Effective Damping Value of Piezoelectric Transducer Determined by Experimental Techniques and Numerical Analysis

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Abstract. Piezoelectric transducer design involves mathematical modelling and experimental verification, which are necessary to validate the piezoelectric transducers. To make a precise numerical model by using finite element method (FEM), it is necessary to know dielectric, piezoelectric and mechanical properties. Therefore, damping is a hard property to measure,since it is related to mechanical loss and it is dependent on the frequency. In addition, damping values for piezoelectric and non-piezoelectrics materials, such as, resins, steel, aluminum etc., which are usually applied to assemble these transducers are not appropriately given for FEM software. The objective of this work is to determine damping values of these materials so they can be used in a FEM software. Damping values are determined by combining experimental and numerical techniques. For piezoceramics the damping is determined through the quality factor (Qm) obtained by measuring the admittance curve which are influenced by damping. By using these damping values, harmonic FEM simulations of piezoceramics and piezoelectric transducers are performed and the simulated electrical admittance curve is compared with the measured one. Damping determination for non-piezoelectric materials are done by comparing experimental and simulated results.

Keywords: Damping measurement, piezoceramic, piezoelectric transducer, electrical admittance analysis, finite element method.

1. Introduction

Piezoelectric materials produce a charge proportional to an applied stress and vice-versa, as first discovered in 1880 by Jacques and Pierre Curie, and is natural sensor and electrical generator materials (Jona and Shirane, 1960). The converse piezoelectric effect, as utilized in actuator applications, describes the deformation of the material in response to an applied electric field.

Piezoelectric materials, such as PZT-4A, PZT-5A, PZT-5H, PZT-7A, PZT-8, PMN, PMN-PT, PVDF, etc., have its dielectric, piezoelectric and mechanical properties generally given by manufacturer catalog. Therefore, the damping is an important property is not given by manufacturer. Damping is related to mechanical loss and it is dependent of the frequency. It is important to know a precise damping value and other properties to model piezoelectric transducer, because piezoelectric transducer design involves mathematical modeling and experimental verification, which are necessary to validate the piezoelectric transducers. These analyses allow us to verify experimental boundary conditions influence and how to model them. However, the currently literature does not discuss these topics in a comprehensive way. In addition, the mechanical damping influence is a difficulty to perform numerical modeling of piezoelectric transducer.

Piezoelectric transducer are manufactured by using piezoceramics and non-piezoelectric materials (isotropic), such as epoxy resin, metals etc. These materials have its Young modulus, density, Poisson ratio, and thermic expansion coefficient given by handbooks. Therefore, it is necessary to develop a measurement technique to determine an important mechanical property, which is the damping material. By knowing all piezoelectric and non-piezoelectric property values of piezoelectric transducer, finite element method (FEM) and topology optimization (Silva *et al.*, 1998_A, Silva *et al*, 1998_B, Silva *et al*, 1999, Silva *et al*, 2000) can be applied to design piezoelectric transducers with desired behavior without manufacturing prototypes.

Therefore, the objective of this work is to determine damping values of these materials so they can be used in a FEM software. Damping values are determined by combining experimental and numerical techniques. Numerical analyses of piezoelectric transducers are done by using ANSYS™. Experimental analyses are done by using an impedance analyzer (HP4194A) to determine the electrical admittance curves of piezoceramics and piezoactuators. Simulations are done in 2D since 3D simulations are computationally expensive. These simulations are done considering plane stress assumption. The damping values are obtained considering damping definitions in ANSYS software.

Next sections will describe theory and analysis done in this work. In section 2, it is shown the viscous damping formulation since the transducer is supposed to vibrate in fluid medium. In section 3, it is shown the piezoelectric equations of motion including damping. Section 4, it describes the techniques applied to analyze piezoelectric transducers. In section 5 experimental and numerical results of electrical frequency responses of piezoelectric transducers are compared. Finally in section 6, some conclusions are given.

