\frac{1}{2},j}}) + \xi_\mathbf{m} \mathbf{A_y} (\mathbf{I}^\mathbf{m}_{\mathbf{i,j+\frac{1}{2}}} - \mathbf{I}^\mathbf{m}_{\mathbf{i,j-\frac{1}{2}}}) = \mathbf{V_{i,j}} (\kappa \mathbf{I_b} - \kappa \mathbf{I^m})_\mathbf{i,j} + \mathbf{S_m} \tag{9}$$

Where $V_{i,j}$ is the control volume i,j in m^3 .

Figure (1) shows the stencil of interpolation. Assuming that the boundary conditions are given the system of equations is closed defining an interpolation system, which relates the intensities at the face with the nodal values.



(a) (b)

Figure 1. Interpolation stencil (a) CLAM scheme, (b) GM scheme

2.1 Interpolation schemes at the faces

In this work the CLAM scheme of Fiveland (1997) permitting one directional interpolation of the second order and the multidimensional scheme GM of Balsara (2001) are used. In the single direction scheme as CLAM, the interpolation in a given face involves three nodes and can be represented by three-point stencil as shown in figure (1.a) for the face f [global face: (i+1/2,j)]. The corresponding intensity can be expressed as

$$\mathbf{I_f} = \mathbf{f}(\mathbf{I_U}, \mathbf{I_C}, \mathbf{I_D}) \tag{10}$$

Having the intention to apply the so-called high resolution schemes in a domain with frontiers, the general interpolation expression is restricted by using the Leonard normalized variable formulation (NVF), Fiveland (1997):

$$\tilde{\mathbf{I}}_\mathbf{f} = \mathbf{f}(\tilde{\mathbf{I}}_\mathbf{C}) \quad \text{e defined as} \quad \tilde{\mathbf{I}} = \frac{(\mathbf{I} - \mathbf{I_U})}{(\mathbf{I_D} - \mathbf{I_U})} \tag{11}$$

The conditions to ensure the discretization limits of the first order operators are as bellow.

The first condition requires that the intensity in the face to be limited by the intensity in the adjacent nodes. This is considered as an interpolating limit and can be expressed in normalized variables as $\tilde{\mathbf{I}}_\mathbf{C} \le \tilde{\mathbf{I}}_\mathbf{f} \le 1$ for $0 \le \tilde{\mathbf{I}}_\mathbf{C} \le 1$.

A second condition states that if a nodal value is not in the monotonic range, the face value is then restricted to the previous nodal value $\tilde{\mathbf{I}}_\mathbf{f} = \tilde{\mathbf{I}}_\mathbf{C}$ for $\tilde{\mathbf{I}} \notin [0,1]$.

The normalized format variable in the scheme of high order CLAM is

$$\tilde{\mathbf{I}}_\mathbf{f} = \begin{cases} \tilde{\mathbf{I}}_\mathbf{C} \left(2 - \tilde{\mathbf{I}}_\mathbf{C}\right) & \text{for} \quad 0 < \tilde{\mathbf{I}}_\mathbf{C} < 1 \\ \tilde{\mathbf{I}}_\mathbf{C} & \text{for the} \quad \text{rest} \end{cases} \tag{12}$$

Frontiers near a node, require a special treatment. In the faces into a control volume with frontiers coincident with the previous face, the HR scheme is applied taking the value of the previous node $\mathbf{I}_U$, as the frontier value and adjusting the weight factors. In the faces coinciding with the next frontiers, an upwind scheme is used. The HR multidimensional non-linear scheme of Balsara (2001), allows working with grids of different aspect ratio. The scheme uses also the limiting flux technique and the Van Albada limiter, in the form: $\text{Limiter}[x,y]=\dfrac{xy^2+yx^2}{x^2+y^2}$, as proposed by Balsara (2001).

