MODELING DELAMINATION IN LAMINATE STRUCTURES UNDER TORSION LOADING

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Abstract. *An efficient method to analyze delamination in laminated composite beams, under torsion effects, is presented. The equilibrium equations were derived by Sankar (1991) from the classical shear deformable laminated plate theory (Whitney, 1987). These equations are assumed to be satisfied, in an average sense, over the width of the beam. From this assumption results a new set of force and moment resultants, which amplify the possibilities for modeling beams under torsion loading. A new offset beam finite element is developed for modeling the problem. Initially, the capability of the new finite element is verified solving the problem of a cantilever beam under different end loading conditions. Next, the strain energy release rate for a delaminated specially orthotropic beam under torsion is calculated. The delamination is assumed to be in the middle plane of the beam. The results for strain energy release rate are compared with the closed form solution for the same problem. The effect of delamination in different interfaces is also studied. Some of the results obtained in this research would be useful in explaining delamination propagation in composite beams due to quasi-static impact.*

Keywords. Laminate strutures, delamination, torsion, strain energy, finite element method

1. Introduction

As the use of composite materials in structural applications increases, more is the need for structural analysis. Unlike an isotropic beam, laminated composite materials bear coupling among their modes of deformation, which is the major obstacle to precisely modeling the mechanical behavior of composite structures. The application of composite materials is increasing day by day and in some areas, like aerospace and automobile structures, robotics, marine industries and medical devices and prosthesis, the structures use to work under significant torsion loading conditions.

Among the fracture modes of this kind of material, the occurrence of delamination in free edges has been receiving increasing attention from investigators in their effort to understand and prevent delamination in composite structures. The reason is the presence of high interlaminar stresses, especially peel stresses, in the neighborhood of a free boundary.

An efficient method to analyze delamination in laminated composite beams, under torsion effects, is presented. The equations of equilibrium were derived by Sankar (1991) from the classical shear deformable laminated plate theory (Whitney, 1987). These equations are assumed to be satisfied in an average sense over the width of the beam. From this assumption results a new set of force and moment resultants, which amplify the possibilities for modeling beams under different loading conditions, including torsion. The equilibrium equations are derived from the Minimum Potential Energy Principle and a new offset beam finite element is developed for modeling the problem. Initially, the capability of the new finite element in modeling the problem is verified solving the problem of a cantilever beam under different end loading conditions. The results are compared with those from beam theory solutions found in Timoshenko (1970), Reismann and Pawlik (1980), Whitney (1987) and Sankar (1991). Next, the strain energy release rate for a delaminated specially orthotropic beam under torsion is calculated. The delamination is assumed to be in the middle plane of the beam. The results for strain energy release rate are compared with the closed form solution for the same problem. The effect of delamination in different interfaces is also studied. Some of the results obtained in this research would be useful in explaining delamination propagation in composite beams due to quasi-static impact.

2. Beam equations

Considering the laminated beam shown in Fig.(1), the traditional Shear Deformation Theory (Whitney, 1987) with its displacements and rotations expanded as a Taylor series in the direction of the width of the beam, with just the first term retained, and also that there is no deformation in cross sections normal to this direction, the displacement field can be represented in terms of seven functions of the x coordinate (Sankar,1991):

$$
\begin{aligned}
u(x,y,z) &= U(x) + yF(x) + z\phi(x) + yz\alpha(x) \\
v(x,y,z) &= V(x) - z\theta(x) \qquad (1) \\
w(x,y,z) &= W(x) + y\theta(x)
\end{aligned}
$$

Where u_0, v_0 and w_0 are the displacements in the middle plane of the plane in the x, y and z directions, respectively, so, are Ψ_x and Ψ_y with respect to the rotations around the x and y axes and.

