FILTERING GPS NAVIGATION SOLUTIONS FOR STATIC POSITIONING

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Abstract. *The purpose of this work is to estimate in real time the positioning vector of a static landmark from the navigation solutions obtained by GPS receivers. In this work the Kalman filter will be used due to its robustness in real time applications, recursive and sequential nature, without unnecessary storage of observations, as they can be processed while being collected, and the convergence is obtained while observations are processed. A comparison will be done between the estimated state vector and the landmark geodetic coordinates which were previously surveyed by IBGE. This work aims at performing a preliminary investigation on real time positioning techniques, with can thereafter be extended to navigation of space vehicles, including more complex features of non linear dynamic and the measurements.*

Keywords. *Global Positioning System (GPS), Real time StateEstimation, Kalman Filter.*

1. Introduction

The Global Positioning System (GPS) is a satellite navigation system that allows the users to determine position, velocity and the time with high precision. Its main purposes are aid to radionavigation in three dimensions with high precision positioning, navigation in real time, global coverage and quick acquisition of data sent by the GPS satellites (Parkinson, 1996). The aim of this work is to develop a the Kalman filter to deliver real time estimates of the position of a static landmark from the navigation solutions obtained or computed by GPS receivers. It is expected that this preliminary investigation on real time positioning techniques can thereafter be extended to navigation of space vehicles, including more complex features of non linear dynamics and measurements.

2. Navigation Solution by GPS

The geometric position determination (navigation solution) by GPS is done through a trilateration method (see Fig.1). The GPS receiver of the user (in this example is a satellite) receives GPS signal which allow to compute the position of the GPS satellites in relation to a system of reference, and measures the signal transmission time. It is therefore possible to calculate the distance between the receiver and the corresponding GPS satellite transmitting the signal. Thus, the receiver is able to calculate its position in relation to a reference system. Nevertheless, because in general the receiver clock is not so accurate as the GPS clock, the clock offset must also be computed. In this case at least four GPS satellites are needed to fully obtain the geometrical solution, including the user clock offset estimation. Every GPS receiver has its own proprietary software to generate navigation solutions. Several other algorithms are listed in literature (Bancroft, 1985; Kleusberg, 1994; Lopes and Kuga, 1988), and Gomes et al. (2002) give an account for several of them.

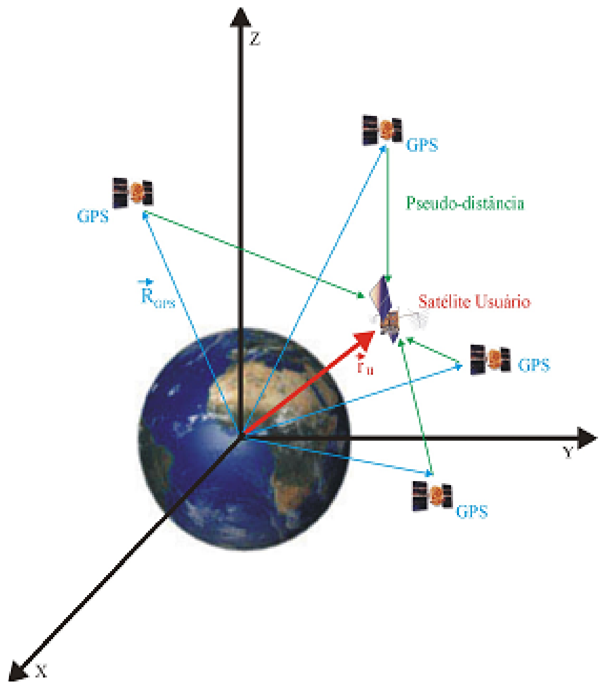


Figure 1. Ilustration of Triangulation Method

3. Dynamical Model

The discrete state dynamic modeling, which models the positioning of the GPS receiver is formally given by:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\varphi}_{k+1,k}\ \boldsymbol{x}_k + \boldsymbol{\omega}_k\ , \tag{1}$$

