

Influence of the anisotropy on the mechanical behavior of laminated beams

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Abstract*. The study of anisotropy is very important in structural engineering to specifically understand the behavior of the fiber-reinforced materials. This paper deals with a theoretical study on anisotropy and on its influence in the stress distribution, and displacement field, in plane laminated beams, constituted by fiber-reinforced materials. Laminated or composite beams are considered anisotropic due to the use of the laminae of the different organization and with fiber in different directions as well. In this context, to study the mechanical behavior of this kind of beam, stress functions were analyzed, and determined, according to those developed by Lekhnitskii et al. (1968) and Hashin (1967). In order to analyze the obtained solutions, using an analytical procedure, it was performed comparisons, in terms of stresses and displacements, between beams of orthotropic materials and isotropic ones, using both a commercial program of finite elements and an approximated analytical method. For this, it was necessary to use the concepts of elastic property transformation to obtain anisotropic beams from the homogeneous laminated beams. It was considered in this analysis the mutual influence coefficients, characteristic of anisotropic solids. The obtained results contribute for a better understanding and application of these anisotropic laminated beams in engineering.*

Keywords*: anisotropy, elasticity, laminated beams.*

1. Introduction

The study of elastic behavior of anisotropic solids is very important in engineering. As structural elements, anisotropic solids are applied in some areas of modern technology. In general, an anisotropic material presents no elastic symmetry and its mechanical properties are different in different directions. In this way, the study of these solids becomes more complex than the study of other solids that possess other types of elastic symmetry, as for example, isotropic solids. Some works that evidence this complexity can be cited, as for example, the work of Carrier and Ithaca (1943), Green and Zerna (1954), Nair and Reissner (1976), Kilic et al. (2001) and Mascia and Vanalli (2002).

In fact, the study of the anisotropy implies in the knowledge of the constitutive law that consequently governs the elastic behavior of the material and in determining the components of the constituent tensor S_{ijkl}. In an elastic model, completely anisotropic, this constitutive tensor possesses 81 unknown constants. Using convenient simplifications, this number can be reduced for 9 constants, which is called orthotropic model, or for 3 constants, called isotropic model.

Treating particularly about anisotropic beams, when these are subjected to the loading situations, it appears other deformations besides the ones presented in isotropic beams, as for example, in steel beams. These deformations are associated to the certain constants of elasticity called of the mutual influence coefficients that quantify the participation of normal stresses in shear strains or shear stresses in normal strains.

During many years the studies in this area have been focused on the analysis of the behavior of the stresses and strains of anisotropic beams. As for example the application of the polynomial forms of the stress function of Airy, where Lekhnitskii et al. (1968) studied the displacements and the stresses in some examples of anisotropic beams subjected to the different conditions of loadings and restrains. When particularized for isotropic beams, these solutions become similar those of Timoshenko and Goodier (1970). Hashin (1967), also using of the stress function of Airy, developed a systematic way of develop polynomial solutions for anisotropic plane beams under the polynomial loads.

Murakami et all (1996), using variational methods, investigated the effect of the constitutive coupling, and the effect caused for the participation of the coefficients of mutual influence on stresses and strains of a cantilever anisotropic beam. In this way, the authors also studied the influence of the relation between the height of the cross section (h) and the span of the beam (*l*) on the behavior of anisotropic beams considering the theory of beam of Euler-Bernoulli and the theory of Timoshenko too. This study demonstrated that the effect of the constitutive coupling increases linearly with the increase of the relation (h / *l*).

In this context, considering the anisotropic formalism of Lekhnitskii (1981), the goal of this paper is to analyze the influence of the anisotropy in the mechanical behavior of laminated beams, considered anisotropic and homogeneous. In order to carry out this, beyond a mathematical description about this subject, it is used the equations developed for Lekhnitskii et al.(1968) for analysis of displacements in cantilever anisotropic beam, subjected to a polynomial loading, as well as, the method developed for Hashin (1967) for analysis of stresses in anisotropic plane beams under the uniform distributed loads . The obtained solutions are compared, by means of diagrams, with solutions for isotropic beams considering the theories of beam of Bernoulli-Euler, and also, with numerical solutions for orthotropic beams derived of a finite element commercial program.

2. Description of problem

On the whole, according to Lekhnitskii (1981), all materials can be divided in homogeneous and not homogeneous, and on the other hand, in isotropic and anisotropic. Most of structural materials show some degrees of anisotropy. Materials, as wood, are naturally anisotropic, others, as laminated composites, are anisotropic due to the manufacture process.