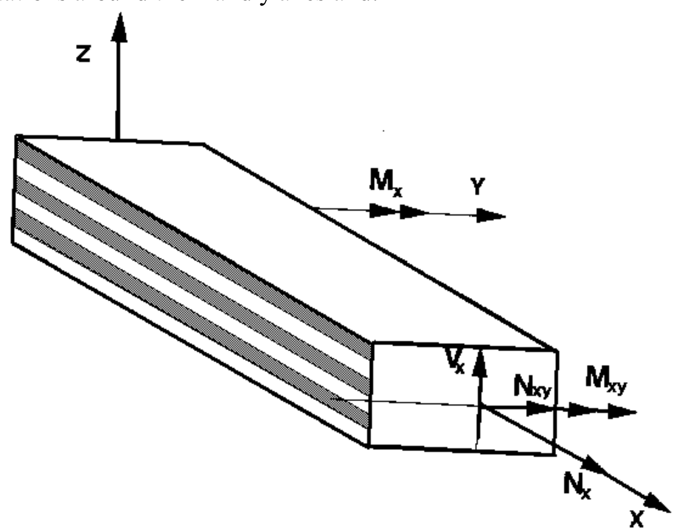

Figure 1. Laminated beam, with force and moment resultants

The deformation field can be expressed as

$$E = \overline{E} + y\hat{E} \qquad (2)$$

$$
(E) = \begin{pmatrix} e_{x0} \\ \gamma_{xy0} \\ \kappa_{xy} \\ \kappa_{xy} \\ \gamma \end{pmatrix} ; \left(\bar{E}\right) = \begin{pmatrix} U' \\ V' + F \\ \phi' \\ \alpha - \theta' \\ \phi + W' \end{pmatrix} ; \left(\hat{E}\right) = \begin{pmatrix} F' \\ 0 \\ \alpha' \\ 0 \\ \alpha + \theta' \end{pmatrix} \qquad (3)
$$

3. Euilibrium equation

A new set of force and moment resultants is defined from the integration of the column vector of forces along the width of the beam:

$$
\bar{F} = \int_{-\frac{b}{2}}^{\frac{b}{2}} F dy = \int_{-b/2}^{b/2} C\,\bar{E}\,dy = bC\bar{E} \quad \text{and} \quad \hat{F} = \int_{-\frac{b}{2}}^{\frac{b}{2}} F\,y dy = \int_{-b/2}^{b/2} C\,\hat{E}\,y^2\,dy = \left(\frac{b^3}{12}\right) C\hat{E} \qquad (4)
$$

where b is the width of the beam and the vector of forces F is defined (Whitney, 1987) as

$$(F)^T = \left(N_x, N_{xy}, M_x, M_{xy}, V_x\right) \qquad (5)$$

and the matrix C is present in the explicit beam constitutive relations (Pinheiro, 1991).

The Principle of Minimum Potential Energy (Trauchert, 1970). is applied in the derivation of the equilibrium equations. The total potential energy of the structure (π) is obtained from the sum of the strain energy of the beam (ϕ) and the potential of the external force (χ), and is defined as (Sankar, 1991):

$$\phi = \int_0^L \phi_L \, dx = \frac{1}{2}\int_0^L \left(b\bar{E}^T C\bar{E} + \frac{b^3}{12}\hat{E}^T C\hat{E} \right) dx \quad \text{and} \quad \chi = -\int_0^L \int_{-\frac{b}{2}}^{\frac{b}{2}} q(x,y)\, w(x,y)\, dydx \tag{6}$$

The potential of the external force (χ) is expressed considering only the transverse loading q (x,y) and the displacements w (x,y) on the beam surface and, similar to the procedure adopted to define a new set of force resultants in Eq. (4), the transverse loading q (x,y) is divided in two parts and defined by:

$$q(x,y) = \bar{q}(x) + y\hat{q}(x) = \int_{b}^{\frac{b}{2}} q(x,y)\, dy + \int_{b}^{\frac{b}{2}} q(x,y)\, ydy \tag{7}$$

Hence,

$$\pi = \frac{1}{2}\int_0^L \left(b\bar{E}^T C\bar{E} + \frac{b^3}{12}\hat{E}^T C\hat{E} - \bar{q}(x)W(x) - \hat{q}(x)\theta(x) \right) dx \tag{8}$$

According to the Principle of Minimum Potential Energy,

$$\delta\pi = \delta\phi + \delta\chi = 0 \tag{9}$$

where δ is the variational operator symbol. From the application of this Principle results seven Equilibrium Equations and, from each one of them, one corresponding natural boundary condition (Pinheiro, 1991). Finally, the new set of force and moment resultants can be expressed in terms of seven unknown functions: U, V, W, F, ϕ, α and θ. Substituting for the resultants in terms of displacements results in a system of seven ordinary equations for the displacements.

