ON THE DYNAMICS OF A FLEXIBLE PORTAL FRAME SUBJECTED TO NON-IDEAL EXCITATIONS

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Abstract. *The dynamical coupling between two non-ideal power sources and a flexible supporting structure is examined via numerical simulations. The considered model consists of a flexible portal-frame displaying internal resonance between its first two vibrations modes and two unbalanced direct current motors with limited power supply.*

Keywords. *Non-ideal system, plane portal frame, internal resonance, self-synchronization.*

1. Introduction

The study of non-ideal vibrating systems, that is, when the excitation is influenced by the response of the system, has been considered a major challenge in theoretical and practical engineering research. When the excitation is not influenced by the response, it is said to be an ideal excitation or an ideal source of energy. On the other hand, when the excitation is influenced by the response of the system, it is said to be non-ideal.

Then, depending of the excitation, one refers to vibrating systems as ideal or non-ideal. The behavior of the ideal vibrating systems is well known in current literature, but there are few results on non-ideal ones. Generally, non-ideal vibrating systems are those for which the power supplies is limited. The behavior of the vibrating systems departs from the ideal case as power supply becomes more limited.

For non-ideal dynamical systems, one must add an equation that describes how the energy source "supplies the energy to the equations" that govern the corresponding ideal dynamical system. Thus, as a first characteristic, the non-ideal vibrating system has one more degree of freedom than its ideal counterpart.

If we consider the region before resonance on a typical Frequency-Response curve, we note that as the power supplied increases, the speed of rotation of the motor increases accordingly. However, this behavior doesn't continue indefinitely. The closer the motor speed moves toward the resonant frequency the more power is required to increase the motor speed, as part of the energy is consumed moving the supporting structure. A large change in the power supplied to the motor results in a small change in its frequency and a large increase in the amplitude of the resulting oscillations. Thus, near resonance, it appears that additional power supplied to the motor only increases the amplitude of the response of the structure while having little effect on the RPM of the motor.

Jump phenomena and the increase in power required by a source operating near resonance are manifestations of a non-ideal energy source and are often referred as the SOMMERFELD EFFECT, in honor of the first man who observed it Sommerfeld (Sommerfelf, 1904). We notice that (Nayfeh, Mook, 1979) presented a review of different theories of non-ideal problems during the period of 1904-1973 and (Balthazar et al. 2002) during the period 1904-2002.

Synchronization phenomena have attracted the attention of many researchers in mathematics and physics as well as in various fields of mechanical and electrical engineering. A formal definition of synchronization means corresponding time behavior of two or more processes. The classical example is the synchronization between the moon orbital motion and its rotational motion. In some cases the synchronous regime arises due to natural properties of the processes themselves and their natural interaction. A well-known example is frequency synchronization of oscillating or rotating bodies. This phenomenon is called self-synchronization (Blekhman, 1997).

In this paper, we present a study of self-synchronization problem of a vibrating problem composed by two unbalanced dc motors with limited power supply that is mounted on the horizontal beam of a flexible portal frame (a non-ideal system) by using numerical simulations. Recently, similar approach was applied in a non-ideal Duffing equation (Palacios et al., 2003) considering that the self-synchronization is influenced by the nonlinear stiffening of a flexible support and its response.

We remark that this kind of phenomenon was considered before in the literature in others ideal problems, distinct of the present one by Profs. Blekhman (Blekhman, 1988; 2000) and Prof. Dimentberg (Dimentberg, 2001) among others

2. Mathematical Model

A schematic diagram of a flexible portal frame excited by two unbalanced DC motors of limited power supplies is shown in Fig. 1. The portal frame model consist of two columns clamped in their bases with h height and the horizontal beam is pinned to the columns at both ends with L length. Both columns and beam have flexural rigidity EI. M is the mass of a dead weight at mid span of the beam. The structure is modeled as a two degrees of freedom system. The coordinate q_1 is related to the horizontal displacement in the sway mode (with natural frequency ω_1) and q_2 to the mid-span vertical displacement of the beam in the first symmetrical mode (with natural frequency ω_2). The two non-dimensional generalized coordinates of this model are

$$q_1 = \frac{u_2}{h},\ q_2 = \frac{v_2}{L} \tag{1}$$

where u_2 is the lateral displacement of the mid-span section of the beam and v_2 is its vertical displacement.