The following interpolation relations define this scheme:

for: $\mu_m \geq 0$; $\xi_m \geq 0$; $\dfrac{\mu_m}{h_x} \leq \dfrac{\xi_m}{h_y}$

$$\mu_m \mathbf{I}^m_{i-\frac{1}{2},j} = \mu_m \mathbf{I}^m_{i-1,j-1} + 0.5\times \text{Limiter}[\mu_m(\mathbf{I}^m_{i,j}-\mathbf{I}^m_{i-1,j-1}) - h_x \mathbf{S}^m_{i,j}, (\mu_m - \frac{\xi_m h_x}{h_y})(\mathbf{I}^m_{i-1,j}-\mathbf{I}^m_{i-1,j-1})]$$

$$\xi_m \mathbf{I}^m_{i,j-\frac{1}{2}} = \xi_m \mathbf{I}^m_{i,j-1} \tag{13}$$

with $\mathbf{S}^m_{i,j} = -(\kappa_{i,j}+\sigma_{i,j})\mathbf{I}^m_{i,j} + \kappa_{i,j}\mathbf{I}_{bi,j}$

For $\mu_m \geq 0$; $\xi_m \geq 0$; $\dfrac{\mu_m}{h_x} \geq \dfrac{\xi_m}{h_y}$

$$\mu_m \mathbf{I}^m_{i-\frac{1}{2},j} = \mu_m \mathbf{I}^m_{i-1,j}$$

$$\xi_m \mathbf{I}^m_{i,j-\frac{1}{2}} = \xi_m \mathbf{I}^m_{i-1,j-1} + 0.5\times \text{Limiter}[\xi_m(\mathbf{I}^m_{i,j}-\mathbf{I}^m_{i-1,j-1}) - h_y \mathbf{S}^m_{i,j}, (\xi_m - \frac{\mu_m h_y}{h_x})(\mathbf{I}^m_{i,j-1}-\mathbf{I}^m_{i-1,j-1})] \tag{14}$$

3. Methods of solution

The radiative transport equations in discrete ordinates spatially discretized are obtained by substituting the corresponding expressions in equation (9) and is denoted as

$$\mathbf{K_m}(\mathbf{I_m}) = \mathbf{F_m} \qquad \text{for } m=1,.........M \tag{15}$$

Where $\mathbf{K}_m$ is a matrix NxN representing the discrete form of the continuous transport operator $\mathbf{L}_m$; $\mathbf{I}_m$ is the vector solution for the ordinate in the direction m_{th}; $\mathbf{F}_m$ is the vector containing the volumetric emission, the scattering and reflection frontiers terms. For the HR schemes, $\mathbf{K}_m$ depends on $\mathbf{I}_m$ and the coupling between the directions is incorporated in $\mathbf{F}_m$. The step scheme equation (15) can be solved using combined upstream–downstream approach. This is analogous to organizing the upstream to downstream equation to obtain a triangular matrix and solving the resulting system of equations. In the global solution, equation (15) is solved individually for each direction and upon including the reflection and scattering terms, the equations are solved again. The iterations are continued until convergence is achieved. The HR schemes are non–linear and must be treated differently. Here, we apply a correction procedure in which the operator of first order is treated implicitly while the difference between the HR operator and the first order operator is treated explicitly:

$$\mathbf{K}^1_\mathbf{m}(\mathbf{I_m})^{\mathbf{n}+1} = \left[\mathbf{K}^1_\mathbf{m}(\mathbf{I_m}) - \mathbf{K}^{\mathbf{HR}}_\mathbf{m}(\mathbf{I_m})\right]^\mathbf{n} + \mathbf{F_m} \qquad \text{for } m=1, ...,M \tag{16}$$

Where $\mathbf{K}^1_m$ and $\mathbf{K}^{HR}_m$ represent the transport operator for the first order and HR schemes respectively and n the global number of iterations over the angular-spatial grade.