4. The finite element solution

A new finite element has been developed to model the beam problem solving the system of ordinary differential equations obtained from the equilibrium equation. This finite element has three nodes and seven degrees of freedom (U, V, W, F, ϕ, α, θ) at each node. The middle node is statically condensed when solving the problem for displacements, but it is considered when calculating the strain energy release rate, in order to obtain a more accurate solution. The nodal forces and moments for the i^{th} node of the structure are F_x, F_y, F_z, M_2, M_x, W and T. A quadratic variation of all seven displacements is assumed along the element length. Denoting by X any specific displacement, and by X_i this specific displacement at the i^{th} node, the displacement and deformation within the element are defined as

$$X = a_1X_1 + a_2X_2 + a_3X_3 \text{ and}$$
$$X' = b_1X_1 + b_2X_2 + b_3X_3 \tag{10}$$

In the last equation, the terms a_1, a_2 and a_3 are interpolation functions and b_1, b_2 and b_3 their derivatives with respect to X direction, all in the form of

$$a_1 = 1 - 3\bar{x} + 2\bar{x}^2; \quad a_2 = \bar{x}\left(2\bar{x} - 1\right); \quad a_3 = 4\bar{x}\left(1 - 2\bar{x}\right)$$

$$b_1 = \left(4\bar{x} - 3\right) / L; \quad b_2 = \left(4\bar{x} - 1\right) / L; \quad b_3 = 4\left(1 - 2\bar{x}\right) / L, \qquad \text{where } \bar{x} = x / L \tag{11}$$

where L is the element length and X is the coordinate along the beam axis.

Using the two above equations, the deformation field within each finite element as a function of the nodal displacement is expressed in the form:

$$\left(\bar{e}\right)=\left[\bar{a}\right](q) \quad \text{and} \quad (\hat{e}) = [\hat{a}]\,(q) \tag{12}$$

or

$$e = \left(\bar{a} + y\,\hat{a}\right) q \quad \text{where} \quad e = \bar{e} + y\,\hat{e} \tag{13}$$

where $\bar{a}$ e $\hat{a}$ are strain-displacements (5x21) matrices and q is the vector of displacements

$$[q]^T = [U_1, V_1, W_1, F_1, \phi_1, \alpha_1, \theta_1, U_2, V_2, W_2, F_2, \phi_2, \alpha_2, \theta_2, U_3, V_3, W_3, F_3, \phi_3, \alpha_3, \theta_3] \tag{14}$$

The element matrix K is defined as

$$k = \bar{k} + \hat{k} = b \int_0^L \bar{a}^T D \bar{a}\, dx + \frac{b^3}{12} \int_0^L \hat{a}^T D \hat{a}\, dx \tag{15}$$

where D is the elasticity matrix for composite materias.

The final representation of the problem is in the form:

$$F = k\,q \tag{16}$$

where k is the total stiffness matrix, and F and q are vectors with the nodal forces and displacements, respectively. In the present work, the Eq.(16) is solved using the Gauss Elimination Method (Bathe, 1982).

5. The strain energy release rate (G)

The strain energy release rate is calculated using the expression for J-integral. A zero volume path is delineated surrounding the crack tip, as shown in Fig. (2) , and the J-integral expressed in terms of the strain energy per unit length (U) of each element around the crack as

$$J = U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)} \tag{17}$$

Using Eq.(8) and Eq.(13, the strain energy per unit length of the Ith element can be expressed as

$$U^{(i)} = \frac{1}{2} q^T \left(b\,\bar{a}^T D \bar{a} + \frac{b^3}{12} \hat{a}^T D \hat{a} \right) q \tag{18}$$

In the above equation, the vector q contains the displacements of the nodes of the ith element, including the middle node.