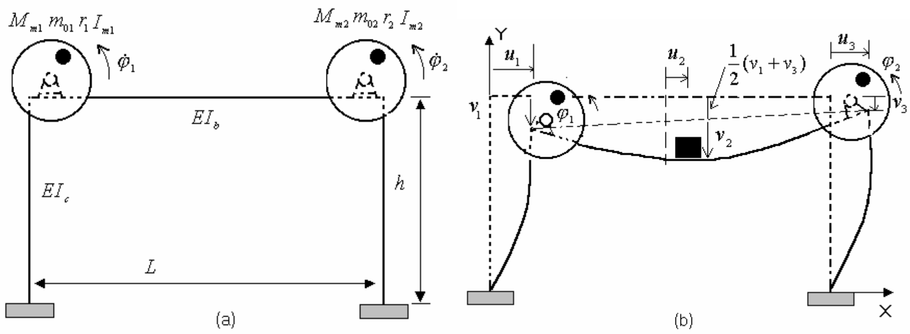

Figure 1. (a) Schematic diagram of a non-ideal system and (b) deformed configuration of the non-ideal system.

The linear stiffness of the columns and the beam can be evaluated by a Raleigh- Ritz procedure from cubic trial functions. Geometric nonlinearly is introduced by considering the shortening due to bending of the columns of the beams as

$$\Delta h = \frac{1}{2}C(u_{1,3})^2 \quad \Delta L = \frac{1}{2}B(v_2)^2 \tag{2}$$

where C, B are obtained from the same cubic functions.

Two unbalanced dc motors of mass M_{ms} are placed on the top of the columns The energy sources turn each rotor whose moment of inertia is I_{ms}. Each rotor has an unbalanced mass m_s at a distance r_s from the center of rotation, which produces forces of inertia.

Two other generalized displacements are the rotation angles of the rotors, φ_1 and φ_2.

$$u_{01} = u_1 + r\cos\varphi_1,\ v_{01} = v_1 + r\sin\varphi_1 + h,\ u_{02} = u_3 + r\cos\varphi_2 + L,\ v_{02} = v_3 + r\sin\varphi_2 + h \tag{3}$$

3. Equations of Motion

The equations of motion can be written in adimensional form (Palacios, 2002)

$$\begin{aligned}
q_1'' + \hat{\omega}_1^2 q_1 &= \hat{\alpha}_1(q_2'' q_2 + q_2'^2) + \hat{\alpha}_2(\varphi_1''\sin\varphi_1 + \varphi_1'^2\cos\varphi_1 + \varphi_2''\sin\varphi_2 + \varphi_2'^2\cos\varphi_2) \\
&\quad + \hat{\alpha}_3 q_2^2 - \hat{\mu}_1 q_1' - \hat{\alpha}_4 q_1 q_2'' \\
q_2'' + \hat{\omega}_2^2 q_2 &= \hat{\alpha}_5 q_1'' q_2 + \hat{\alpha}_6 q_1 q_2 - \hat{\mu}_2 q_2' - \hat{\alpha}_{10} - \hat{\alpha}_7(q_1'^2 + q_1 q_1'') \\
\varphi_1'' &= \hat{\alpha}_8 q_1''\sin\varphi_1 - \hat{\alpha}_9\cos\varphi_1 + \hat{a}_1 - \hat{b}_1\varphi_1' \\
\varphi_2'' &= \hat{\alpha}_8 q_1''\sin\varphi_2 - \hat{\alpha}_9\cos\varphi_2 + \hat{a}_2 - \hat{b}_2\varphi_2'
\end{aligned} \tag{4}$$

where

$$\tau = \omega_1 t \,,\ \hat{\omega}_1 = 1,\ \hat{\omega}_2 = \frac{\omega_2}{\omega_1} \approx 2 \,,\ \hat{\alpha}_1 = \alpha_1 \,,\ \hat{\alpha}_2 = \alpha_2 \,,\ \hat{\alpha}_3 = \frac{\alpha_3}{\omega_1^2} \,,\ \hat{\alpha}_4 = \alpha_4 \,,\ \hat{\alpha}_5 = \alpha_5 \,,$$

$$\hat{\alpha}_6 = \frac{\alpha_6}{\omega_1^2} \,,\ \hat{\alpha}_7 = \alpha_7 \,,\ \hat{\alpha}_8 = \alpha_8 \,,\ \hat{\alpha}_9 = \frac{\alpha_9}{\omega_1^2} \,,\ \hat{\alpha}_{10} = \frac{\alpha_{10}}{\omega_1^2} \,,\ \hat{\mu}_1 = \frac{\mu_1}{\omega_1} \,,\ \hat{\mu}_2 = \frac{\mu_2}{\omega_1} \qquad (5)$$