Lekhnitskii (1981) classifies as isotropic the solid whose properties of elasticity are constant for any directions established from one determined point, or either, the transformations of coordinates are invariant for all. An anisotropic solid, in general, shows different properties of elasticity for different directions associates to a given point. The directions in which the elasticity properties are constant are called elastically equivalent directions or principal directions of elasticity.

Apart from this if the structure of an anisotropic body presents some type of symmetry, its properties of elasticity also show it. The elastic symmetry expresses the fact that in each point of the solid exists equivalent symmetrical directions with respect the elastic properties. If the symmetry of the elastic properties of an anisotropic body exists, the equations of Hooke´s Law can be simplified, also occurring many simplifications in the constituent tensors S_{ij} (or C_{ij}), Mascia (1991).

Dealing particularly with laminated beams, constituted by laminae of materials strengthened for fibers, the stress and strain analysis becomes more or less complex whether considering the coincidence or not between the geometric axis of the beam and the principal axis of elasticity of the laminae, that compose the structure. Thus, the constitutive tensor for the plane case, in case of having this coincidence, is simplified, becoming orthotropic with four independent terms and different of zero. Here, we have that:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \tag{1}$$

However, the simplifications presented in the orthotropic tensor disappear when the coincidence between the geometric axis of the beam and the principal axis of the laminae did not considered, generally related to the directions of strengthened fibers, appearing then, other elements in the constitutive tensor, which becomes anisotropic. In a general point of view, the angles that relate reinforcement fibers of the laminae and the geometric axis of the analyzed structure establish the anisotropic behavior of the laminated beams (Fig. 1).

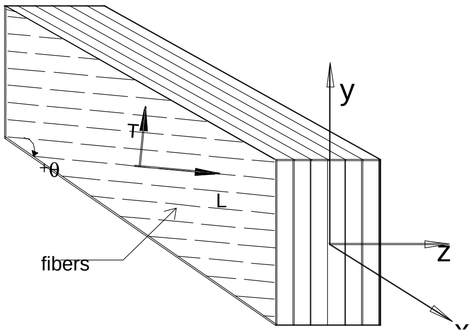

Figure 1. Laminated beam related to the systems geometric and local (of the fibers).

Thus, considering the angle θ among the fibers of reinforcement of the laminae and the geometric axis of the beam, the constitutive tensor becomes:

$$\begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} = [T]^T \cdot \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \cdot [T] \tag{2}$$

where the constants $\bar{S}_{16}$ and $\bar{S}_{26}$ are present in anisotropic materials and are dependent of the coefficients of mutual influence. The matrix of transformation, present in Eq. (2), is given by:

$$[T]=\begin{bmatrix} \cos^2\theta & \text{sen}^2\theta & 2\cdot\text{sen}\,\theta\cdot\cos\theta \\ \text{sen}^2\theta & \cos^2\theta & -2\cdot\text{sen}\,\theta\cdot\cos\theta \\ -\text{sen}\,\theta\cdot\cos\theta & \text{sen}\,\theta\cdot\cos\theta & \cos^2\theta-\text{sen}^2\theta \end{bmatrix} \tag{3}$$

Consequently, the anisotropic constitutive relations, or stress (σ or τ) and strain (ε or γ) relationship, that are involved in the analysis of a laminated beam, where it does not have coincidence between the local axis of the laminae and the geometric axis of the beam, for a plane case, can be written as:

$$\varepsilon_x = \bar{S}_{11}\,\sigma_x + \bar{S}_{12}\,\sigma_y + \bar{S}_{16}\,\tau_{xy} \tag{4.1}$$

$$\varepsilon_y = \bar{S}_{12}\,\sigma_x + \bar{S}_{22}\,\sigma_y + \bar{S}_{26}\,\tau_{xy} \tag{4.2}$$

$$\gamma_{xy} = \bar{S}_{16}\,\sigma_x + \bar{S}_{26}\,\sigma_y + \bar{S}_{66}\,\tau_{xy} \tag{4.3}$$

Thus, it is evident that beams constituted by orthotropic materials can become anisotropic from these considerations, and that with the use of the transformation matrices, Eq. (3), it is possible to get all the constants that are necessary for the development of the anisotropic constitutive relations, including the coefficients of mutual influence ($h_{xy,x}$ and $h_{xy,y}$), which are of difficult experimental determination (See Eq. 5.1).