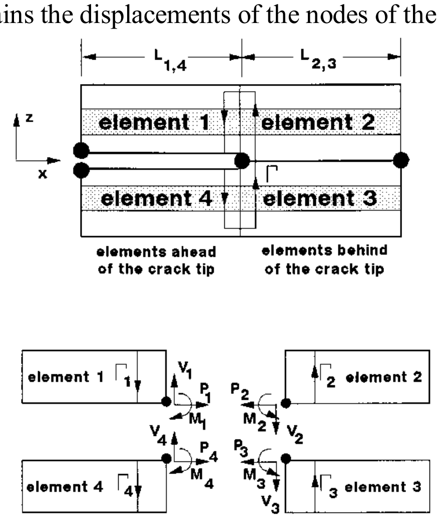

Figure 2. zero-volume path for J-integral

6. The rigid and gap elements

The possibility of interference between the crack surface in the delaminated region, when the beam is under loading, principally under torsion loading, requires that gap elements or rigid elements be placed in appropriate positions in order to monitor the contact.

Two types of rigid elements are anticipated to be used depending on which side, left or right, of the beam presents the interference phenomenon. The Fig. (3) illustrates the interference situation and the rigid elements action.

The gap element is placed in the structure to avoid the interference in the delaminated region while the structure is under vertical loading. Both, the rigid element and gap element matrices are assembled in the global stiffness matrix using the same procedure adopted to assemble the element stiffness matrices, and their terms can be found in Pinheiro (1991).

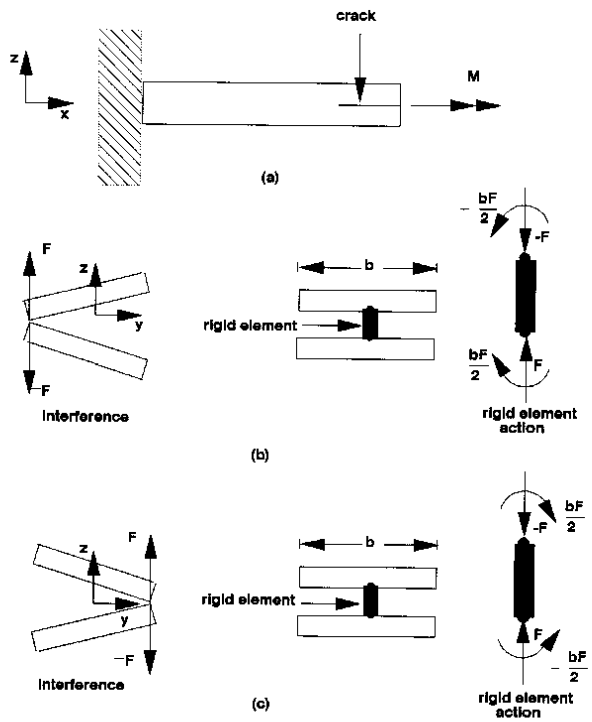

Figure 3. Rigid elements: (a) beam under torsion (b) interference on left side (c) (b) interference on right side

7. Numerical results

The capability of the new finite element in modeling the problem is verified solving the problem of a cantilever beam under different end loading conditions. The results are compared with those from beam theory solutions found in Timoshenko (1970), Reismann and Pawlik (1980), Whitney (1987) and Sankar (1991).