where $\boldsymbol{x}$ is the vector of n-states to be estimated, $\boldsymbol{\varphi}$ is the n x n transition matrix, and $\boldsymbol{\omega}$ is the vector of discrete white noise with n dimension. In the case of estimating a static landmark, the dynamics model for the Kalman filter simplifies to:

$$\begin{aligned} \dot{\boldsymbol{r}} &= \boldsymbol{0} + \boldsymbol{\omega_r} \\ \dot{\boldsymbol{b}} &= \boldsymbol{0} + \boldsymbol{\omega_b} \end{aligned} \tag{2}$$

where $\boldsymbol{r}$ is the position vector $[x \quad y \quad z]^T$, and $\boldsymbol{b}$ is a vector of parameters $[b_o \quad b_1]^T$ which models linearly the receiver clock offset by:

$$\delta t = b_o + b_1\, t \tag{3}$$

It is known that clock offset can present discontinuities. However this problem is treated elsewhere (Marques Filho et al., 2003), and we focus here on the dynamical modelling of the Kalman filter. The dynamical model in (2) simply states that $\boldsymbol{r}$ and $\boldsymbol{b}$ are constant, except for the dynamical noise levels $\boldsymbol{\omega_r}$ and $\boldsymbol{\omega_b}$. Therefore the state to be estimated is given explicitly by $\boldsymbol{x} = [x \quad y \quad z \quad b_o \quad b_1]^T$.

The observations model (measurements) is given by:

$$\boldsymbol{y}_k = \boldsymbol{H}_k\ \boldsymbol{x}_k + \boldsymbol{v}_k\ , \tag{4}$$

where $\boldsymbol{y}$ is the vector of m collected observations, $\boldsymbol{H}$ is the m x n matrix relating the observations to the state and $\boldsymbol{v}$ is the vector of m discrete white noise. The observation vector $\boldsymbol{y}$ is composed of the navigation solution $[x \quad y \quad z \quad \delta t]^T$ that is either supplied by the receiver or computed by the user. The matrix $\boldsymbol{H}$ models how the observations are related to the state. In this case, the relation is linear and quite simple.

4. The Kalman Filtering Technique

The Kalman filter purpose is to estimate the state vector based on a set of observations. Such a computational algorithm processes measurements to produce a minimum variance estimates of a system using knowledge of the dynamics and of the measurements, stastistics of the measurement errors, and information about initial conditions. In this case the state vector is updated right after processing each observation, in real time, and the process has sequential and recursive characteristics (Gelb *et. al.,*1974).

In this work the Kalman filter will be used due to its suitability to real time applications, without unnecessary storage of observations, as they can be processed while being collected. Also, and the convergence is obtained while observations are processed, if the filter is correctly tunned. The dynamical model considers the state vector to be estimated and the dynamic noise is assumed to be a discrete white noise, to cover unmodelled errors. The observations modelling considers the *m* collected observations vector, the matrix relating the observations to the state and the observations noise vector.

4.1 Kalman filter time update cycle

This phase uses the dynamic model, to propagate the state and the covariance between the instants *k-1* and *k*, by:

$$\begin{aligned} \bar{\boldsymbol{x}}_k &= \boldsymbol{\varphi}_{k,k-1}\,\hat{\boldsymbol{x}}_{k-1} \\ \bar{\boldsymbol{P}}_k &= \boldsymbol{\varphi}_{k,k-1}\,\hat{\boldsymbol{P}}_{k-1}\,\boldsymbol{\varphi}^t_{k,k-1} + \boldsymbol{Q}_k \end{aligned} \tag{5}$$

where $\bar{\boldsymbol{x}}_k$ and $\bar{\boldsymbol{P}}_k$ represent the time updated state and covariance for the instant *k*.

