# Nonlinear Analysis of Flapping Motion of Helicopter Rotor Blades in Hover with Gust Effects

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**Abstract.** This work presents a study of the nonlinear flapping motion of rotor blades in hover considering gust effects. The nonlinear differential equation for the flapping motion is obtained and the computational simulations show that above a critical collective pitch angle, large flapping oscillations can occur, particularly under high winds and with reduced rotational speeds. These large, possibly destructive, oscillations are not predicted by the linear model and may be related to the phenomenon of blade sailing.

**Keywords.** *helicopter, flapping, nonlinear, dynamics, gust.*

## 1. Introduction

Helicopter response in gusty air has been poorly studied, despite its importance for several applications, particularly in naval operations. This work presents a study of the nonlinear flapping motion of rotor blades in hover considering gust effects. The aim is to compare the linear model commonly used for small flapping oscillations analysis (Kunz, 1998; Bramwell, 1976 and Dowell et al., 1995) to a more general nonlinear model for the rotor blades in hover that allows large flapping oscillations to occur due to gusty air and collective pitch command.

The arising of large destructive oscillations in structures due to a gust input is not a new phenomenon. The famous case of the Tacoma Narrows Bridge, not still completely understood, illustrates that the linear approach, based on resonance, may not be the correct explanation for the observed large oscillations. The resonance phenomenon requires stringent conditions of damping and gust/structure frequencies to take place. A recent and more plausible hypothesis is based on the nonlinearity of the system, which obviously cannot be captured by linearizing the model, under the small angle assumption (McKenna, 1999).

## 2. The nonlinear flapping model with gust effects

The rotor blade is assumed to be rigid and the nonlinear flapping model is derived using the blade element theory, including a simplified sinusoidal vertical gust and collective pitch inputs with constant inflow, neglecting the couplings with other degrees of freedom. The inflow angle obtained in (Dowell et al., 1995), made up of the effect of induced velocity (downwash) and the induced angle due to flapping velocity, can be modified by including gust effects, as follows:

$$\phi = \frac{r\dfrac{d\beta}{dt} + v_i - w}{\Omega r} \tag{1}$$

where $\phi$ is the new inflow angle, assumed to be relatively small, $\beta$ is the flapping angle, $v_i$ is the induced velocity, $w$ is the vertical gust velocity, $\Omega$ is the rotor rotational speed and $r$ is the radial coordinate of the blade element.

Therefore, the nonlinear differential equation for the flapping motion in hover with gust effects is:

$$\frac{d^2\beta}{dt^2} + \frac{\gamma\Omega}{8}\frac{d\beta}{dt} + \lambda_1^2\Omega^2 \sin\beta\cos\beta = \left(\frac{\gamma\theta_0}{8} - \frac{\gamma v_i}{6\Omega R}\right)\Omega^2 + \frac{\gamma\Omega}{6R}w \tag{2}$$

where

$\beta$ - flapping angle, rad
$\gamma$ - Lock number
$\Omega$ - rotor rotational speed, rad/s
$\lambda_1$ - nondimensional flapping frequency ratio
$\theta_0$ - collective pitch angle, rad
$v_i$ - induced velocity, m/s
$R$ - rotor radius, m
$w$ - vertical gust velocity, m/s

The gust effects are modeled by a simplified sinusoidal wave actuating uniformly over the rotor blades (Riaz et al., 1993):

$$w = \alpha \sin\left(\frac{2\pi}{\lambda} V_{mw} t\right) \tag{3}$$

where
$\alpha$ - gust amplitude, m/s
$\lambda$ - wavelength, m
$V_{mw}$ - mean wind velocity, m/s

The term $\lambda_1^2 \Omega^2 \sin\beta\cos\beta$ of the Equation (2) represents the nonlinearity of the flapping motion and it is usually approximated by the term $\lambda_1^2 \Omega^2 \beta$ for small amplitude oscillations (Bramwell, 1976; Dowell et al., 1995). While this approximation seems reasonable for stability analysis purposes around an equilibrium position, the gust response may require a nonlinear analysis considering the possibility of the arising of large oscillations.

**3. Simulation of the nonlinear flapping equation**

A common approach for rotor stability analysis is to find the equilibrium points using the complete nonlinear equations and then linearize the equations around these points. However, though commonly assumed, this static nonlinear approach may not be adequate for response problems, for, in fact, the modeling requires the use of non-homogeneous differential equations with forcing terms that interact in a complex way.