The convergence is measured by the normalized difference between the incident energy is two successive angular-spatial sweeps, that is

$$||R||_\infty \equiv \text{Max}\ (\ R_i\ ;\ i=1,.....,N_{node}\ ) < 10^{-6} \tag{17}$$

Where $$\mathbf{R_i} = \frac{|\mathbf{G}^{\mathbf{n}+1}_\mathbf{i} - \mathbf{G}^\mathbf{n}_\mathbf{i}|}{\mathbf{G}^{\mathbf{n}+1}_\mathbf{i}} \tag{18}$$

Where G_i is the incident energy = $\int_{4\pi}\mathbf{I}(\mathbf{r},\Omega)\mathbf{d}\Omega$ w/m^2 (19)

The subscripts *n* and *i* denote the iteration and the node index respectively.

The first order step scheme is simple and involves only the preceding nodal value: $\mathbf{I}_f = \mathbf{I}_C$

The corresponding formulation in normalized variable notation (NVF) for this scheme is : $\tilde{\mathbf{I}}_{\mathbf{f}} = \tilde{\mathbf{I}}_{\mathbf{C}}$, This scheme is inconditionally unlimited, quick but has first order precision.

3.1 Deferred correction

In a general manner the deferred correction procedure is denoted as

$$\phi_{\mathbf{f}} = \phi_{\mathbf{f}}(\mathbf{U}) + \phi_{\mathbf{f'}} \tag{20}$$

With: $\phi_{\mathbf{f'}} = \phi_{\mathbf{f}}(\mathbf{H}) - \phi_{\mathbf{f}}(\mathbf{U})$ (21)

Where: ϕ = variable; ϕ_f = variable value in the face f; $\phi_f(u)$ = upwind step scheme; $\phi_f(H)$ = higher order scheme HR
The discretized equation of ϕ is

$$(\mathbf{J_h} - \mathbf{J_l}) + (\mathbf{J_n} - \mathbf{J_s}) + (\mathbf{J_e} - \mathbf{J_w}) = \mathbf{S_p} \tag{22}$$

Where $J_f = C_f \phi_f$ and $\phi_f = P(\phi_{nb})$; S_P is a source term (23)

$$\left(\mathbf{C_h}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{n}} - \mathbf{C_l}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{l}}\right) + \left(\mathbf{C_n}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{n}} - \mathbf{C_s}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{s}}\right) + \left(\mathbf{C_e}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{e}} - \mathbf{C_w}[\mathbf{P}(\phi_{\mathbf{nb}})]_{\mathbf{w}}\right) = \mathbf{S_p} \tag{24}$$

Substituting equation (20) into equation (24)

$$\left(\mathbf{C_h}\phi_{\mathbf{h}}(\mathbf{U}) - \mathbf{C_l}\phi_{\mathbf{l}}(\mathbf{U})\right) + \left(\mathbf{C_n}\phi_{\mathbf{n}}(\mathbf{U}) - \mathbf{C_s}\phi_{\mathbf{s}}(\mathbf{U})\right) + \left(\mathbf{C_e}\phi_{\mathbf{e}}(\mathbf{U}) - \mathbf{C_w}\phi_{\mathbf{w}}(\mathbf{U})\right) = \mathbf{S_p} + \mathbf{B_p} \tag{25}$$

where B_P is

$$\mathbf{B_p} = \left(\mathbf{C_l}\phi_{\mathbf{l'}} - \mathbf{C_h}\phi_{\mathbf{h'}}\right) + \left(\mathbf{C_s}\phi_{\mathbf{s'}} - \mathbf{C_n}\phi_{\mathbf{n'}}\right) + \left(\mathbf{C_w}\phi_{\mathbf{w'}} - \mathbf{C_e}\phi_{\mathbf{e'}}\right) \tag{26}$$