4.2 Kalman filter measurement update cycle

This phase is used to update the state and covariance of the instant *k* due to the measurement vector $\boldsymbol{y}_k$. The equations are:

$$\begin{aligned} \boldsymbol{K}_k &= \bar{\boldsymbol{P}}_k \boldsymbol{H}^t_k \left(\boldsymbol{H}_k \bar{\boldsymbol{P}}_k \boldsymbol{H}^t_k + \boldsymbol{R}_k\right)^{-1} \\ \hat{\boldsymbol{P}}_k &= \left(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k\right) \bar{\boldsymbol{P}}_k \\ \hat{\boldsymbol{x}}_k &= \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k \left(\boldsymbol{y}_k - \boldsymbol{H}_k \bar{\boldsymbol{x}}_k\right) \end{aligned} \tag{6}$$

where $\boldsymbol{K}$ is the Kalman gain, $\hat{\boldsymbol{x}}_k$ and $\hat{\boldsymbol{P}}_k$ are the updated state and covariance for the instant *k,* respectively.

4.3 Kalman Filter setup

The measurements supplied by the GPS receiver provides the GPS time *t*; *x, y, z* coordinates; a *b* bias due to user clock offset; and an information about the geometry of the GPS constellation expressed by the so named GDOP (Geometrical Dilution of Precision). Thus, the initial conditions for the state $\boldsymbol{x}$, to initialize the filter, can be generated by:

$$\bar{\boldsymbol{x}}(t_o) = \begin{bmatrix} x(t_o) \\ y(t_o) \\ z(t_o) \\ b_o(t_o) \\ b_1(t_o) \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}(t_o) \\ \cdots \\ 0 \end{bmatrix} \tag{7}$$

where $\boldsymbol{y}(t_o)$ is the measurement vector at the t_0 instant (x_o, y_o, z_o, xb_o). Notice that the b_1 term which represents physically the bias rate is initialized null. For the initial error estimates (covariance), it was assumed a rather large error level:

$$\boldsymbol{P}_o = diag\left(100^2,\ \ 100^2,\ \ 100^2,\ \ 100^2,\ \ 100^2\right) \tag{8}$$

in m^2, m^2, m^2, m^2, and $(m/s)^2$. Those are the initial state and covariance matrix of the filter.

The matrix $\boldsymbol{H}$ relating the observations to the state is given by:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & t-t_o \end{bmatrix} \tag{9}$$

The measurement noise covariance matrix $\boldsymbol{R}$, uses the accuracy information contained in the DOP value, and assumes a conventional pseudorange measurement error. Let the error of the navigation solution be given by:

$$\begin{bmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\sigma}_z \\ \hat{\sigma}_b \end{bmatrix}^2 = \sigma_\rho^2 \begin{bmatrix} \sigma_x^2 & \cdots & \cdots & \cdots \\ \cdots & \sigma_y^2 & \cdots & \cdots \\ \cdots & \cdots & \sigma_z^2 & \cdots \\ \cdots & \cdots & \cdots & \sigma_b^2 \end{bmatrix}$$

The parameter GDOP is normally a mesure of accuracy of the navigation solution. Formally it is the root of the trace of the scaled covariance matrix, namelly:

$$gdop = \left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_b^2\right)^{1/2}$$

In the absence of any other information one can assume $\sigma_x = \sigma_y = \sigma_z = \sigma_b \equiv \sigma$ so that an individual component σ contributes to the error as $\sigma^2 = gdop^2 / 4$. Assuming that each CA pseudorange measurement has a 5m standard deviation one has that $\hat{\sigma} \cong \sigma_\rho \; \sigma = 5(gdop/2)$. Therefore we can on the safe side build the measurement covariance error matrix as:

$$\boldsymbol{R} = diag\left\{\left(\frac{5\text{dop}}{2}\right)^2, \left(\frac{5\text{dop}}{2}\right)^2, \left(\frac{5\text{dop}}{2}\right)^2, \left(\frac{5\text{dop}}{2}\right)^2\right\} \tag{10}$$