2. Damping

Damping is a material property, which is very important to vibration control in engineering. In addition, numerical results of vibration and acoustical analysis are very sensitive to this parameter. For the mechanical damping treatment of structure is necessary to consider three parameters (Nashif *et al.*, 1985, Cook, 1995, Cai *et al.*, 2002): damping; mass, and stiffness. These three parameters are needed to design and optimize piezoelectric transducers by using numerical modeling since all of them have some effect in piezoelectric transducer dynamic response. In addition, most part of systems that dissipate energy by vibration is non-linear. Therefore, it is necessary to develop models of ideal damping with satisfactory approximation.

The several types of damping are (Nashif *et al.*, 1985, Cai *et al.*, 2002):

- Viscous damping, due to energy dissipation;
- Structural damping, due to the material properties;
- Friction damping, due to mechanical sliding between surfaces.

Normally, viscous damping occurs in piezoelectric transducer vibration at low kHz range. Therefore, in this work only the viscous damping is explained and it is shown how to use FEM to model it.

2.1. Viscous Damping

When mechanical systems oscillate in a fluid medium, such as air, gas, water and oil, the fluid resistance to body movement causes energy dissipation. The amount of dissipated energy is dependent on factors such as size, body shape, fluid viscosity, frequency vibration and body vibration velocity. In the viscous damping, the damping forces can be expressed as:

$$F = c\dot{u}, \tag{1}$$

where c is a proportionality constant and $\dot{u}$ is the body velocity. When the single mass-spring system is freely oscillating, the motion equation is

$$m\ddot{u} + c\dot{u} + ku = 0, \tag{2}$$

where $\ddot{u}$ is the body acceleration, m is the mass and k is the modal stiffness. Equation (2) has the follow solution

$$u = \left[A\exp\left(i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}\, t \right) + B\exp\left(-i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}\, t \right) \right] \exp\left(-\frac{c}{2m} t \right), \tag{3}$$

where A and B are arbitrary constants, which are dependent on motion starting. Analysis of influence of stiffness, mass and damping, in the resonance frequency ω, of the system, can be obtained from exponential radical of Eq.(3)

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}\, t, \tag{4}$$

the contribution of each term in the response curve of piezoelectric material and non-piezoelectric, in terms of the resonance frequency is:

- Decrease mass → increase resonance frequency [Fig.(1a)];
- Increase stiffness → increase resonance frequency [Fig.(1b)];
- Increase damping → decrease (a little) resonance frequency and displacement amplitude [Fig.(1c)].

Notice that, Fig.(1) shows resonance and anti-resonance frequencies. These response curves are for piezoelectric material.

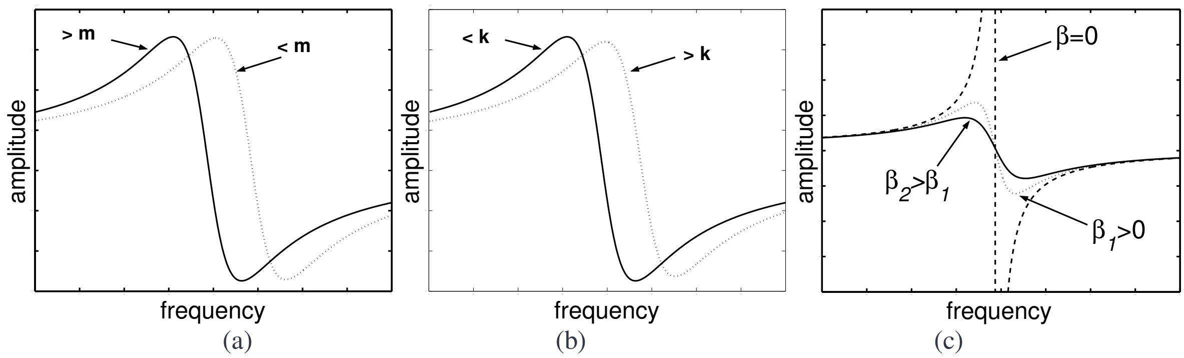


Figure1. Frequency displacement amplitude under influence: (a) mass (m); stiffness (k); damping (β).