Therefore, the aeroelastic investigation developed in the present work uses the fully nonlinear model describing the dynamic flapping behavior and is based on the Runge–Kutta simulation of the Equation (2). Fig. 1 shows the differences between the solutions predicted by the nonlinear model and the linear one, for the following set of parameters and inputs:

$R = 5.7m$
$\gamma = 8$
$\Omega = 10rad/s$
$\lambda_1 = 1$
$\theta_0 = 4\deg$
$v_i = 0m/s$
$\alpha = 21m/s$
$\lambda = 15m$
$Vmw = 3m/s$

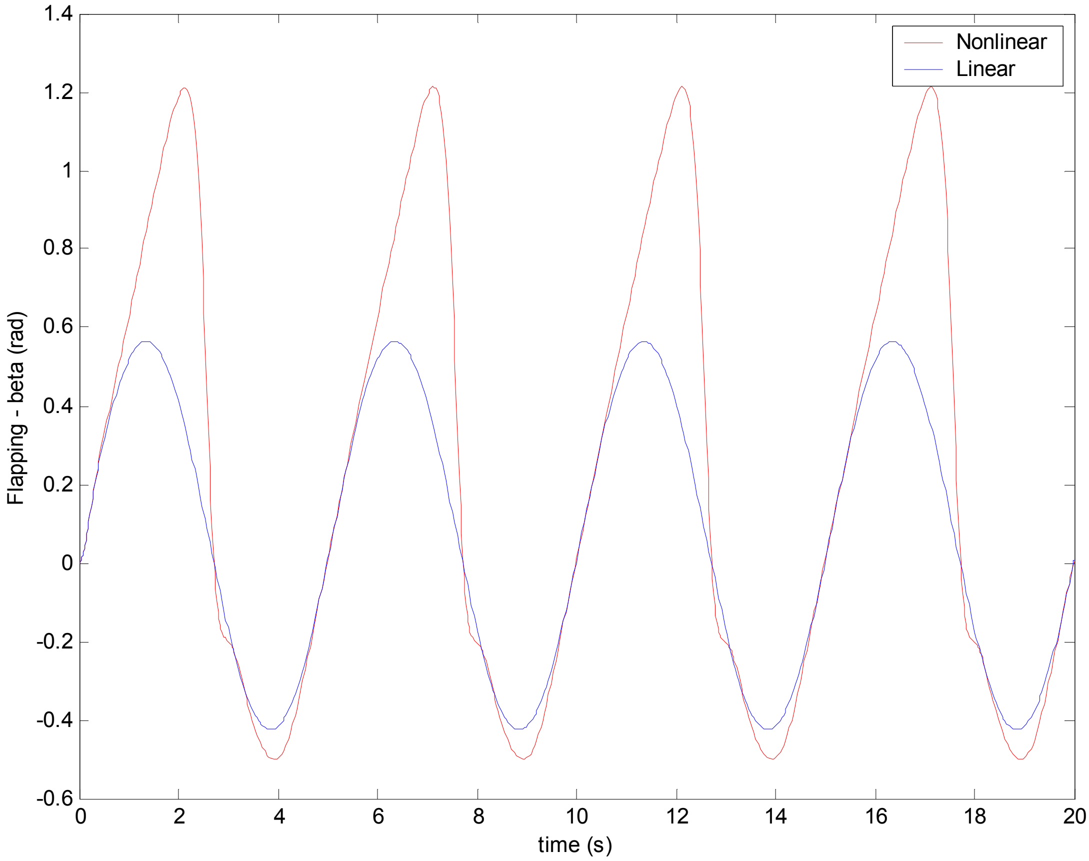


Figure 1. Nonlinear and linear gust / collective pitch response.

The low rotational speed used in the simulation represents a condition of startup or shutdown of the rotor system, when the centrifugal force is small. This condition is also associated with a low stiffness of the rotor blades. The simulation considered a high amplitude sinusoidal gust input, thus generating a large aerodynamic force. The combination of low stiffness and large aerodynamic force can give rise to excessive flapping of the rotor blades. This phenomenon is called blade sailing and can yield severe damage to the helicopter (Geyer Jr. et al., 1998).

A linear model can be used to study this phenomenon, if relatively small flapping angles are considered. However, for large angles the nonlinearity becomes important and a new class of phenomena can occur, including bifurcations and, possibly, chaos.

This work is limited to a numerical analysis of the Equation (2) and the next session discuss the results illustrated in Fig. 1.

**4. Discussion of the simulation results**

The computational simulations show that above a critical collective pitch angle, the gust input can give rise to increasing, possibly destructive, flapping oscillations, not predicted by the linear model.

The origin of the discrepancies between the two models is that the principle of superposition does not apply for the nonlinear flapping and, thus, the gust and collective pitch input contributions are not additive, yielding the large oscillations. The analysis of the Equation (2) reveals that a helicopter rotor, as a nonlinear dynamic system, may be extremely sensitive to gust input, despite the large flapping damping. Probably a careful nonlinear analysis will be specially important for naval operations, where high winds are common.

Future work includes a nonlinear analysis of the coupling of the flapping motion with other degrees of freedom, particularly the torsional motion, including gust effects. Eventually, this aeroelastic coupling may constitute itself as the basis for a control method to reduce rotor oscillations.

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