The dynamical noises are added in equation (5) whenever the covariance matrix reaches a lower limit, in order to avoid the divergence phenomenon:

$$\boldsymbol{Q} = \begin{cases} Q_{xx} = 3^2\,\text{m}^2 & \text{if} \quad P_{xx} < 3^2 \\ Q_{yy} = 3^2\,\text{m}^2 & \text{if} \quad P_{yy} < 3^2 \\ Q_{zz} = 3^2\,\text{m}^2 & \text{if} \quad P_{zz} < 3^2 \\ Q_{b_o b_o} = 3^2\,\text{m}^2 & \text{if} \quad P_{b_o b_o} < 3^2 \end{cases}$$

and $\boldsymbol{Q} = \boldsymbol{0}$ otherwise. Such values were tunned and yielded a robust filter working for the several different situations.

5. Results

Data were collected using twin dual frequency Ashtech Z-12 GPS receivers of geodetic quality, placed at two known reference landmarks, previously surveyed. The reference landmarks are situated within INPE dependencies and their real reference coordinates are (4084784 m, -4209393 m, -2498492 m) for the first one, called "base" and (4084785 m, -4209388 m, -2498491 m) for the second one, called "user". Data were collected in a campaign on November 29, 2002, from 16:53:09 to 17:25:10 GMT time. The antennas of the receiver were placed on the top of the two landmarks. Figure 2 shows the set of receiver and antenna used.

Figure 2. GPS antenna, Z-12 receiver, battery and computer

In all the tests we recorded the proprietary navigation solution generated by the receiver, consisting of GPS time, position coordinates in WGS-84 system, receiver clock offset, and the DOP parameter. Also, we generated our own navigation solution using a conventional least squares method to compare with the ones from the receiver. The input to the least squares method was the pseudorange measurements obtained by the receivers. Both navigation solutions will be used as measurement inputs to the Kalman filter to check the overall impact on the solution accuracy. Along the period of the data analysed, there were always at least 6 GPS satellites locked in view of the receivers. One of the receivers, on top of a higher landmark was named "base" receiver; the other receiver on a floor landmark was named "user", and they both operated simultaneously during the data collection campaign.

A preliminary view of the data shows that there are some discontinuities in the receivers clock offset, which will not be discussed in this paper, however it was considered in another work (Marques et al., 2003). In this way we have chosen data intervals in which discontinuities were not present, so that we could concentrate on aspects of positioning using the Kalman filter.

First of all we present the characteristics of the navigation solutions. In this case it was chosen 5 minutes of the collected data, which presented no failure or discontinuities. Figures 3 and 4 show the position errors of the "base" landmark receiver with respect to surveyed landmark coordinates. Figure 3 was obtained by a least squares method and Figure 4 from the receiver generated solutions. During the whole interval 6 GPS satellites were in view. The figures seem quite similar, differing essentially in the *z*-coordinate, which however presents the same shape. For Figure 3, the error in meter, for x, y, z coordinates were 1,6149±0,4719, -1,7561±0,6 and –1,2455±0,4543 respectively and for Figure 4 they were 1,7737±0,5633, -1,4722±0,5858 and 0,5426±0,4117.