The damping behavior of damped system is dependent on numerical values of Eq.(4). Considering the resonance frequency expression (ω=k/m) the critical damping c_c can be obtained:

$$\left(\frac{c_c}{2m}\right)^2 = \frac{k}{m}, \tag{5}$$

which can be written as

$$c_c = 2m\omega_n. \tag{6}$$

An important parameter to describe the damping properties is the mechanical damping coefficient:

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n}. \tag{7}$$

The relationship between natural resonance frequency, of a non-damped system, and resonance frequency of a damped system is obtained by Eqs.(4) and (7):

$$\omega_r = \omega_n\sqrt{1-\zeta^2}. \tag{8}$$

The Rayleigh equation is the most common way to describe the damping coefficient:

$$c = \alpha m + \beta k, \tag{9}$$

where α is the mass multiplication factor and β is the stiffness multiplication factor. Thus, different physical damping can be modeled by determining α and β values (Lerch, 1990). These relationships are:

- Non-damped ($\alpha = 0$ and $\beta = 0$);
- Viscous damping ($\alpha = 0$ and $\beta > 0$);
- Damping proportional to the mass ($\alpha > 0$ and $\beta = 0$);
- Rayleigh damping ($\alpha > 0$ and $\beta > 0$).

The α and β values are dependent on energy dissipation characteristic of structure. Currently, these values cannot be obtained through direct calculation and they must be measured. They are determined through modal damping ratio (ζ_{mr}):

$$\zeta_{mr} = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2}, \tag{10}$$

which is the ratio between effective damping and critical damping for a particular mode shape r.

Normally in piezoelectric transducer vibration occurs the viscous damping. Thus, α=0 and the damping β can be determined by known values of ζ_{mr} and ω_r, representing structural mechanical damping. When a structure is under an harmonic excitation, it can be modeled as subjected to viscous damping with coefficient (Naillon *et al.*, 1983}

$$\beta = \frac{1}{\omega Q_m} \tag{11}$$

where Q_m is the mechanical quality factor. Q_m can be experimentally determined through electrical impedance analysis (Holland and EerNisse, 1969} by resonance and anti-resonance frequencies of mode shape *r*.

After determined the Q_m factor and consequently the damping material to mode shape *r*, it is used in FEM simulations. Next section describes the piezoelectric equations of motions, and how to apply the damping in simulations.

3. FEM Piezoelectric Equations of Damping Motion

Piezoelectric equations of motion can be obtained considering the minimum energy principle from variational calculus (Allik and Hughes, 1970, Naillon *et al.*, 1983, Ostergaard and Pawlak, 1986). By using finite element formulation, resulting equations are able to represent piezoelectric medium in a matrix. Matrix equations can be written in terms of displacement $\{\mathbf{u}\}$, electric potential $\{\mathbf{\Phi}\}$, mechanical forces $\{\mathbf{F}\}$ and electrical charges $\{\mathbf{Q}\}$ at nodal points:

$$[\mathbf{M_{UU}}]\{\ddot{\mathbf{u}}\} + [\mathbf{C_{UU}}]\{\dot{\mathbf{u}}\} + [\mathbf{K_{UU}}]\{\mathbf{u}\} + [\mathbf{K_{U\Phi}}]\{\mathbf{\Phi}\} = \{\mathbf{F}\}, \tag{12}$$

$$[\mathbf{K_{U\Phi}}]^{\mathbf{t}}\{\mathbf{u}\} + [\mathbf{K_{\Phi\Phi}}]\{\mathbf{\Phi}\} = \{\mathbf{Q}\}, \tag{13}$$

where $[\mathbf{M_{UU}}]$ is mass matrix, $[\mathbf{K_{UU}}]$ is elastic stiffness matrix, $[\mathbf{K_{U\Phi}}]$ is piezoelectric stiffness matrix and $[\mathbf{K_{\Phi\Phi}}]$ is dielectric stiffness matrix. The term $[\mathbf{C_{UU}}]$ is the damping matrix given by:

$$[\mathbf{C_{UU}}] = \alpha[\mathbf{M_{UU}}] + (\beta + \beta_c)[\mathbf{K_{UU}}] + \sum_{\mathbf{j=1}}^{\mathbf{Nmat}} \beta_j \mathbf{K_j} + \sum_{\mathbf{k=1}}^{\mathbf{Nele}} \mathbf{C_k} + \mathbf{C}_\zeta\ . \tag{14}$$