Using equation (21) in (25)

$$\mathbf{B_p} = \left(\mathbf{C_l}(\phi_{\mathbf{l}}(\mathbf{H}) - \phi_{\mathbf{L}}) - \mathbf{C_h}(\phi_{\mathbf{h}}(\mathbf{H}) - \phi_{\mathbf{P}}\right) + \left(\mathbf{C_s}(\phi_{\mathbf{s}}(\mathbf{H}) - \phi_{\mathbf{S}}) - \mathbf{C_n}(\phi_{\mathbf{n}}(\mathbf{H}) - \phi_{\mathbf{P}}\right) + \left(\mathbf{C_w}(\phi_{\mathbf{w}}(\mathbf{H}) - \phi_{\mathbf{W}}) - \mathbf{C_e}(\phi_{\mathbf{e}}(\mathbf{H}) - \phi_{\mathbf{P}})\right) \tag{27}$$

Writing the equation in a two dimensional form:

$$\left(\mathbf{C_n}\phi_{\mathbf{p}} - \mathbf{C_s}\phi_{\mathbf{S}}\right) + \left(\mathbf{C_e}\phi_{\mathbf{p}} - \mathbf{C_w}\phi_{\mathbf{W}}\right) = \mathbf{S_p} + \mathbf{B_p} \tag{28}$$

$$\mathbf{B_p} = \left(\mathbf{C_s}(\phi_{\mathbf{s}}(\mathbf{H}) - \phi_{\mathbf{S}}) - \mathbf{C_n}(\phi_{\mathbf{n}}(\mathbf{H}) - \phi_{\mathbf{P}}\right) + \left(\mathbf{C_w}(\phi_{\mathbf{w}}(\mathbf{H}) - \phi_{\mathbf{W}}) - \mathbf{C_e}(\phi_{\mathbf{e}}(\mathbf{H}) - \phi_{\mathbf{P}})\right) \tag{29}$$

or in terms of ϕ_P

$$\phi_{\mathbf{P}} = \frac{\mathbf{C_s}\phi_{\mathbf{S}} + \mathbf{C_w}\phi_{\mathbf{W}} + \mathbf{S_p} + \mathbf{B_p}}{\mathbf{C_n} + \mathbf{C_e}} \tag{30}$$

Two methods are used in treating equation (30); First approach:

$$\phi_{\mathbf{P}}^{\mathbf{n+1}} = \frac{\mathbf{C_s}\phi_{\mathbf{S}}^{\mathbf{n+1}} + \mathbf{C_w}\phi_{\mathbf{W}}^{\mathbf{n+1}} + \mathbf{S_p} + \mathbf{B_p^n}}{\mathbf{C_n} + \mathbf{C_e}} \tag{31}$$

Where n indicates the number of iterations and B_p is evaluated in the n iteration by making

$$\mathbf{S_{df}} = \mathbf{B_p} = \mathbf{C_s}(\phi_{\mathbf{s}}(\mathbf{H}) - \phi_{\mathbf{S}})^{\mathbf{n}} + \mathbf{C_w}(\phi_{\mathbf{w}}(\mathbf{H}) - \phi_{\mathbf{W}})^{\mathbf{n}} + \mathbf{C_n}(\phi_{\mathbf{P}} - \phi_{\mathbf{n}}(\mathbf{H}))^{\mathbf{n}} + \mathbf{C_e}(\phi_{\mathbf{P}} - \phi_{\mathbf{e}}(\mathbf{H}))^{\mathbf{n}} \tag{32}$$

Second approach

$$\phi_P^{n+1} = \frac{C_s\phi_S^{n+1} + C_w\phi_W^{n+1} + S_p + B_p^{n'}}{C_n + C_e} \tag{33}$$

Where n means n'= n+1 for points upstream of P and n'= n for points P and downstream of P.
Hence

$$S_{df} = B_p^{n'} = C_s\phi_s(H)^{n'} - C_s\phi_S^{n+1} + C_w\phi_w(H)^{n'} - C_w\phi_W^{n+1} + C_n\phi_P^n - C_n\phi_n(H)^{n'} + C_e\phi_P^n - C_e\phi_e(H)^{n'} \tag{34}$$

and

$$\phi_P^{n+1} = \frac{C_s\phi_S^{n'}(H) + C_w\phi_W^{n'}(H) + S_p + [C_n\phi_P^n - C_n\phi_n^{n'}(H) + C_e\phi_P^n - C_e\phi_e^{n'}(H)]}{C_n + C_e} \tag{35}$$