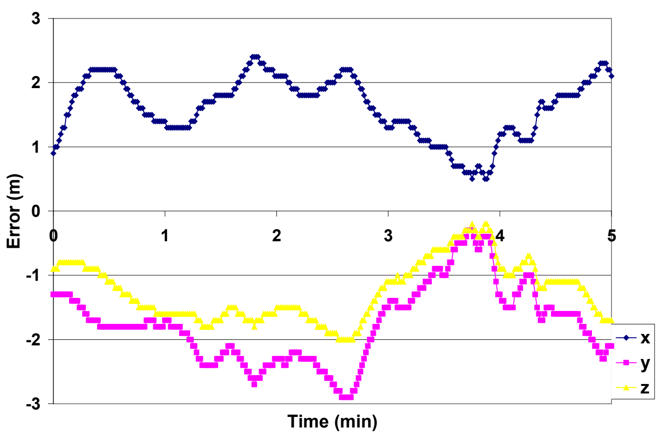


Figure 3: Least Squares Navigation Solution Error

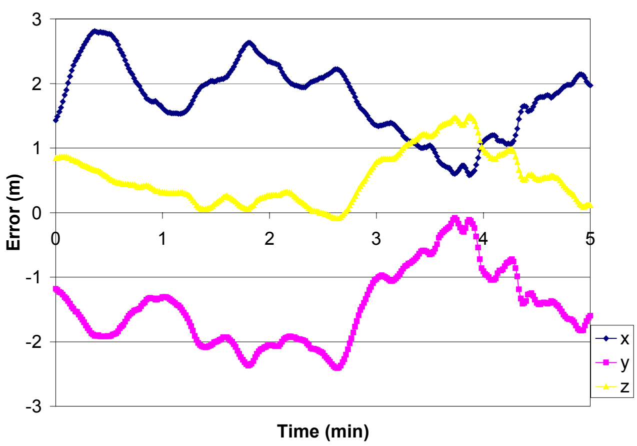


Figure 4: Receiver Navigation Solution Error

Both navigation solutions were input to the Kalman filter. Figures 5 and 6 show the errors of the Kalman filter when inputing navigation solutions generated by the Least Squares method and by the receiver, respectively. It is possible to see the smoothing effect of the filter, in real time, nevertheless no gain in precision is perceptible. It was obtained an error in position of 2.65±0.75m for the first filter and 2.43±0.53m for the second one. Figures 7 and 8 show the residuals (inovation sequence) of the observation vector (*x, y, z, b*) along time for both cases. They are mostly confined to less than 0.5m and, particularly, the residual of the receiver clock offset measurement is nicely fit around zero. This depicts the quality of the Kalman filter fitting to the observations. Quantitatively, for Figure 7 the residuals statistics in meters were 0.04±0.24 (mean and standard deviation) for *x*, -0.03±0.27 for *y*, -0.02±0.19 for *z*, and –0.01±0.11 for bias *b*; whereas for Figure 8 they were 0.01±0.27 for *x*, -0.01± 0.27 for *y*, -0.02±0.18 for *z*, and –0.01±0.12 for the bias.