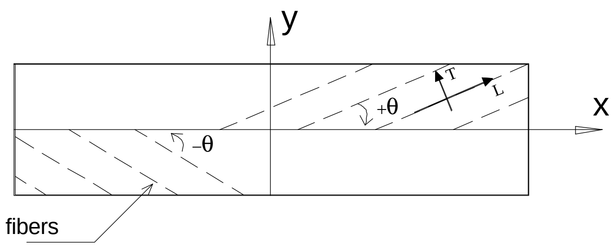

Figure 2. Angles of the fibers in the lamina.

To illustrate the transformations of the mentioned elastic properties, it is made, as follows, diagrams that present the variations of the values of the elastic constants, in function of the angle of inclination of reinforcement fibers, of the laminae considered orthotropic, as illustrated in Fig. (2). For this analysis, it is considered that the laminae are strengthened with carbon fibers and possess the following elastic properties:

Table 1. Elastic constants of laminae strengthened with carbon fibers.

Material	**Elastic Constants**			
	E_x (GPa)	**E_y (GPa)**	**G_{xy} (GPa)**	**ν_{xy}**
Carbon- polymeric Composite	140,00	14,00	5,00	0,2

Thus, using Eq. (2) and considering that:

$$\bar{S}_{16} = \frac{\eta_{xy,x}}{E_x}\,; \qquad \bar{S}_{26} = \frac{\eta_{xy,y}}{E_y} \tag{5.1}$$

$$\bar{S}_{11} = \frac{1}{E_x}\,; \qquad \bar{S}_{22} = \frac{1}{E_y}\,; \qquad \bar{S}_{12} = -\frac{\nu_{xy}}{E_x} \tag{5.2}$$

the following diagrams with the values for the variations of the elastic constants in relation to the angles of carbon fibers can be obtained:

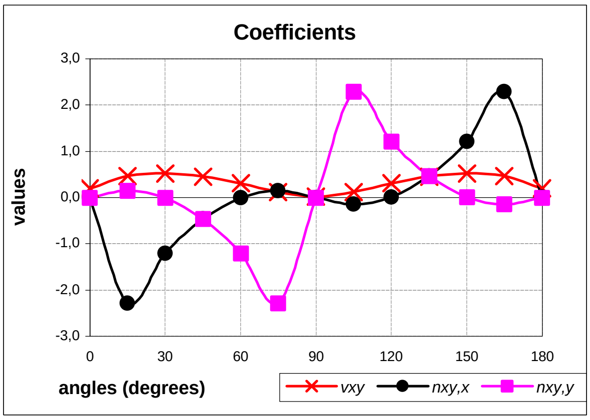

Figure 3. Values of the coefficients of mutual influence and Poisson´s ratio.

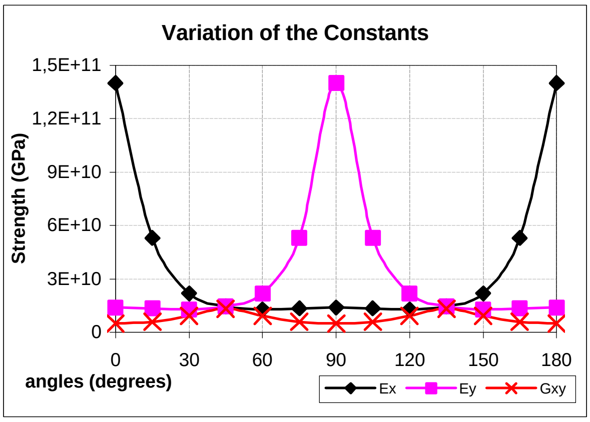

Figure 4. Values of the longitudinal elastic moduli of and the shear modulus.

Considering the variations of the presented elastic properties for the strengthened polymeric laminae with carbon fibers, it is observed that for an angle of fibers inclination of 45° degrees, the values of the longitudinal and transversal moduli of elasticity are equal. This is also noted for the two coefficients of mutual influence. Another characteristic to be observed is that the longitudinal moduli of elasticity are complementary and the coefficients of mutual influence as well. The knowledge of the behavior of the elastic constants with the variation of the angles of fibers is important to optimize the performance of the laminated beams.

3. Displacements and stresses in anisotropic beams

It is analyzed in the following sections the displacements and the stresses in anisotropic laminated beams, subjected to uniformly distributed load, by means of the equations developed for Lekhnitskii et al. (1968) and using of the method developed for Hashin (1967) too.