ANSYS[TM] damping matrix $[\mathbf{C_{UU}}]$ can be written as a function of the structure damping properties, which normally are dependent on the frequency. Equation (14) is the most common damping matrix (Cook, 1995, Kohnke, 2001). Each term of Eq. (14) is used in ANSYS[TM] as follow: α is constant mass matrix multiplier; β and β_j are constant stiffness matrix multiplier of system and of each *j* material, respectively; β_c is variable stiffness matrix multiplier; $[\mathbf{C_\zeta}]$ is structure damping matrix; $[\mathbf{C_K}]$ is element damping matrix..

The damping value of each material used in ANSYS[TM] model is β_j. When damping is applied to simulations, it is more convenient to approximate it by Rayleigh damping, which relates the matrix:

$$[\mathbf{C_{UU}}] = \alpha[\mathbf{M_{UU}}] + \sum_{\mathbf{j=1}}^{\mathbf{Nmat}} \beta_j \mathbf{K_j}\ . \tag{15}$$

Therefore, for viscous damping $\alpha = 0$, thus:

$$[\mathbf{C_{UU}}] = \sum_{\mathbf{j=1}}^{\mathbf{Nmat}} \beta_j \mathbf{K_j} = \sum_{\mathbf{j=1}}^{\mathbf{Nmat}} \frac{1}{\omega_{Rj} Q_{mj}} \mathbf{K_j} \quad , \tag{16}$$

Next section shows piezoelectric transducers analyzed (flextensional actuators) and the techniques applied to analysis. These analyses are done by electrical admittance curves and numerical techniques, which describes considerations done in simulations.

4. Analysis

To check the damping results, experimental electrical analysis is applied and it is compared with numerical analysis. Analyses are done in piezoceramics to determine the damping value and in flextensional piezoactuators to verify the damping value obtained for piezoceramic and to determine the aluminum and epoxy resin damping. Epoxy resin (Araldite 502/956) is applied to bond the aluminum endcap on piezoceramic. The endcap cover flextensional actuator and produces displacement and sound amplification (Rolt, 1990, Nader *et al.*, 2001_A, Nader *et al.,* 2001_B, Silva *et al.*, 2003).

Flextensional piezoactuators analyzed in this work are shown in Figs.(2) and (3). These actuators are denominated f1a20827 and f1c0815, and they have a piezoceramic block as active element. In Figs. (2) and (3) are shown the approximately piezoceramic dimensions applied in flextensional actuators. Table 1 shows the physical dimensions of piezoceramics analyzed in this work to determine the damping coefficients. Directions assumed to ANSYS[TM] simulations are shown in Fig. (3).

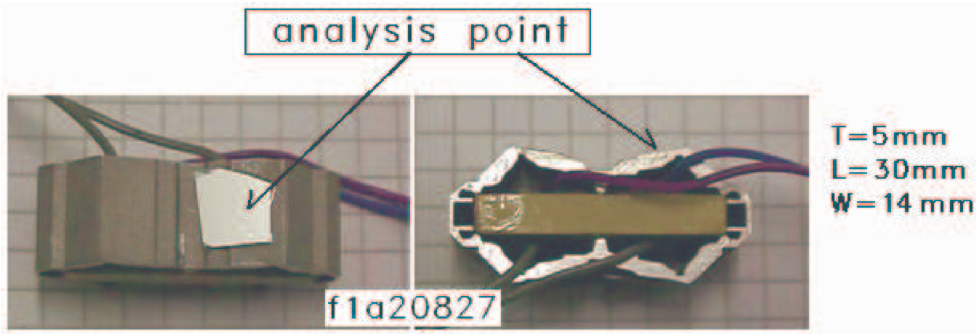


Figure 2. Flextenxional piezoactuator f1a20827.