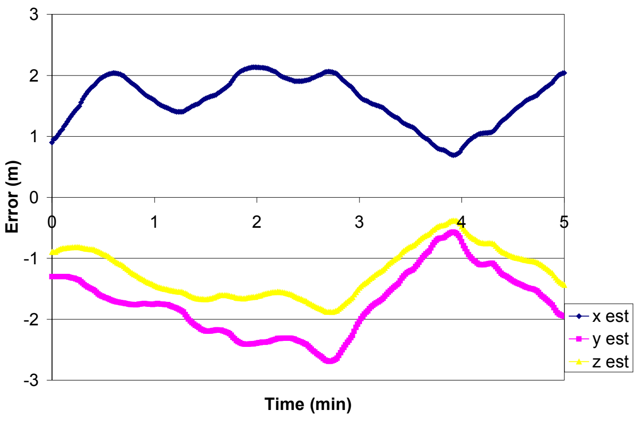


Figure 5: Kalman Filter applied to Least Squares Navigation Solution

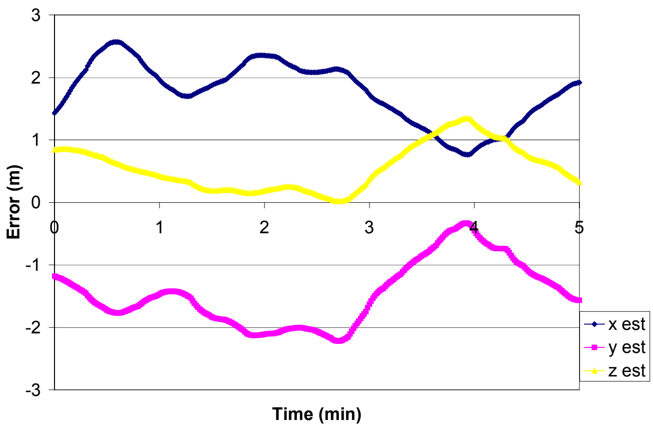


Figure 6: Kalman Filter applied to proprietary Navigation Solution

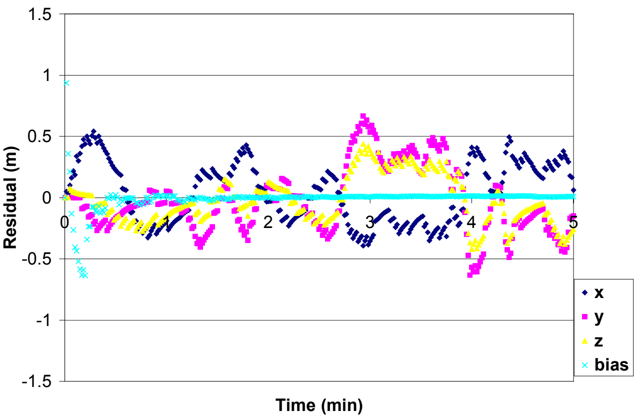


Figure 7: Kalman Filter Residuals for the Least Squares Navigation Solution

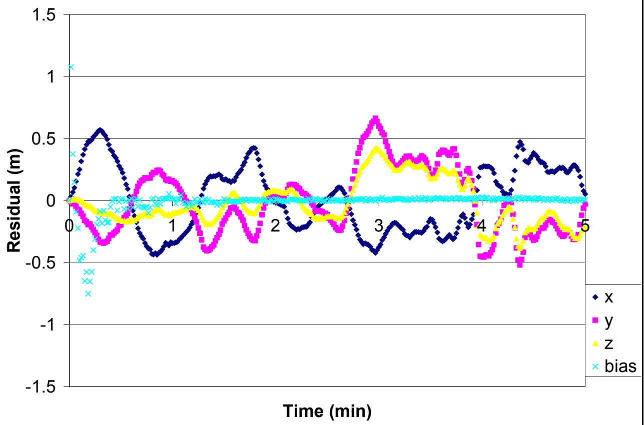


Figure 8: Kalman Filter Residuals for the Receiver Navigation Solution

For the other landmark named "user", the results are shown in the same way as the former landmark (called "base"). In this case, another time interval of 3 minutes was used..Figures 9 and 10 show the position errors of the navigation solutions, for the least squares method and the receiver proprietary method respectively. For Figure 9, the error in meter, for x, y, z coordinates were –0,2754±2,2919, -0,0902±2,3172 and –0,1937±0,8680 respectively and for Figure 10 they were –6,5465±4,3764, 5,5409±4,6935 and 4,2412±3,0592. in this case,there is a pronounced difference in the profiles. A sudden jump appears at 2min. in Figure 9, due to a new GPS satellite entering to the view of the receiver. There were 6 GPS satellites up to 2min., then a new one was locked (summing up 7). In many occasions, the satellite which enters in the field of view of the receiver, comes with low elevation and perturbations as multi path can be bigger. In this case the error can be worse if its measurement is unduly overweighted when the constellation changes, although the covariance decreases. The Least Squares method processes all available GPS satellite measurements and the change of constellation was quickly noticed and translated to the jump. However the proprietary receiver software disregarded this fact and most probably stayed with what it considered the 4 best GPS satellites in terms of geometry (DOP). A bad initial choice of the best 4 GPS satellites certainly might cause this problem, which was magnified further when the new GPS satellite information was disregarded and the DOP information did not trigger a new best set of 4 GPS satellites.