3.1 Displacements in anisotropic cantilever beams in accordance with Lekhnitskii et al. (1968)

In this section, the displacements of laminated cantilever beams are analyzed, considered homogeneous (method of the homogeneity) and anisotropic, through the solutions obtained for Lekhnitskii et al. (1968) for the analysis of anisotropic beams, subjected to the uniformly distributed load, as Fig. (5):

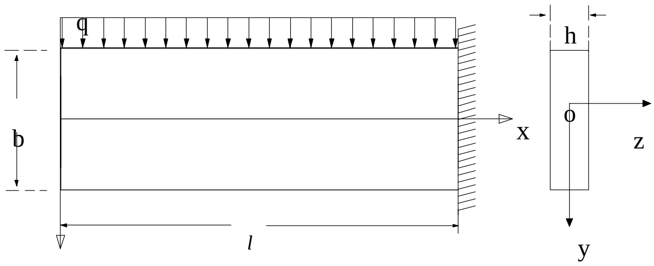

Figure 5. Cantilever beam subjected to uniformly distributed load.

This analysis is made by means of comparative diagrams between the solutions of displacements for an anisotropic beam, according to Eq. (7), and solutions obtained for isotropic beams, submitted and not submitted the influence of the shear, and also, with numerical solutions for deriving orthotropic beams of one commercial program of finite elements with linear approximation. According to Lekhnitskii et al. (1968), the displacements (y – direction) for cantilever anisotropic beams, Fig.(5), can be given by Eq. (6):

$$v = \frac{q}{24E_x I}(x^4 - 6l^2x^2 + 5l^4) + \frac{qh^2}{80I}\left(-3\frac{\nu_{xy}}{E_x} + 4\cdot\frac{1}{G_{xy}} - \frac{32}{3}\cdot\frac{\eta_{xy,x}^{\;2}}{E_x}\right)\left(l^2 - x^2\right) \tag{6}$$

In Eq. (6), E_x is the longitudinal elastic modulus, G_{xy} is the shear modulus, ν_{xy} is Poisson´s ratio and $h_{xy,x}$ is the coefficient of mutual influence of 1st kind that quantifies the influence of shear stresses in normal deformations. For the isotropic beams, when it is considered influence of the shear, the same equations of displacements of the anisotropic beams are used, however, omitting the terms where the coefficients of mutual influence of 1st kind ($h_{xy,x}$) are present. In case of not considering the influence of the shear, the first term of Eq. (6) is used only.

Taking the elastic constants presented in Tab. (1) for a strengthened polymeric lamina with carbon fibers into account, it is necessary to adopt a value for the angle of inclination of fibers, and thus, to determine the values of these constants for the geometric system of reference of the beam. Hence, an angle of inclination (θ) of +15° was adopted and considering Eq. (2) and Tab. (1), the results of elastic constants are shown in Tab. (2):

Table 2. Elastic constants of laminae strengthened with carbon fibers (θ = +15°).

Material	**Elastic Constants**				
	E_x (GPa)	**E_y (GPa)**	**G_{xy} (GPa)**	**ν_{xy}**	**$\eta_{xy,x}$**
Carbon- polymeric Composite	53,00	13,40	5,92	0,47	-2.28

In the analysis, it is also important to evaluate the influence of the cross section height and the beam span relation (h / *l*) in the displacement field. Thus, it is necessary to define the geometric parameters of the beams. Based on Figure 5, it is adopted:

b = 1 m; q = 10 kN/m,

and still, for the relation (h / *l*), two beams were analyzed, according to Table 3:

Table 3. The cross section height –beam span (h / *l*) relations.

Ratio (h/*l*)	**Beam Span (m)**	**Section Height (m)**
1/10	3,00	0,30
1/5	3,00	0,60

In this way, having the presented data, it is possible to make the comparative diagrams of displacements (y – direction) of the laminated beams (LEK) with the isotropic ones, considering the influence of the shear (Timoshenko´s theory - ISO1) or not (Euler-Bernoulli´s theory - ISO2) and with the orthotropic ones whose solutions are numerical (ANSYS ® a Commercial program of finite elements with linear approximation - COM).