In all the numerical tests performed in the present study, the beam is supposed to be made up of material with the following properties, which are typical values for high performance graphite/epoxy unidirectional composites:

Longitudinal Elastic Modulus	(E_1):	14.00 GP_a
Transverse Elastic Modulus	(E_2):	1.00 GP_a
Shear Modulus	(G_{12}):	0.53 GP_a
Poison Ratio	(V_{12}):	0.30
Poison Ratio	(V_{23}):	0.55

Specifically Fig. (4) shows the agreement among solutions for the angle of twist values along the beam length for a cantilever beam (regular elements) with different lengths. The beam is under an unit torque and was modeled by ten finite elements. From these figures, we can see that in the region close to the support the agreement is not good, and probably it will require use of larger number of finite elements near the fixed support. The figure also shows the results with the beam modeled using top offset elements, which are elements that had your neutral axis dislocated to the bottom portion of the beam. The offset elements are shear deformable beam finite elements with nodes offset to either the top (bottom elements) or bottom side (top elements) and their formulation can be found in (Sankar and Pinheiro ,1990).

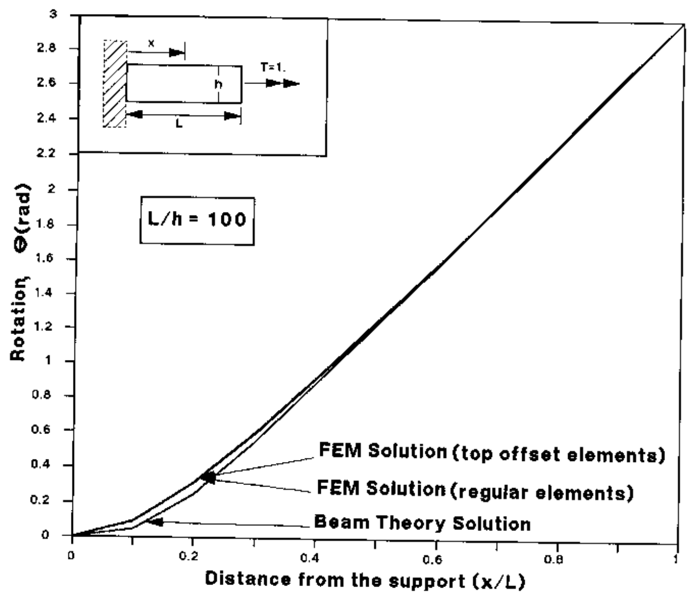

Figure 4. Angle of twist along the length (L / h = 100)

In a second numerical test, a specially orthotropic delaminated cantilever beam is put under torsion loading. Initially, the delamination is supposed to be in the middle plane of the beam, and the result in terms of the strain energy release rate is compared with the closed-form solution. Then, the delamination is placed between in different layers along the beam thickness, and a study is performed to understand the effect of the delamination position on the strain energy released rate. The finite element model used to solve the case of delamination supposed to be in the middle plane of the beam has forty-six nodes and forty-six elements, as shown in Fig. (5). There are twenty-two regular finite elements modeling the uncracked area and twenty-four offset elements (twelve top elements and twelve elements) used for modeling the delaminated region.

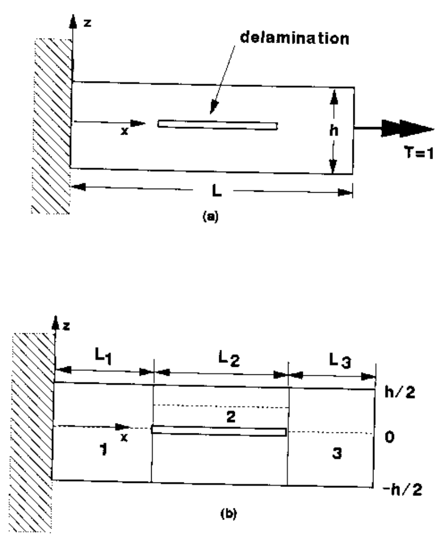

Figure 5. Delaminated beam: (a) Under torsion (b) Separated by parts

Referring to the strain energy release rate (G) definition (Hellan, 1984), and the condition shown in Fig. (5), it is obtained for the strain energy release rate the value of 2,6367.0 J/m^2., while the numerical result for the present case is 2,545.5 J/m^2. The relative difference between numerical and closed-form solutions is 3.45% and both results can be said being in good agreement with each order.