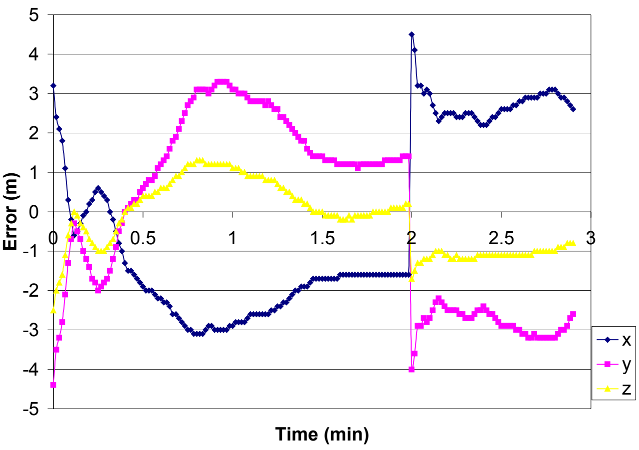


Figure 9: Least Squares Navigation Solution Error

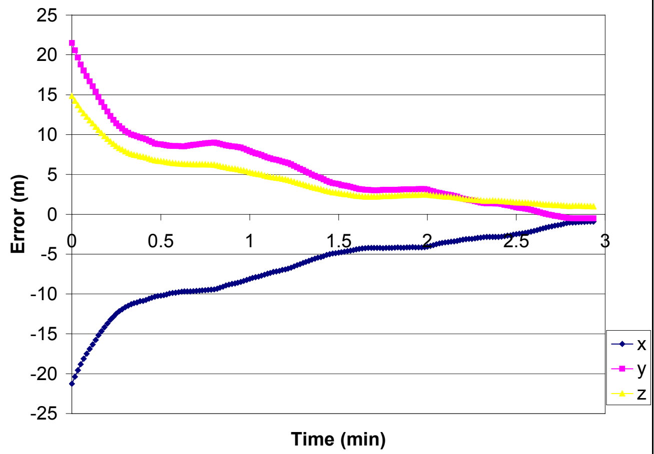


Figure 10: Receiver Navigation Solution Error

Figures 11 and 12 shows the Kalman filter performance for both inputs. The smoothing effect of the filter is pronounced around the jump region in Figure 11, and almost unnoticed in Figure 12. It was obtained an error in position of 2.6±1.0m for the first filter and a large dispersion of 12.8±8.1m for the second one. Figures 13 and 14 show the residuals for both cases. Statistics for Figure 13 were 0.05±1.25m (mean and standard deviation) for x, 0.05±1.32m for y, 0.06±0.54 for z, and 0.08±0.32 for bias b; whereas they were 1.94±1.00 for x, -2.09±1.15 for y, -1.38±0.84 for z, and –0.04±0.16 for the bias, for Figure 14.

It is clear that the performance of the Kalman filter is affected by the quality of the navigation solution data which is input. Statistically the filter fed by the Least Squares navigation solution presented neat performance with residuals being minimized on average. The proprietary navigation solution presented strange results, which should be furtherly investigated, as well as understanding of its embedded software package which unfortunately is not disclosed.

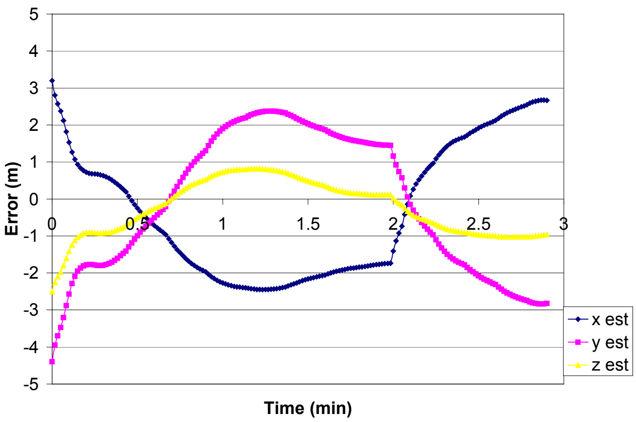
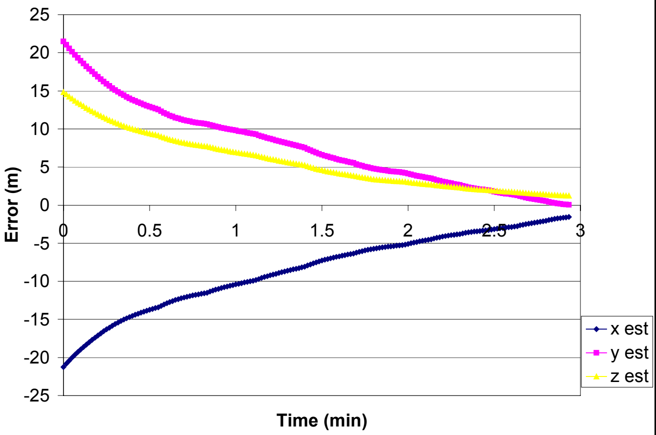