For the laminated beam whose relation h / *l* = 1/10, it has the following diagrams:

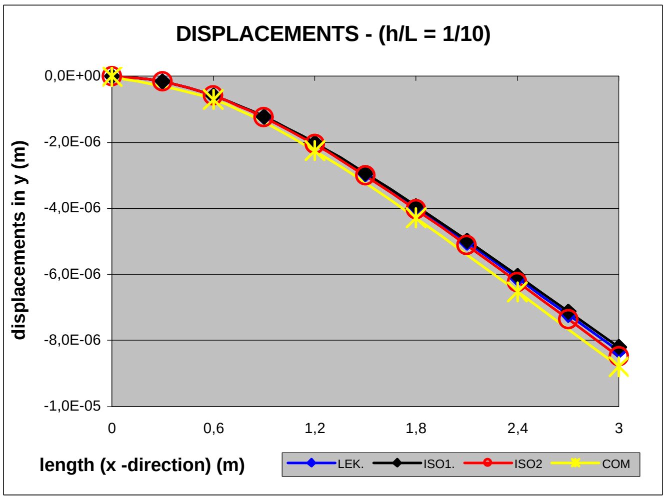

Figure 6. Displacement of a laminated cantilever beam, relation (h / *l* = 1/10), of two isotropic ones, considering the shear (ISO1) and not considering (ISO2), and of an orthotropic one.

For the beams whose relation h / *l* = 1/5, it is obtained the following diagrams:

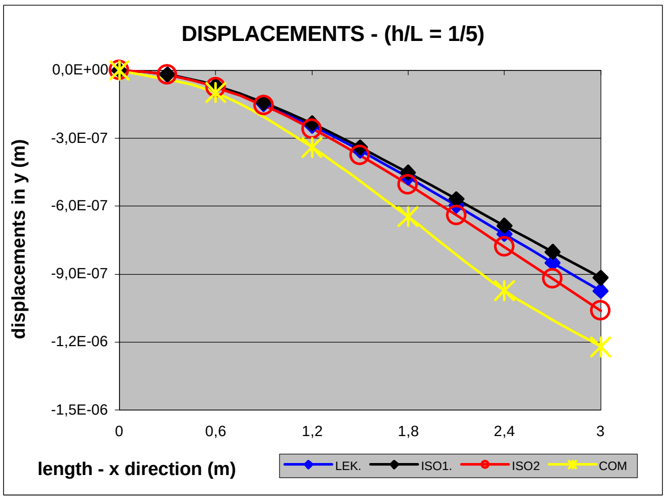

Figure 7. Displacement of a laminated cantilever beam, relation (h / *l* = 1/5), of two isotropic ones, considering the shear (ISO1) and not considering (ISO2), and of an orthotropic one.

After presenting the comparative diagrams of the displacements, it can be observed that, proportional to the increase of the relation (h / *l*), it exists an increase in the differences among the displacements of the cantilever beams analyzed here. Taking the influence of the shear into account, the isotropic beams present the lesser displacements. The laminated beams considered anisotropic (evidencing the presence of the coefficient of mutual influence $\eta_{xy,x}$) also possess lesser displacements than the isotropic beams (theory of Bernoulli-Euler) or the orthotropic ones.

Pointing out still that in the numerical response function of the orthotropic beam it was reckoned the influence of the modulus of elasticity E_y (lesser that the E_x) in the solution, what does not occur with the analytical expressions of displacements, for the plane case of the isotropic elasticity. In the following section, it is analyzed the stresses in laminated beams, using of Hashin´s method (1967).

3.2. Stresses in anisotropic cantilever beams – Analytical Method of Hashin (1967)

Here, the distributions of normal and shear stresses in laminated cantilever beams by means of an analytical method are analyzed, based on the stress function of Airy (ϕ), considered for Hashin (1967) for the analysis of plane anisotropic beams, subjected to the uniformly distributed loads, as Fig. (8):

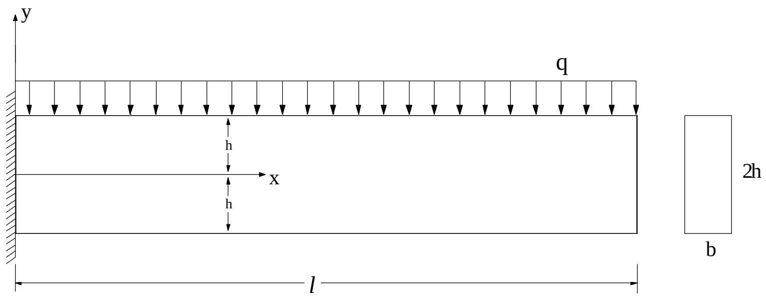

Figure 8. Cantilever beam, subjected to the uniformly distributed loads.