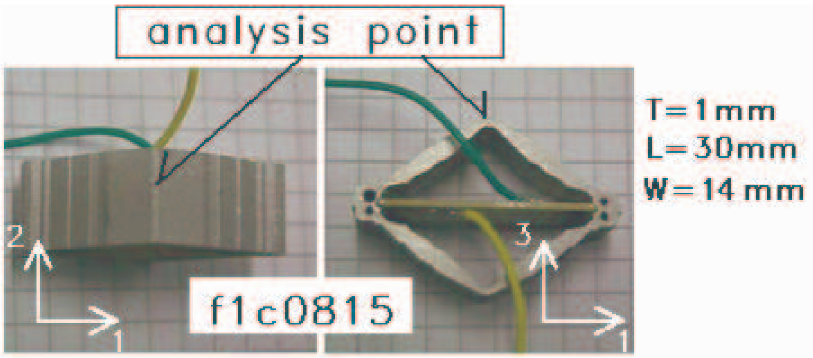


Figure 3. Flextensional piezoactuator f1c0815.

Table 1. Piezoceramic block dimensions.

	thickness	width	length
ceramic 1	4.97 mm	13.90 mm	29.90 mm
ceramic 2	2.95 mm	13.95 mm	29.95 mm
ceramic 3	0.99 mm	13.95 mm	29.87 mm

4.1 Electrical Admittance Analysis

PZT5A piezoceramics used to build prototypes are initially analyzed by using the impedance analyzer (HP4194A) to determine their resonance frequencies and electrical admittance curves. These curves allowed us to determine the mechanical quality factor (Q_m). Electrical admittance curves are acquired by HP4194A between 10 to 60 kHz, and they are compared with simulated curves calculated by ANSYS^{TM} in the same frequency range, considering damping of piezoceramic, aluminum and epoxy resin. This frequency range was chosen due to bellow 10 kHz, for piezoceramics or piezoactuators, there are resonance frequencies, and besides 60 kHz is larger than first resonance frequency and less than other resonance frequencies.

4.2 Finite Element Method

Piezoceramic and flextensional actuators are modeled by finite element using the software ANSYS. Since the piezoceramic has a prismatic shape and it is symmetric, 2D FEM models are built with 1/4 of symmetry. Once the depth of piezoceramic is small in relation to their other dimensions the plane stress assumption should be adopted. This avoids the need of building a 3D FEM model, which is computationally expensive. However, when the electrical admittance is measured by using the impedance analyzer (HP4194A), piezoceramic and the entire actuator are free, and thus the plane stress assumption prevails. Therefore, for the electrical admittance calculations, a plane stress condition was assumed for the 2D model. A 1/4 symmetry model of flextensional piezoactuator is built considering the symmetry of the actuator with electrodes on both surfaces normal to the 3-direction (poling direction), as shown in Fig. (3).

Material properties used in ANSYS^{TM} of piezoceramic PZT5A, aluminum and epoxy resin are shown in Tab.(2).

Table 2: Material Properties of PZT5A, aluminum and epoxy resing used in FEM.

PZT5A	
elastic constants	(10^{10} N.m^{-2})
c_{11}^E	12.1
c_{12}^E	7.54
c_{13}^E	7.52
c_{33}^E	11.1
c_{44}^E	2.11
c_{66}^E	2.26
piezoelectric constants	(C.m^{-2})
e_{31}	-5.4
e_{33}	15.8
e_{51}	12.3
ε^S: dielectric constants	
$\varepsilon^S{}_{11}/\varepsilon_0$	916
$\varepsilon^S{}_{33}/\varepsilon_0$	830
density (ρ)	7750 kg.m^{-3}

aluminum	
Young's modulus (E)	71 x 10^9 N.m^{-2}
density (ρ)	2700 kg.m^{-3}
damping (β_j)	15 x 10^{-8}
Poisson's ratio (σ)	0.33