To discretize the radiative transport equation, one can rewrite equation (9) based upon the first approach,

$$\mu_m A_x (I_{i+\frac{1}{2},j}^m - I_{i-\frac{1}{2},j}^m)^{n+1} + \xi_m A_y (I_{i,j+\frac{1}{2}}^m - I_{i,j-\frac{1}{2}}^m)^{n+1} + V_{i,j}(\kappa I_{i,j}^m)^{n+1} = V_{i,j}(\kappa I_{bi,j})^n + S_m^n + S_{df}^n \tag{36}$$

Using the step scheme for the fluxes $(I_{i+\frac{1}{2},j}^m)^{n+1}$ and $(I_{i,j+\frac{1}{2}}^m)^{n+1}$

$$\mu_m A_x (I_{i,j}^m - I_{i-\frac{1}{2},j}^m)^{n+1} + \xi_m A_y (I_{i,j}^m - I_{i,j-\frac{1}{2}}^m)^{n+1} + V_{i,j}(\kappa I_{i,j}^m)^{n+1} = V_{i,j}(\kappa I_{bi,j})^n + S_m^n + S_{df}^n$$

By separating the variables, one has

$$(\mu_m A_x I_{i,j}^m + \xi_m A_y I_{i,j}^m + V_{i,j}\kappa I_{i,j}^m)^{n+1} = V_{i,j}(\kappa I_{bi,j})^n + \mu_m A_x (I_{i-\frac{1}{2},j}^m)^{n+1} + \xi_m A_y (I_{i,j-\frac{1}{2}}^m)^{n+1} + S_m^n + S_{df}^n \tag{37}$$

From which one can obtain

$$(I_{i,j}^m)^{n+1} = \frac{V_{i,j}(\kappa I_{bi,j})^n + |\mu_m| A_x (I_{i-\frac{1}{2},j}^m)^{n+1} + |\xi_m| A_y (I_{i,j-\frac{1}{2}}^m)^{n+1} + S_m^n + S_{df}^n}{|\mu_m| A_x + |\xi_m| A_y + \kappa V_{i,j}} \tag{38}$$

Where

$$S_{df}^n = |\mu_m| A_x (I_{i,j}^m - I_{i+\frac{1}{2},j}^m)^n + |\xi_m| A_y (I_{i,j}^m - I_{i,j+\frac{1}{2}}^m)^n \tag{39}$$

The values of the fluxes in the faces $(I_{i+\frac{1}{2},j}^m)^n$ and $(I_{i,j+\frac{1}{2}}^m)^n$ are interpolated using the CLAM scheme or the multidimensional scheme while the values of the fluxes $(I_{i-\frac{1}{2},j}^m)^{n+1}$ and $(I_{i,j-\frac{1}{2}}^m)^{n+1}$ are interpolated using a step scheme.

In the second approach the procedure is the same and the final discretized equations are also the same except that the flux values in the faces $(I_{i-\frac{1}{2},j}^m)^{n+1}$ and $(I_{i,j-\frac{1}{2}}^m)^{n+1}$ are interpolated using a higher order scheme with the upstream nodal values as the iterate actual values and the nodal values downstream are the correspondents to the previous iteration. We had to do several numerical experiments in symmetrical cases to be sure that the algorithm does not have directional march run error and also to determine the adequate value of over-relaxation.

4. Results and discussion

Two cases having exact solution are presented here for comparison and validation of the model. The first test case is pure absorption equivalent to the case of pure scattering. The medium is absorbing and emitting with three black and cold walls while the top wall is black and hot with diffuse emission I_D = 1. Here the convergence of the numerical method can be evaluated from the results shown in figures (2) and (3). To study the behavior of the converged solution for different quadratures S_n and T_n, the results shown in figure (4), show that for higher order quadrature the solution is smooth and the comparison with the exact solution, Crosbie(1984), indicates a good agreement as show in figure (5).