When considering delamination position varying along the thickness of the beam, the procedure adopted is similar to that used to calculate the strain energy release rate with the delamination placed in the middle plane, except for the integration limits when calculating the stiffness coefficients A_{ij} , B_{ij} and D_{ij}, which are the elasticity coefficients of the matrix D, for composite materials.

The Fig. (6) illustrates the adopted model for the case of delamination placed between layers close to the top.

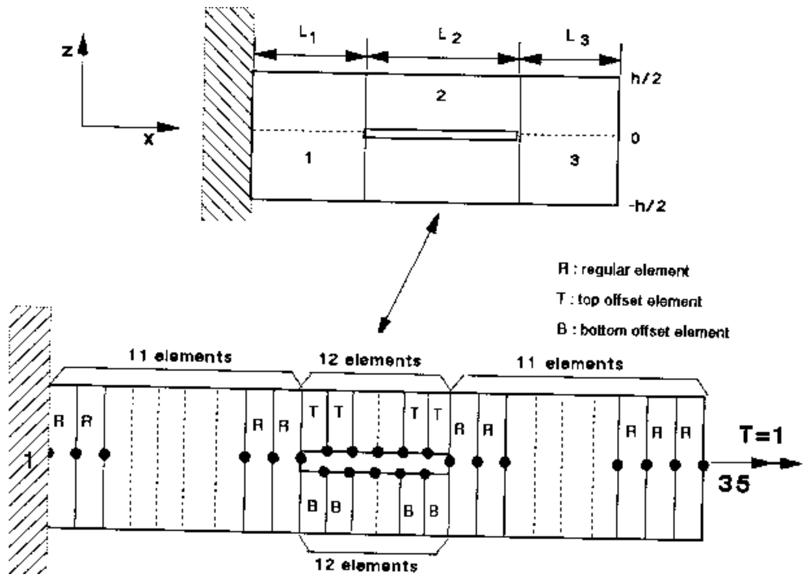

Figure 6. Delamination close to the top plane

Figure 7 shows the obtained results for different positions of the delamination along the thickness of the beam. From this figure we can conclude that the possibility of crack propagation will be increased as the crack position approaches the middle plane of the beam. As the crack plane approaches the top or bottom faces of the beam, strain energy release rate becomes negative. This could be due to errors in the energy calculations, and can be avoided by having shorter elements near the crack tip.

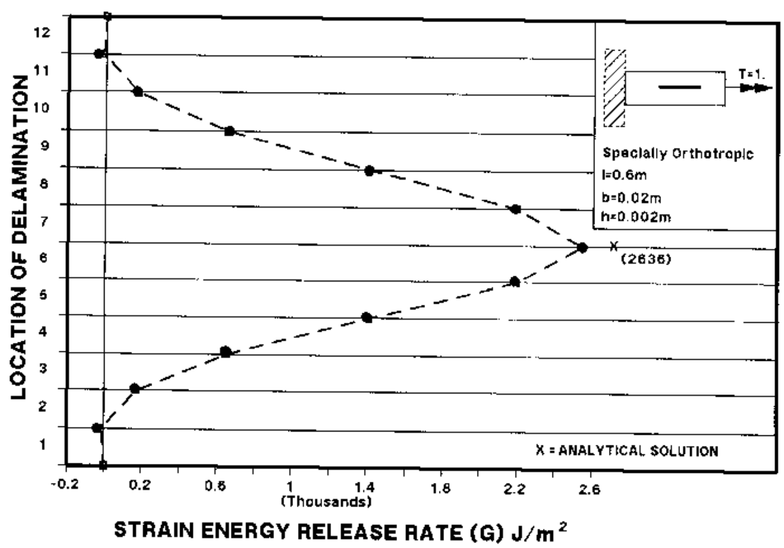

Figure 7. Strain energy release rate for cantilever beam

The last numerical test consists in the simulation of a quasi-static impact test in a quasi-isotropic simply supported beam. Again, the focus was the effect of the delamination position on the values of the strain energy release rate, and the present numerical study can help to interpret experimental results (Knon, 1991).