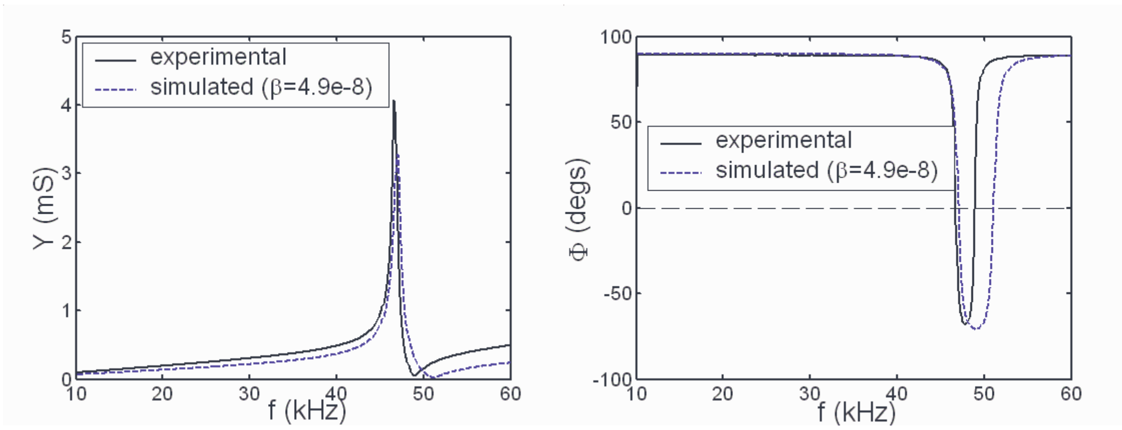
epoxy resin	
Young's modulus (E)	4.25 x 10^9 N.m^{-2}
density (ρ)	1160 kg.m^{-3}
damping (β_j)	40 x 10^{-8}
Poisson's ratio (σ)	0.38

In this work it is assumed the viscous damping, which it is related to vibration in a fluid medium. It is enough to simulate the physical piezoelectric transducer damped. Through these hypotheses, harmonic analysis are performed. Resonance frequencies and admittance curves are obtained for each piezoceramic and piezoactuator.

5. Results

5.1. Piezoceramic with 5 mm thickness

From experimental analysis the 5 mm thickness piezoceramic density value was determined to be equal to ρ=7591 kg.m^{-3}, first resonance frequency to length mode shape (*1-direction*) is f_0 = 46.6 kHz and mechanical quality factor has value equal to Q_m=69. By using Eq.(11), it is calculated the damping for this resonance frequency (*1-direction*). This value is β_j=4.9x10^{-8}. Electrical admittance results (Y) obtained from experimental and numerical techniques are shown in Fig. (4a). In simulations by ANSYS™ was only applied β_j to each material. The mass matrix multiplier (α) is considered null to this frequency range.



(a) (b)

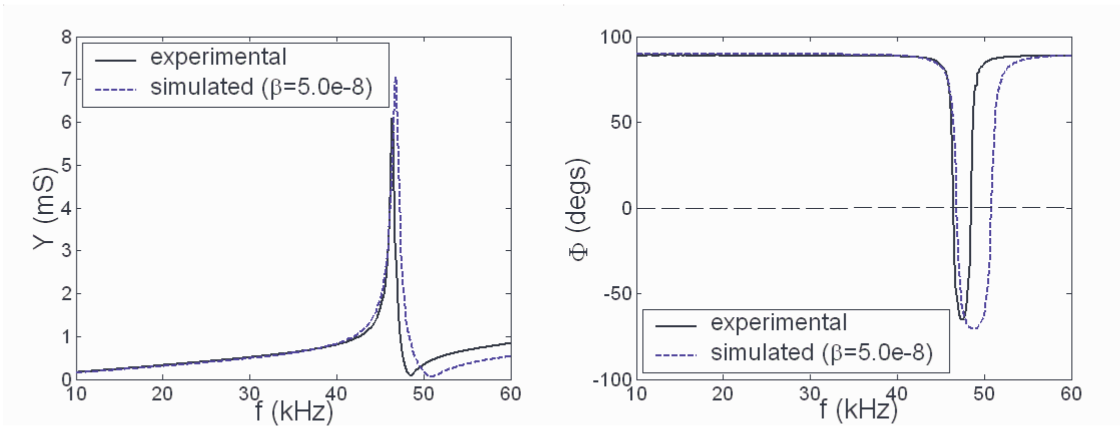
Figure 4. Electrical frequency response of piezoceramic 1 with 5 mm thickness. (a) electrical admittance and (b) electrical phase.