The second test case is a difficult one to show if the method and the adopted scheme minimize the ray effect. The medium is absorbing and emitting with three black and cold walls while the top wall is black and hot in one strip of the wall with diffuse emission I_D = 1. The results shown in figure (6) indicate that when using quadratures of order less as S_6, S_8, the numerical solution is poor when using CLAM or multidimensional scheme GM and upon increasing the order of quadratures T_7, T_8, T_9 the solution becomes smooth and converges to Crosbie (1884) exact solution. When comparing solutions of CLAM and GM scheme it is found that GM scheme solution is more accurate and smooth than CLAM scheme.

When using computational grids of aspect ratio different from unity as shown in figure (7), the results indicate that the model using GM scheme is adequate and converge for rectangular grids.

The algorithm is then used to calculate radiative transfer in an enclosure with reflecting walls, the results are shown in figure (8). The geometry is the same as the test problems while the east and west wall are reflecting. One can observe that as the wall emissivity decrease the ray effect also decreases.

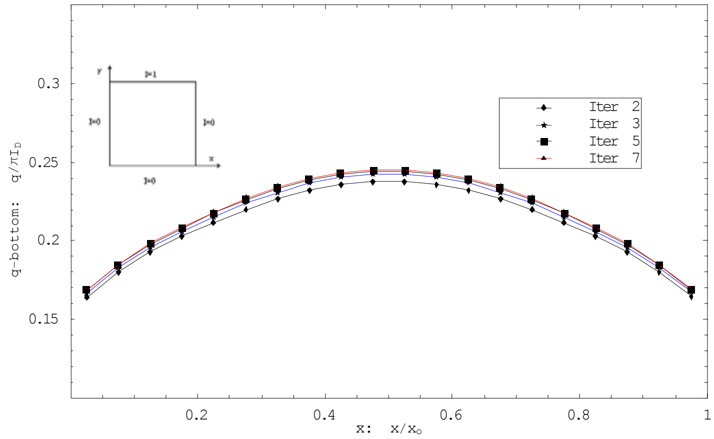


Fig. 2. Predicted heat flux at the bottom (cold) wall obtained by T_9 , 20x20 grid, isotropic scattering case –Solution Convergence.

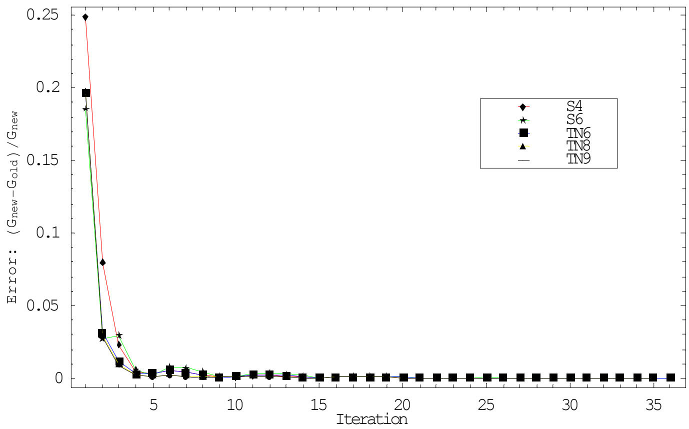


Fig. 3. Convergence histories in different quadratures without over relaxation.