Figure 11: Kalman Filter Applied to Least Squares Navigation Solution



Kalman Filter Applied to proprietary Navigation Solution

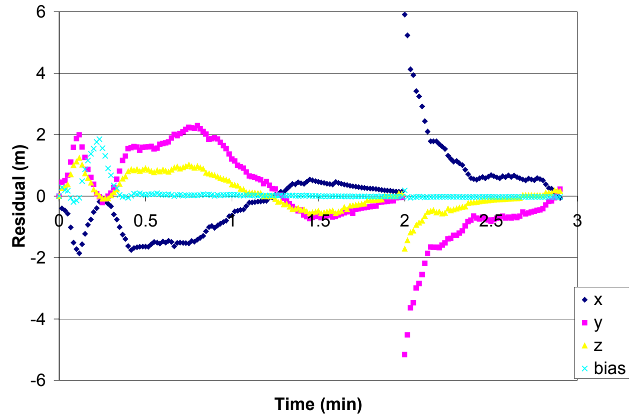


Figure 13: Kalman Filter Residuals for the Least Squares Navigation Solution

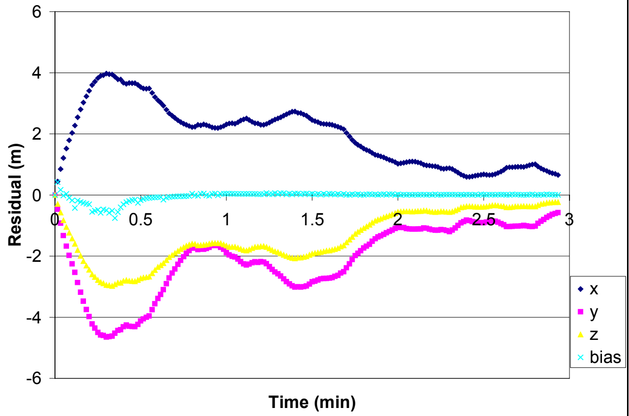


Figure 14: Kalman Filter Residuals for the Receiver Navigation Solution

6. Conclusions

Two surveyed landmarks were chosen to test positioning using the navigation solution as input to a Kalman filter. In the cases studied here it was shown that the receiver proprietary navigation solution accuracy was comparable when the constellation is stable, but might become worse than the one obtained by a conventional least squares method when there are changes of the constellation geometry. Such bad navigation solution input degraded the Kalman filter accuracy, although not to the extent of provoking divergence. Convergence was not a problem in the two test case as the filter is initialized with the initial navigation solution. The smoothing effect of the filter was always perceptible in all situations tested. Peaks of 4m error coming from the navigation solution was smoothed out to less than 3m, in a situation of geometry change. It is concluded that the quantitative figures obtained here are typical of the possible accuracy obtainable in real time using the navigation solution. It seems that the smoothing effect is the main characteristic of he Kalman filter, which can be advantageous for smoothing undesiraded sudden jumps in an on board real time environment. Resource to differential GPS techniques certainly could improve the accuracy of real time positioning and has been investigated in another work (Baroni et al.,2003). An extension work intends to include mobile users as car, airplane, ships and high dynamics satellite motion, but then investigating the usage of more complex dynamic modeling.

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