From the electrical admittance curve [Fig. (4a)], it can be noticed that the experimental and simulated values up to the anti-resonance frequency (approximately 50 kHz) are close. Beyond this value there is an increase in the difference between experimental and simulated electrical admittance values and resonance frequency. It is possible due to the fact the coupling factor value (k_{31}) measured (Holland *et al.*, 1969, IEEE, 1987) to this piezoceramic shows a deviation of 5.5% in relation to manufacturer catalog (manufacture 0.36, measured 0.34). By considering this deviation, experimental and simulated results show a good agreement. Thus, the value found for damping is satisfactory.

Electrical phase curve (Φ) is also shown experimental and numerical results approximately equal. [Fig.(4b)]. However, numerical phase results, obtained by ANSYS™, shown the phase between 180° and 0°. Therefore, experimental results obtained by using HP4191A always shown phase results between 90° and -90°. Then, the numerical phase curves showed in this work are displaced in -90°.

5.2. Piezoceramic with 3 mm thickness

From experimental analysis the 3 mm thickness piezoceramic density value was determined to be equal to ρ =7680 kg.m^{-3}, first resonance frequency to length mode shape (*1-direction*) is f_0 = 46.4 kHz and mechanical quality factor has value equal to Q_m=69. The damping coefficient value is β_j=5.0x10^{-8}. Electrical admittance results obtained from experimental and numerical techniques are shown in Fig.(5a).



(a) (b)

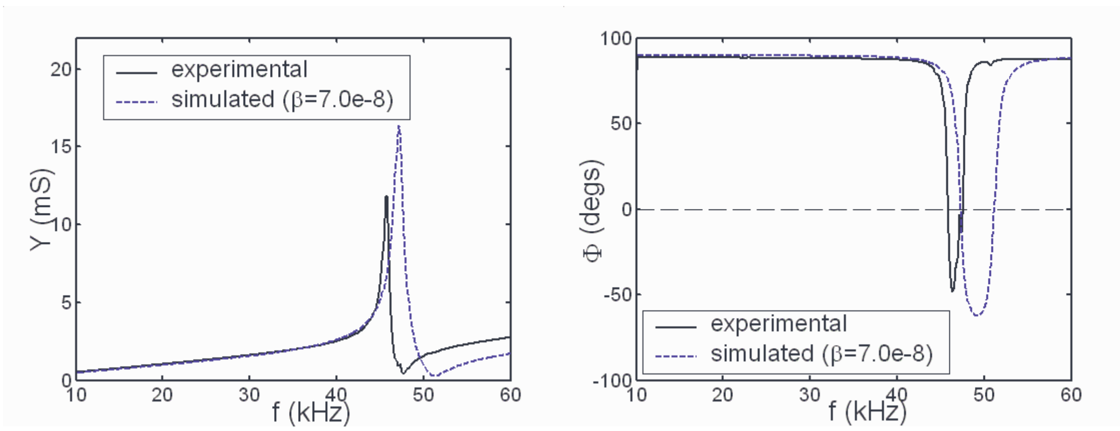
(b)

Figure 5. Electrical frequency response of piezoceramic 2 with 3 mm thickness. (a) electrical admittance and (b) electrical phase.

From the electrical admittace curve [Fig.(5a)], it can be noticed that the value found for damping is satisfactory. Electrical phase curve is also shown experimental and numerical results approximately equal [Fig.(5b)].