Figure 8 shows the numerical model for representing the simulation of a quasi-static impact test in a quasi-isotropic simply supported beam.

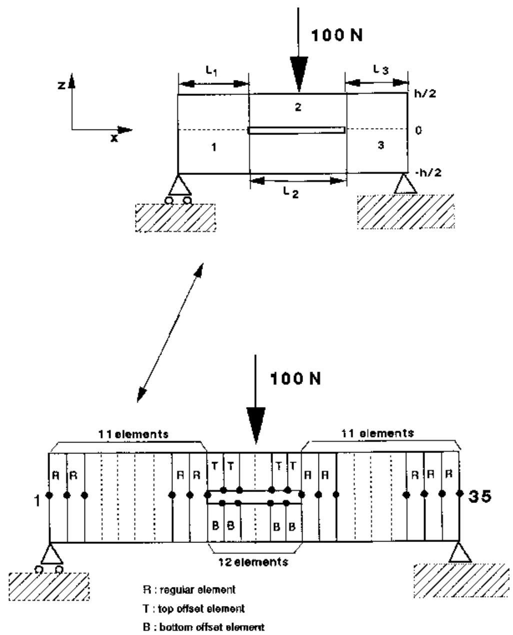

Figure 8. Simply supported beam

The dimensions of the beam, the length of the delamination, the material and the finite element model are the same as the preceding example. A force equal to 100 N is applied to the center of the beam. The finite element model is also shown in Figure 8. Two classes of laminates were analyzed in this case, a specially orthotropic laminate and a [0/45/-45/90]s laminate. The delamination was assumed in different positions along the laminate thickness, and the results for each class of laminate are showed in Fig.(9).

8. Summary

The delamination in anisotropic beams was analyzed using a new beam finite element to model the problem. The formulation of this new beam finite element is based on a beam theory for laminated beam sin which the equilibrium equations are assumed to be satisfied in an average sense over the width of the beam. A new set of force and moment resultants for the beam were introduced from this assumption.

Two problems of practical interest were solved using this method. One of them was the problem os a specially orthotropic delaminated cantilever beam under torsion loading, and the other was the case of a specially orthotropic and a quasi- orthotropic simply supported beams under transverse loading.

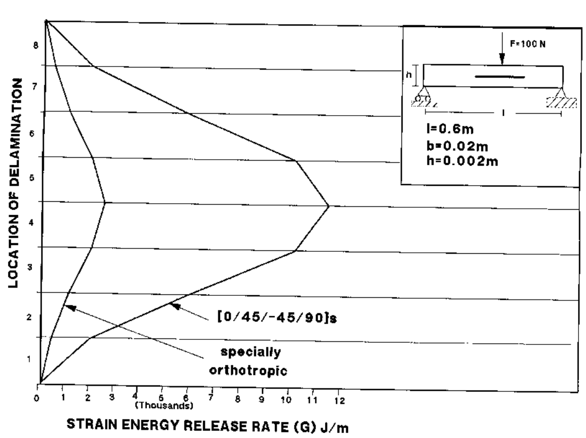

Figure 9. Strain energy release rate for simply supported beam

In both problems the delamination was assumed to be in different positions along the thickness of the beam, and the strain energy release rate for each position was computed. The strain energy release rate was found to have the maximum value in the middle plane of the beam, for both cantilever beam under torsion and the simply supported beams under transverse force. Pratically, this indicates that the probability of crack propagation becomes higher as the position of the delamination approaches the middle plane of the beam.

Similarly to the cantilever beam case, from the present results we can conclude that the possibility of crack propagation will be increased as the crack position approaches the middle-plane of the beam. This phenomenon is less pronounced in the specially orthotropic beam than in the other beam. Another conclusion is that for a given transverse force, the possibility of crack propagation is more in the [0/45/-45/90]s laminate. These results have an important application in understanding the propagation of delamination damage in compasite laminates due to low-velocity impact as well as quasi-static indentation types of loading (Kwon, 1991).

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