The constant torques of each DC motor (control parameter) will be represented by the dimensionless parameters:

$$\hat{a}_1 = \frac{k_m i_{01} \hat{V}_{a1}}{p_1 (I_m + m_0 r^2)\omega_1^2} \,,\ \hat{a}_2 = \frac{k_m i_{02} \hat{V}_{a2}}{p_3 (I_m + m_0 r^2)\omega_1^2} \qquad (6)$$

and the motor type of the two DC motors are represented by the parameters:

$$\hat{b}_1 = \frac{k_m i_{01} p_2}{p_1 (I_m + m_0 r^2)\omega_1} \,,\ \hat{b}_2 = \frac{k_m i_{02} p_4}{p_3 (I_m + m_0 r^2)\omega_1} \qquad (7)$$

Note also that the two dc motors are distinct in the elements of characteristic curves (straight lines).

4. Numerical Simulations

Next, we carried out a number of numerical simulations in order to observe the interaction between the two non-ideal dc motors and the portal frame structure with controlled synchronization by using the MATLAB-SIMULINK®.

Here, we have taken the following numerical values for the non dimensional parameters: $\hat{\alpha}_1$=5.76, $\hat{\alpha}_2$=0.0025, $\hat{\alpha}_3$=18, $\hat{\alpha}_4$=0.96, $\hat{\alpha}_5$=3, $\hat{\alpha}_6$=18.76, $\hat{\alpha}_7$=0.5, $\hat{\alpha}_8$=4.8, $\hat{\alpha}_9$=0.049, $\hat{\alpha}_{10}$=0.0042, $\hat{\mu}_1$=0.014, $\hat{\mu}_2$=0.019, $\hat{\omega}_1$=1, $\hat{\omega}_2$=2.

Note that the dimensional natural frequencies of the analyzed problem are ω_1=49.08 rad/s, ω_2=98.27 rad/s, satisfying the internal resonance condition $\omega_2 \approx 2\omega_1$.

Next, we present three sets of numerical simulations obtained taking equal and different torques for the same kind of dc motors in order to obtain synchronization control in the pos-resonance and resonance regions.

4. 1 First set of numerical simulations

Now, in order to carry out the numerical simulations, we take the following numerical values (two identical unbalanced motors and quasi equal control parameters): $\hat{a}_1$ = 3.0303, $\hat{a}_2$ = 3.03; $\hat{b}_1$ = 1.19 = $\hat{b}_2$,

with initial conditions: $\varphi_1(0) = \frac{\pi}{2}$, $\varphi_2(0) = -\frac{\pi}{2}$, $\varphi_1'(0)$=1.5, $\varphi_2'(0)$=0.5.

In Figure 2, we show the development of self-synchronization in the non-ideal system when $\hat{a}_1 \approx \hat{a}_2$, and for the same motor type $\hat{b}_1 = \hat{b}_2$.