5.3. Piezoceramic with 1 mm thickness

Analyses of 1 mm thickness piezoceramic shown a low quality factor for first resonance (Q_m = 50). Therefore, by comparing experimental and numerical results to electrical admittance curves and phase [Figs.(6a) and (6b)], it is possible to note that this value will can be less, as is shown in Figure. The piezoceramic density value was determined to be equal to ρ =7674 kg.m^{-3} and first resonance frequency to length mode shape (*1-direction*) is f_0 = 45.7 kHz. The damping coefficient value is β_j=7.0 x10^{-8}.



(a) (b)

(b)

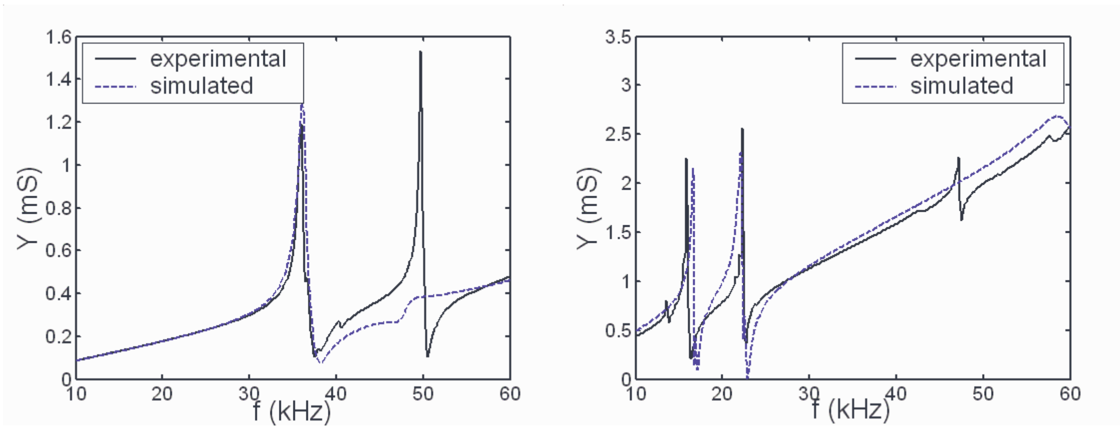
Figure 6. Electrical frequency response of piezoceramic 3 with 1 mm thickness. (a) electrical admittance and (b) electrical phase.

From the electrical admittance curve [Fig. (6a)], it can be noticed that the value found for damping is satisfactory. Electrical phase curve is also shown experimental and numerical results approximately equal [Fig.(6b)]. The deviation between experimental and numerical resonance frequency is due to loss and defects in physical piezoceramic.

Through these three analyses, it is shown that damping hypothesis shows convergence to frequency range at low kHz operation.

5.4. Flextensional piezoactuators

By using these damping value in flextensional models, electrical admittance responses showed in Figs.(7a) and (7b) are obtained for flextensional f1a20827 and f1c0815, respectively. Notice in Figs.(7a) and (7b) have an agreement in first resonance frequency. However, second frequency mode to numerical results does not appear, or light appears. They were manufactures a lot of flextensional transducers and analyzed in other works (Silva *et al.* 2003, Nader *et al.*, 2003). In all cases there is a resonance frequency close to 50 kHz. The effect observed in this work will be analyzed in a future work.



(a) (b)

Figure 7. Electrical frequency response of piezoactuators: (a)f1a20827 and (b) f1c0815

6. Conclusions

It is important to know a precise damping value of piezoelectric transducer components to obtain a more precision modeling. This worked showed that damping value for piezoelectric materials could be very well determined by combining numerical and experimental techniques. Analysis of piezoceramic was done by electrical admittance response, at low kHz, and the decaying displacement of piezoelectric transducers. From these analyses were possible to determine reasonable damping values for FEM simulations.

As a future work, a method for determining the aluminum damping will be developed based on decaying displacement analysis during transient excitation by comparing laser interferometry results with simulated results. The non-present resonance frequency close to 50 kHz in numerical analysis will be analyzed in a future work.

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