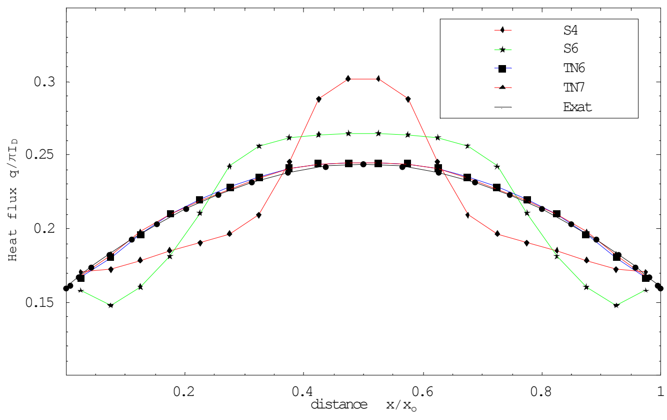


Fig. 4. Predicted heat flux at the bottom (cold) wall, S_N and T_N quadratures and exact solutions Crosbie (1984), Ramankutte and Crosbie (1997) comparison , 20x20 grid, isotropic scattering case , , $\beta_{x/y}$=1.0.

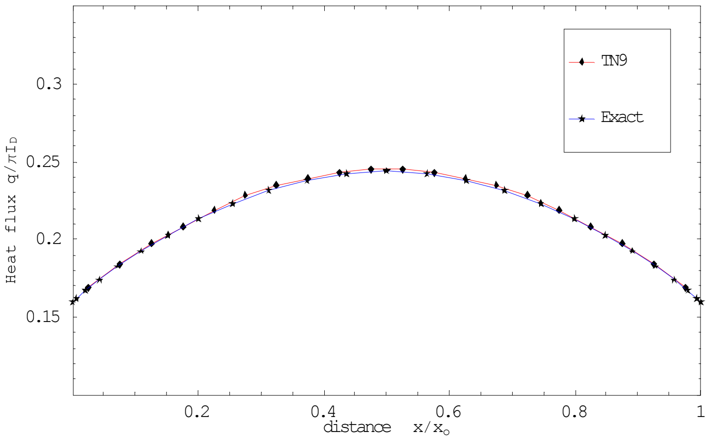


Fig. 5. Predicted heat flux at the bottom (cold) wall, comparison of T_9 and exact solutions, Crosbie (1984), 20x20 grid, isotropic scattering case , $\beta_{x/y}$=1.0.

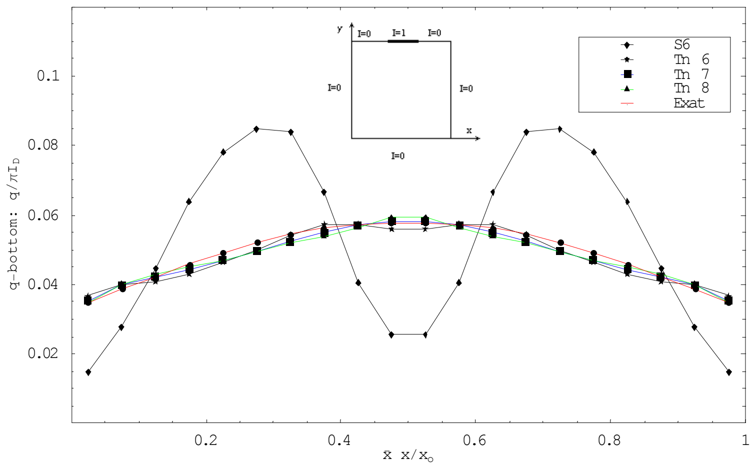


Fig. 6. Predicted heat flux at the bottom (cold) wall obtained for strip diffuse loading case, 0.4 < x < 0.6, 20x20 grid, isotropic scattering, , $\beta_{x/y}$=1.0. – Comparison with exact solution obtained by Crosbie (1984), and Ramankutte and Crosbie (1997) .