Thus, it is necessary to consider the formulation of anisotropic plane problem. According to Ting (1996), it can be written by:

$$\varepsilon_m = S_{mn}\sigma_n \qquad (m, n = 1, 2, 6) \tag{7}$$

Disregarding the action of the volume forces, the equilibrium equations lead to:

$$\sigma_{i1,1} + \sigma_{i2,2} = 0 \qquad (i = 1, 2) \tag{8}$$

The equilibrium equations are satisfied for the stress function of Airy (ϕ**):**

$$\sigma_{11} = \phi,_{22}\ ; \qquad \sigma_{22} = \phi,_{11}; \qquad \sigma_{12} = -\phi,_{12} \tag{9}$$

Substituting Eq. (9) in Eq. (7) and this result in the following compatibility equation:

$$\varepsilon_{1,22} + \varepsilon_{2,11} - \varepsilon_{6,12} = 0 \tag{10}$$

it is obtained the differential equation:

$$S_{22}\phi,_{1111} - 2S_{26}\phi,_{1112} + (2S_{12} + S_{66})\phi,_{1122} - 2S_{16}\phi,_{1222} + S_{11}\phi,_{2222} = 0 \tag{11}$$

that models the anisotropic plane problem. As an application of it, as an example, the study of stresses in laminated beams, where are considered anisotropic and homogeneous. It is important to notice that the solution of Eq. (11) can be given by the stress function of Airy (ϕ**)**. In this way, Hashin (1967) presented an analytical method that allows the construction of a stress function polynomial that satisfies Eq. (11):

$$\phi(x, y) = \sum_{m=0}^{m=M}\sum_{n=0}^{n=N} C_{mn}\, x^m y^n \tag{12}$$

where C_{mn} are constants to be determined by the resolution of a linear system of equations.

Introducing Eq. (12) in Eq. (11) and equating the coefficients $x^m y^n$ of equal powers, Hashin (1967) develops relations, among the C_{mn} coefficients, that are used for assembling of the system of equations:

$$\begin{aligned} &S_{2222}(m+2)(m+1)m(m-1)C_{m+2,n-2} - 4S_{2212}(m+1)m(m-1)(n-1)C_{m+1,n-1} \\ &+ 2(S_{1122} + 2S_{1212})m(m-1)n(n-1)C_{mn} - 4S_{1112}(m-1)(n+1)n(n-1)C_{m-1,n+1} \\ &+ S_{1111}(n+2)(n+1)n(n-1)C_{m-2,n+2} = 0 \end{aligned} \tag{13}$$

where $m \geq 2$ and $n \geq 2$. If in any problem, the biggest power of variable *x* in Eq. (12) is *M*, then the biggest power of the variable *y* is given by *N* = *M*+3 and thus, Eq. (12) assumes the form:

$$\phi(x,y)=\sum_{m=0}^{m=M}\sum_{n=0}^{n=M+3} C_{mn}x^m y^n \tag{14}$$

The number of necessary equations for each specific problem, from the relations of Eq. (13) and of the boundary conditions, can be obtained through the arithmetical series:

$$S=\frac{1}{2}(M+1)(M+8) \tag{15}$$

From this, it is possible to find the coefficients C_{mn} and determined, then, the stress function that is the solution of the problem of stresses. Following this methodology (Hashin´s method (1967)) it is applied for the study of stresses in laminated beams. The obtained results of stresses are compared with solutions for isotropic beams too, considering the influence of the shear, and later, without this consideration, and also with numerical solutions for orthotropic beams. Indeed, for the cantilever beam of Fig. (8) it has the following boundary conditions:

$$\phi(x,-h)=0 \quad ; \quad \phi(x,h)=\frac{q}{2}(l-x)^2 \tag{16.1}$$

$$\phi(x,-h)_{,2}=0\ ; \quad \phi(x,h)_{,2}=0 \tag{16.2}$$

The biggest power of *x* in the boundary conditions (16) is *M* = 2. Thus, *N* = *M*+3 = 5, and the stress function becomes:

$$\begin{aligned}\phi(x,y)&=\sum_{m=0}^{m=2}\sum_{n=0}^{n=5} C_{mn}x^m y^n = \qquad\qquad (\ m+n\ \le 5)\\ &=C_{00}+C_{01}y+C_{02}y^2+C_{03}y^3+C_{04}y^4+C_{05}y^5+C_{10}x+C_{11}xy+C_{12}xy^2+C_{13}xy^3+C_{14}xy^4+\\ &+C_{20}x^2+C_{21}x^2y+C_{22}x^2y^2+C_{23}x^2y^3\end{aligned} \tag{17}$$