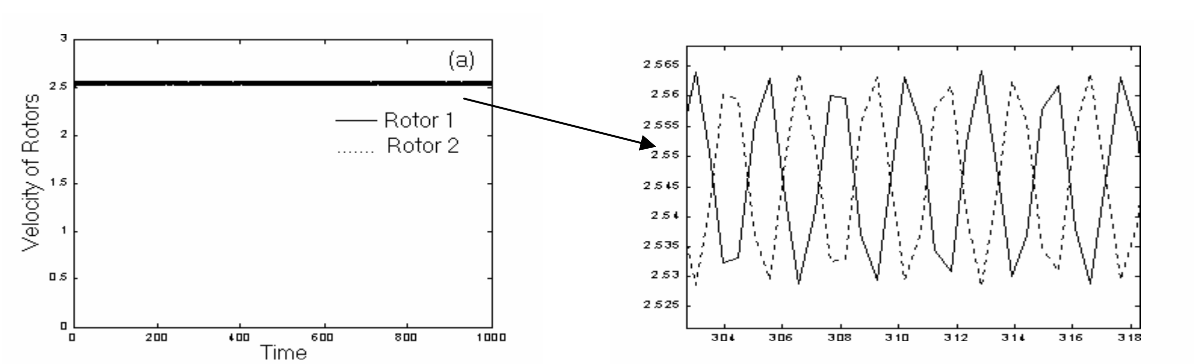

Figure 2. Self-synchronization of the non-ideal sources with equal controlling torques for $\hat{a}_1$ = 3.0303, $\hat{a}_2$ = 3.03.

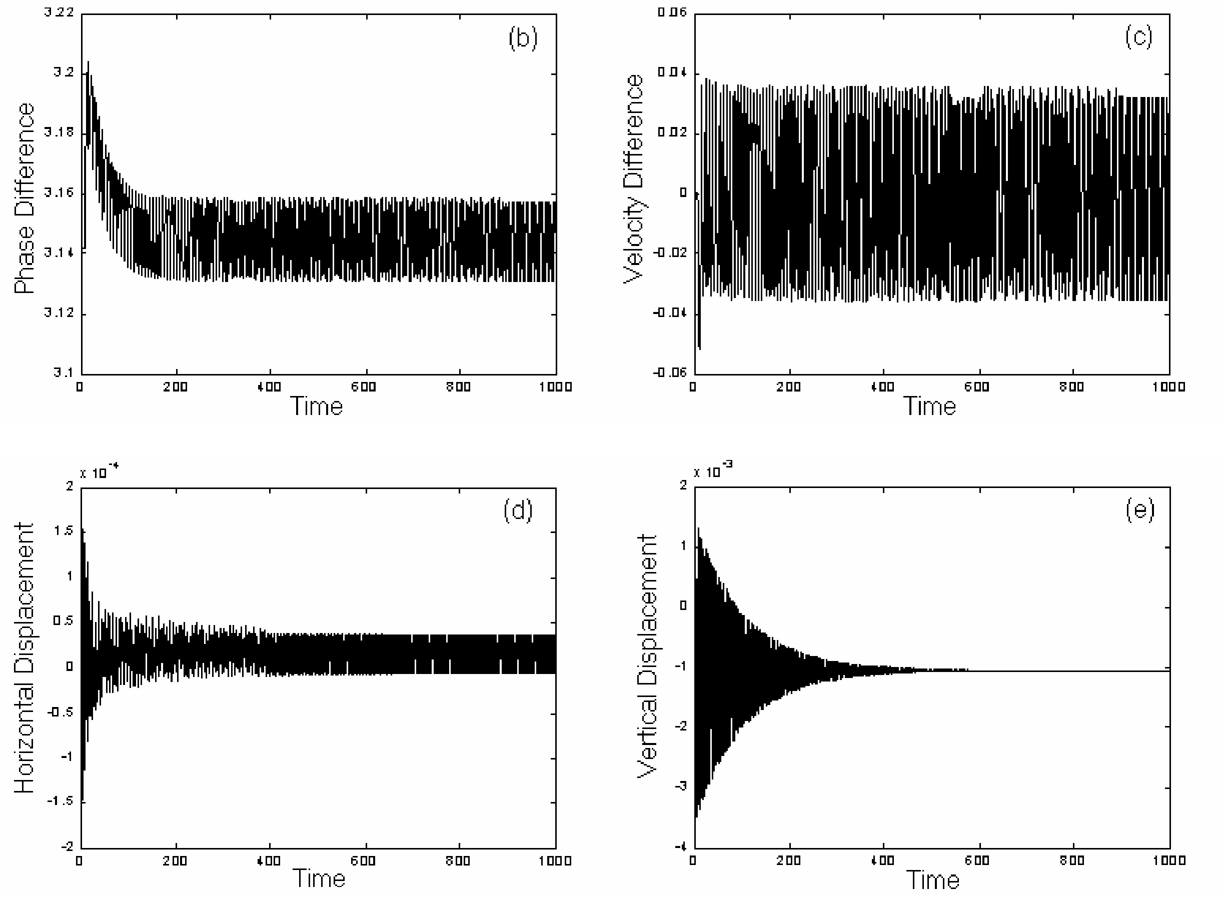

Figure 2. (Continued).