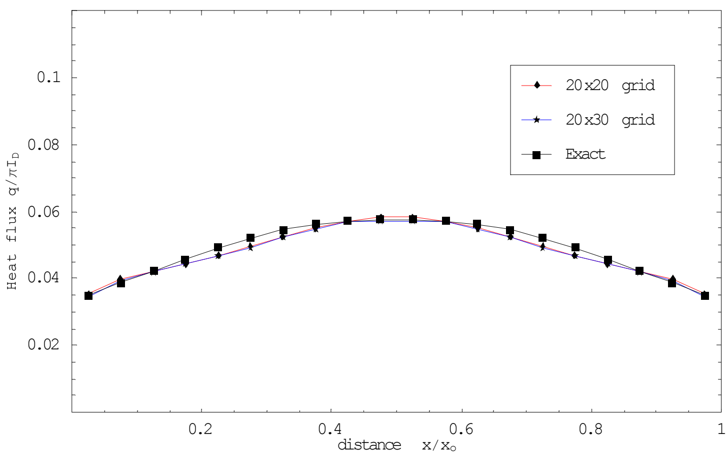


Fig 7. Predicted heat flux at the bottom (cold) wall obtained for strip diffuse loading case, 0.4 < x < 0.6, isotropic scattering, $\beta_{x/y}$=1.0. – Comparison different grids with exact solution of Crosbie (1984) and Ramankutte and Crosbie (1997)

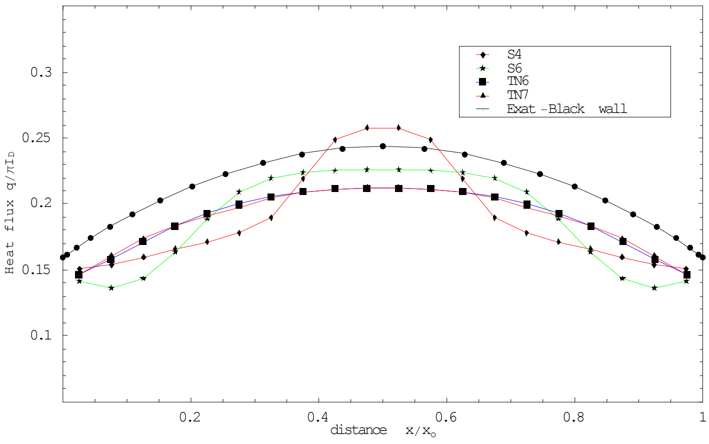


Fig. 8. Heat Flux at south cold wall, reflecting west and east walls case, ε_w=0.8, ε_e=0.8, $\beta_{x/y}$=1.0.. Comparison of quadratures

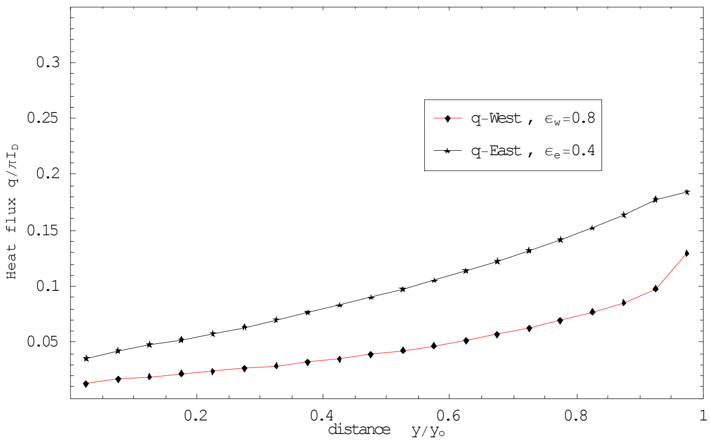


Fig 9. Heat Flux at west and east wall, different reflecting west and east walls case, ε_w=0.8, ε_e=0.4, $\beta_{x/y}$=1.0.

Figure (9) shows the case of reflecting walls with different emissivity but the same geometry a case 1, and having the west and east walls as diffusing and reflecting surfaces. The converged solution is found to be smooth even for less order angular T_N quadrature.

5. Conclusions

In this study the one multidimensional scheme in the classical discrete ordinates method was applied and it was found that it is suitable for accurate calculations in radiative transfer and minimizing the ray effect in complex geometrical situations. The algorithm can be used for calculating the radiative source term in combined heat transfer problems using CFD codes.

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