The number of necessary equations for the determination of the C_{mn} coefficients is given by:

$$S=\frac{1}{2}(M+1)\cdot(M+8)=15 \tag{18}$$

Inserting the stress function (17) in the boundary conditions (16) and equating the coefficients *x* of equal powers it has 12 equations. The three remaining equations come from the relations of Eq. (13), respecting thus the inequality $m+n\le 5$. From the 15 equations it is possible to solve a system to find the 15 unknown coefficients C_{mn}. Substituting the coefficients, the stress function (17) becomes:

$$\phi(x,y)=\frac{-q(h+y)^2\begin{pmatrix}10\alpha_{04}^2(2h-y)(l-x)^2-2\alpha_{13}^2(h-y)^2y+\alpha_{04}(h-y)^2\cdot\\ \cdot(5\alpha_{13}(l-2x)+2\alpha_{22}y)\end{pmatrix}}{80\cdot\alpha_{04}^2\cdot h^3} \tag{19}$$

where the constants α_{ij} are substituted by the usual constants of engineering:

$$\alpha_{04}=S_{1111}=S_{11}=\frac{1}{E_x};\quad \alpha_{22}=S_{1122}+2\cdot S_{1212}=\frac{-\nu_{xy}}{E_x}+\frac{1}{2\cdot G_{xy}};\quad \alpha_{13}=4\cdot S_{1112}=\frac{\eta_{xy,x}}{2\cdot E_x} \tag{20}$$

By using Airy´s function, it has the following stresses:

$$\sigma_x = \frac{\partial^2\phi(x,y)}{\partial y^2} = \frac{q\left[\left[\left(3E_x y + 4G_{xy}\begin{pmatrix}-12y\cdot\eta_{xy,x}^2 + 5l\cdot\eta_{xy,x} - 10x\cdot\eta_{xy,x} - \\ -3\cdot\nu_{xy}\cdot\eta_{xy,x}\cdot y\end{pmatrix}\right)\right]h^2\right]}{40h^3 G_{xy}} + \tag{21}$$

$$+\frac{q\left[5y\left(2G_{xy}\begin{pmatrix}3l^2 - 6xl - 6\cdot\eta_{xy,x}\cdot yl + 3x^2 + 8\cdot\eta_{xy,x}^2\cdot y^2 + \\ +2\cdot\nu_{xy}\cdot y^2 + 12\cdot\eta_{xy,x}\cdot x\cdot y\end{pmatrix} - E_x y^2\right)\right]}{40h^3 G_{xy}}$$

$$\sigma_{xy} = \frac{\partial^2\phi(x,y)}{\partial y\partial x} = \frac{q\left(3l - 3x - 4\cdot\eta_{xy,x}\cdot y\right)\left(y^2 - h^2\right)}{4h^3} \tag{22}$$

In the comparative diagrams, presented as follows, it is considered the same type of composite laminae, whose elastic constants are presented in Tab. (2). The used geometric parameters are presented in Tab. (3), with a load of 10 kN/m applied in the beams. For the beams whose relation h / l = 1/10, it has the following diagrams of normal and shear stresses in the cross section:

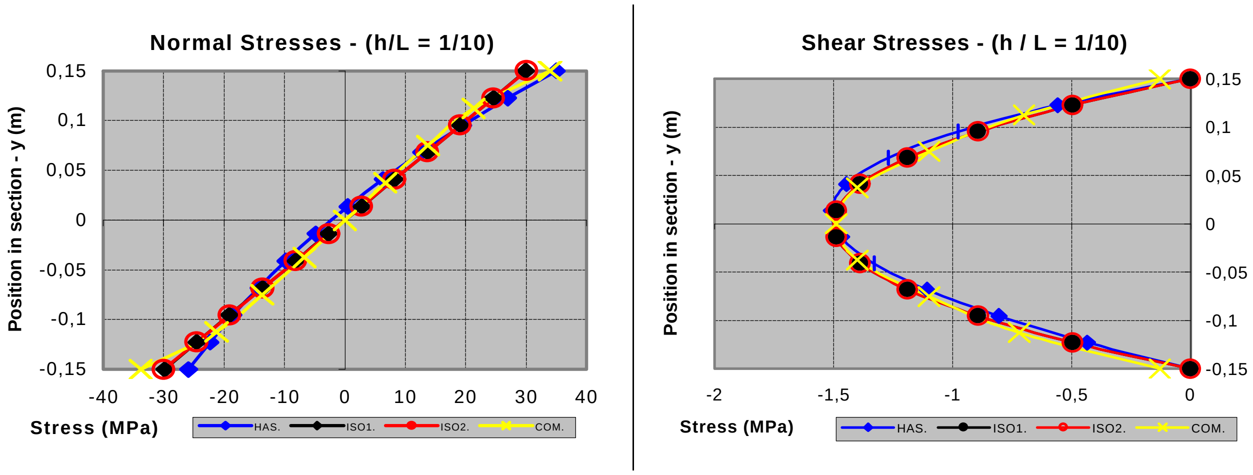

Figure 9. Normal and shear stresses in the laminated cantilever beam, relation (h / l = 1/10), in two isotropic ones, considering the shear (ISO1) and not considering (ISO2), and in an orthotropic one.