It is observed that the rotors are rotating in the same direction above the first resonance region and arrive at some average velocity with anti-phase motion (see Fig. 2(a) and its zoom in the right hand).

For that anti-phase motion to be achieved it is necessary that in steady state motion the average phase difference achieves value π (see Fig. 2(b)) and the rotor average velocity difference approaches the value zero (see Fig. 2(c)) (but we observe that we do not get constant values; oscillations are observed due to the portal frame influence upon the rotation of the rotors).

In Figs. 2(d) and (e)), the vertical and horizontal displacements of the portal frame are decreasing.

4. 2 Second set of numerical simulations

Next, in order to carry out the numerical simulations, we take the following numerical values (two identical unbalanced dc motors and different control parameters): $\hat{a}_1$ = 3.0, $\hat{a}_2$ = 1.3; $\hat{b}_1$ = 1.19, $\hat{b}_2$ = 1.19,

with initial conditions: $\varphi_1(0)=\frac{\pi}{4}$, $\varphi_2(0)=-\frac{\pi}{4}$, $\varphi_1'(0)$=1.5, $\varphi_2'(0)$=0.5.

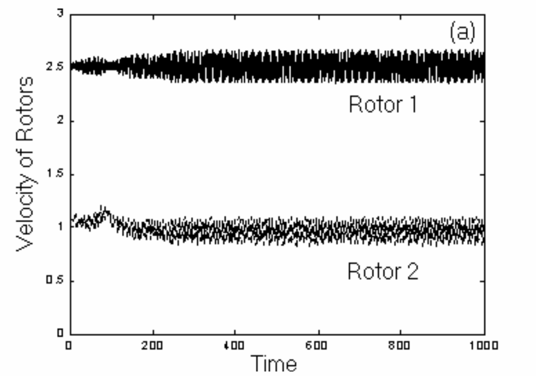

Figure 3. Absence of self-synchronization of the non-ideal sources with unequal controlling torques $\hat{a}_1$ = 3.0, $\hat{a}_2$ =1.3.

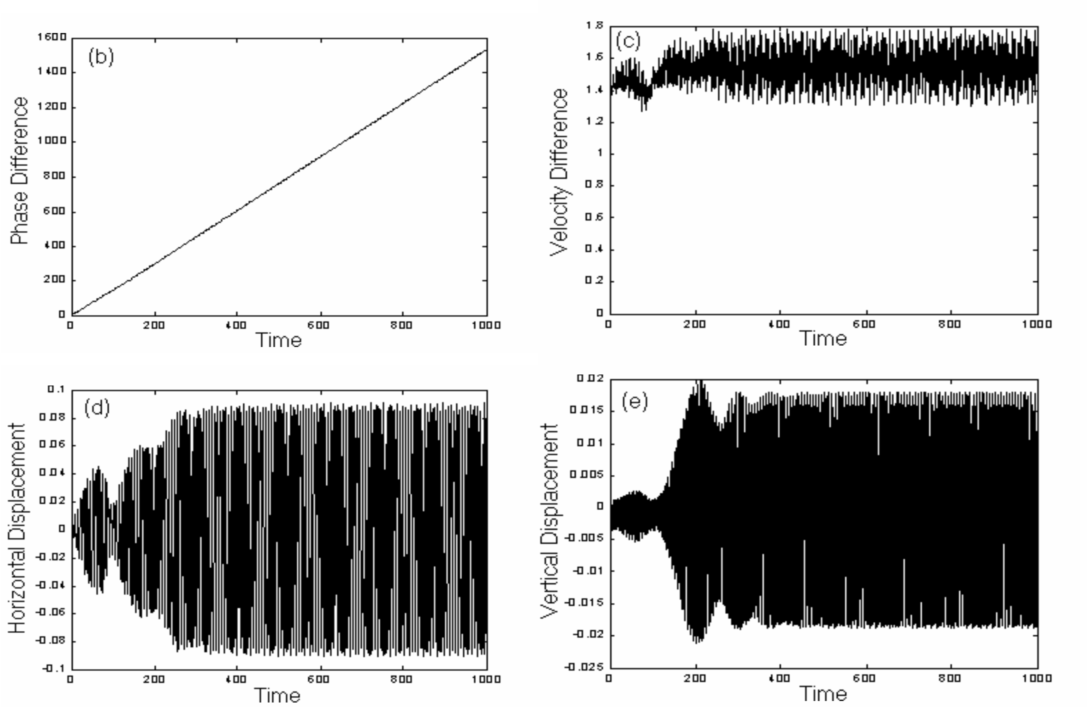

Figure 3. (Continued).

In Figure 3 we show the absence of self-synchronization when both the torques are not equal.

The rotation of first rotor is above of the region of resonance and the second rotor is captured for the first resonance region.

Note that the horizontal and vertical displacements of the frame do not decrease. As the internal resonance is present the response of horizontal motion absorbs energy of the vertical mode response.

Finally we mention that the rotor phase difference increases and rotor velocity difference is different from zero.

4. 3 Third set of numerical simulations

Next, we present numerical results for the particular case when we have unequal torques and two unequal unbalanced dc motors.

In this case, in order to carry out the numerical simulations, we take the following numerical values for the control parameters $\hat{a}_j$ for the two dc motors: $\hat{a}_1$ = 2.39, $\hat{b}_1$ = 1.19; $\hat{a}_2$ = 1.406, $\hat{b}_2$ = 0.7,

with initial conditions: $\varphi_1(0)=\frac{\pi}{2}$, $\varphi_2(0)=-\frac{\pi}{2}$, $\varphi_1'(0)$=2.5, $\varphi_2'(0)$=1.5.

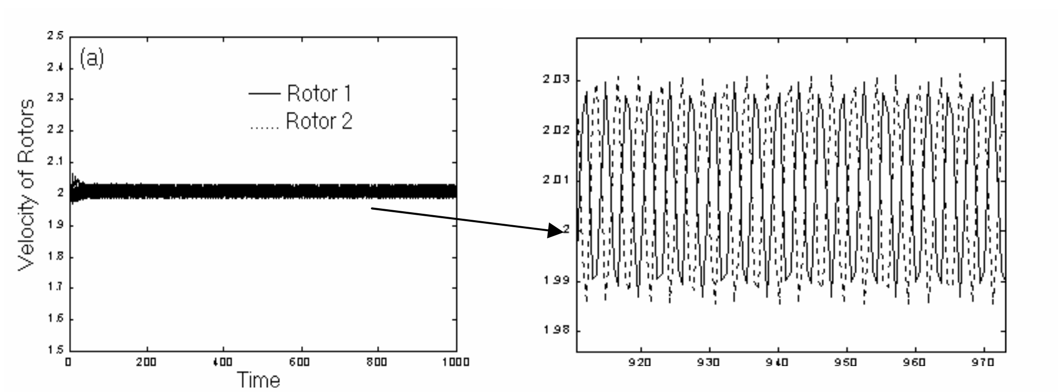

Figure 4. Presence of self-synchronization of the non-ideal sources with unequal controlling torques $\hat{a}_1$=2.30, $\hat{a}_2$=1.406 and unequal motors type $\hat{b}_1$ = 1.19, $\hat{b}_2$ = 0.7.