For the beams whose relation h / l = 1/5, it has the diagrams:

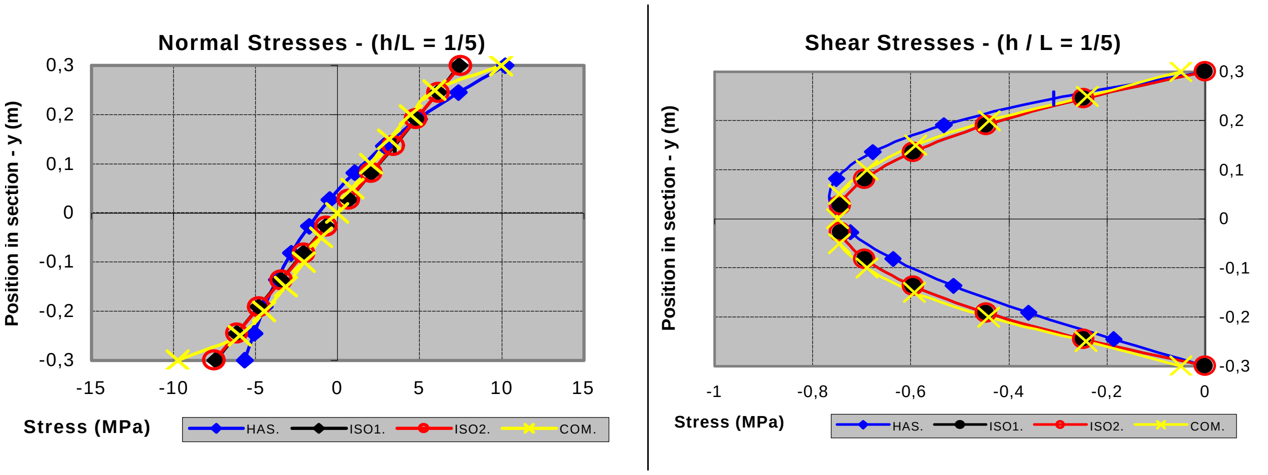

Figure 10. Normal and shear stresses in the laminated cantilever beam, relation (h / l = 1/5), in two isotropic ones, considering the shear (ISO1) and not considering (ISO2), and in an orthotropic one.

It is observed that an increase of the relation (h / *l*) results an increase in the differences between the isotropic and orthotropic elastic behavior of the beams. It also notices that the behavior of the stresses in the laminated beams is not symmetrical.

4. Conclusions

With the intention to contribute for the studies on the application of anisotropic beams in mechanical structures, this paper presented both some general concepts on anisotropy and an analysis of the influence of the anisotropy on distributions of stresses and displacements in fiber-reinforcement laminated beams.

In the analysis of the displacements it was verified that the laminated beams presented greater difference of behavior in relation to the isotropic beams, especially when it is not taken the shear effect into consideration, for the two relations (h / *l*). With respect to the displacements of the isotropic beams, it is noticed that the consideration of the shear effect in these beams (Timoshenko´s theory) leads to the different results of the presented for the isotropic beams where this consideration is not made (Euler-Bernoulli´s theory), mainly for the beams of lesser span. Therefore, depending on the orientation of fibers in the laminae of the analyzed beam, the determination of the displacements of the beam, admitting the theory of Euler-Bernoulli, can originate results very different of whom that will really occur.

Considering the participation of the cross section height – beam span relation (h / *l*) in the behavior of the distributions of displacements and of stresses, it is observed that an increase in this relation produces greater differences in relation to the isotropic beams considered in this work. This is due to the fact that beams with lesser spans, the contribution of the shear in the displacements is bigger, bringing up then as consequence, a bigger participation of the anisotropy, associated with the mutual influence coefficients that relate shear with normal deformation.

In addition to this, it is also emphasized that the influence of these coefficients in numerical way constitutes an important point to be considered in this kind of research as it was presented in this article.

In general, the stress functions have shown efficient to solve of problems of the mechanics of solids considering the analytical development only. However, it is necessary to point out, that the methods of solutions based on the stress functions are approached due to the polynomial form of the stress function can not be admitted as an accurate solution.

5. Acknowledgement

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6. References

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