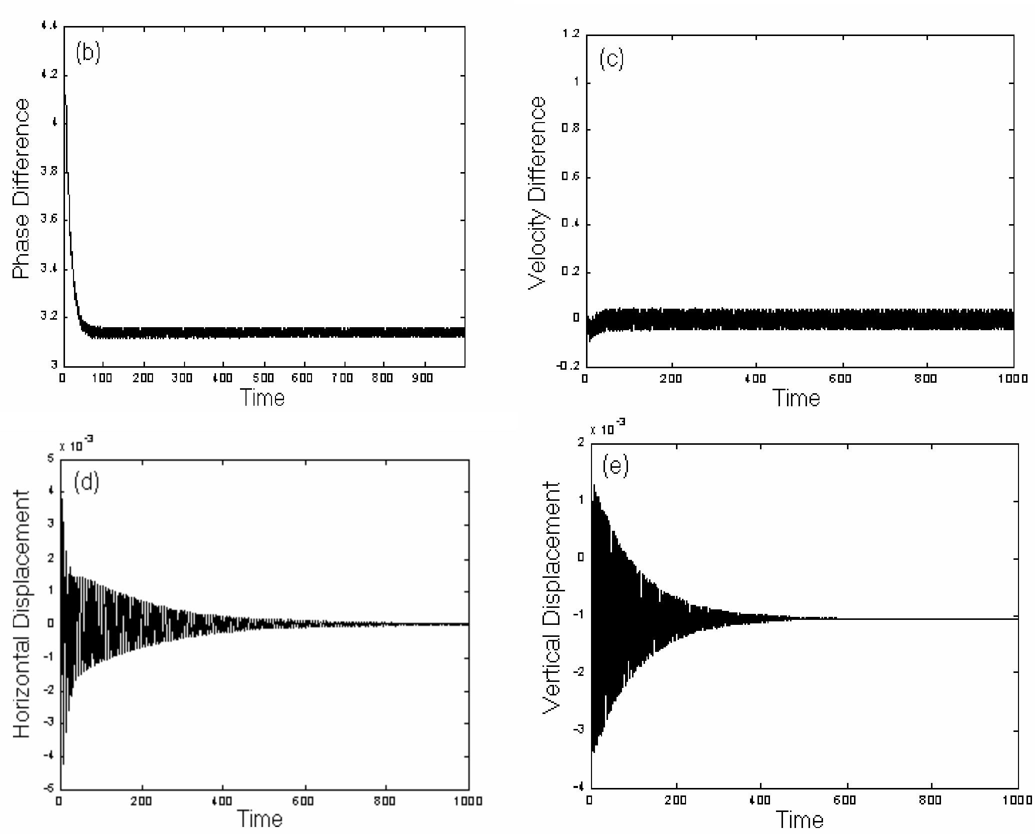

Figure 4. (Continued).

In Figure 4 we show the presence of self-synchronization in the non-ideal system when $\hat{a}_1 \neq \hat{a}_2$ and $\hat{b}_1 \neq \hat{b}_2$. Note that the two dc motors rise through to second resonance region and they arrive at some average velocity with anti-phase motion (see Fig. 4 (a) with illustration of a zoom at right hand).

This fact is justifiable as rotor phase and velocity difference achieve the values shown and zero respectively (see Figs. 4(b) and (c)).

During the passage through second resonance, we may observe low amplitudes of the horizontal and vertical motions of the portal frame structure (around zero).

This fact suggests that the portal frame structure have small amplitudes in both horizontal and vertical modes (see Figure 4 (d) e (e)).

5. Concluding Remarks

A particular non-linear phenomenon of self-synchronization of two unbalanced dc motors influenced by the response of their supporting flexible portal frame has been analyzed through numerical simulation.

The dynamical behavior of the non-ideal system in the resonance regions of the supporting structure was also shown. The presence de internal resonance between the first vibration modes of portal frame was considered in this study.

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7. References

Sommerfeld, A 1904, " Beitrag zum dynamischen asban de festigketslehre,"(in German) Physikalische Zetschr. Vol. 4,1904, 631-636.

Nayfeh, AH, 1979, " Nonlinear Oscillations", John Wiley, New York, 1979.

Balthazar, JM, Mook, DT, Weber HI, Brasil RMLRF, Fenili A, Belato D., Felix JLP, 2002, "An overview on non-Ideal vibrations", MECCANICA, pp. 1-9.

Blekhman, II, 1988, "Synchronization in science and technology", ASME Press, New York.

Blekhman, II, Fradkov, A.L., Nijmeijer, H., 1997, "On self-synchronization and controlled synchronization", Syst. Control Lett. 31, pp. 299-305.

Blekhman, II, 2000, "Vibrational mechanics: nonlinear dynamics effects, general approach, applications", World Scientific

Dimentberg, M., Cobb E. and Mensching, J., 2001, "Self-sinchronization of transient rotation in multiple shaft Systems", Journal of Vibration and Control, 7, pp. 221-232

Palacios JL, 2002, A theory on non-ideal portal frame, excited by non-ideal sources, Ph.D. Thesis, Engineering Mech. College, UNICAMP, Campinas, SP, Brazil (In Portuguese).

Palacios, J.L., Balthazar J.M. , Brasil, and R.M.L.R.F, 2003, "On the non-linear dynamics of a non-linear system under two non-ideals excitations", In: Proceedings of the X DINAME, 10-14 Th. March 2003, Ubatuba, SP, Brazil, and pp. 381-386.