

FINITE ELEMENT METHOD APPLIED TO INTELLIGENT BEAMS WITH SHAPE MEMORY ACTUATORS

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Abstract. *Inspired by nature, researchers are trying to create systems and structures that can repair themselves, presenting an adaptive behavior according to its environment. Among many options of smart sensors and actuators, employed in this kind of system, one can highlight piezoelectric materials, magnetostrictive materials, electrorheological fluids and shape memory alloys (SMAs). SMAs are metallic compounds with the ability to return to a previous shape or dimension, when subjected to an appropriate thermomechanical procedure. This article concerns with the finite element modeling of intelligent shape memory beams. Composite beams reinforced by shape memory actuators in the form of wires or laminae are considered. Finite element method is employed for spatial discretization and a constitutive model with internal constraints is used in order to describe the thermomechanical behavior of shape memory alloys. An iterative numerical method is developed in order to deal with nonlinearities in the formulation. Numerical simulations are carried out illustrating different possibilities using SMAs actuators.*

Key-words: *Shape memory alloys, Finite Element Method, Modeling and simulation, Beams.*

1. Introduction

Shape memory alloys (SMAs) are found in a great number of applications in different fields of science and engineering (Denoyer *et al.*, 2000; Duering *et al.*, 1999; Fujita & Toshiyosh, 1998; Garner *et al.*, 2001; Kibirkstis *et al.*, 1997; Lagoudas *et al.*, 1999; La Cava *et al.*, 2000; Machado & Savi, 2003; Pacheco & Savi, 1997; Pietrzakowski, 2000; van Humbeeck, 1999; Webb *et al.*, 2000). Therefore the modeling and simulation of systems using SMA actuators has a growth importance. This contribution proposes a finite element formulation to deal with intelligent beams with shape memory actuators. Finite element modeling of SMA structures has been previously addressed by Brinson and Lammering (1993), where a constitutive theory based on Tanaka's model (Tanaka, 1986), and later modified by Brinson (1993), has been employed to describe the SMA behavior. More recently, Auricchio and Taylor (1996) have also proposed a three-dimensional finite element model. Savi *et al.* (1998) discuss an iterative numerical procedure that has been developed to deal with both geometrical and constitutive nonlinearities in a finite element model for adaptive trusses with SMA actuators. La Cava *et al.* (2003) consider the analysis of shape memory bars using the constitutive model proposed by Baêta-Neves *et al.* (2003). Lagoudas *et al.* (1997) consider the thermo-mechanical response of a laminate with SMA strips where the thermo-mechanical response is based on Boyd-Lagoudas' polynomial hardening model (Boyd and Lagoudas, 1996). Kouzak *et al.* (1998) also treats SMA beams using a constitutive equation proposed by Brinson (1993). Trochu & Qian (1997), Masud *et al.* (1997), Bhattacharyya *et al.* (2000), Liu *et al.* (2002) are other contributions in this field. Moreover, dual kriging interpolation has been employed with finite element method (FEM) in order to describe the shape memory behavior (Trochu & Qian, 1997; Trochu & Terriault, 1998; Trochu *et al.*, 1999).

Here, the FEM is employed to analyze intelligent beams with shape memory actuators using a constitutive equation proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003) to describe the thermo-mechanical behavior of SMAs. This model is based on Fremond's theory (Fremond, 1987, 1996) and includes four phases in the formulation: three variants of martensite and an austenitic phase. Furthermore, different material parameters for austenitic and martensitic phases are concerned. Thermal expansion and plastic strains are also included in the formulation and hardening effect is represented by a combination of kinematic and isotropic behaviors. A plastic-phase transformation coupling is incorporated into the model allowing a correct description of the thermo-mechanical behavior of SMAs. Moreover, horizontal enlargement of the stress-strain hysteresis loop is considered, allowing better adjustments with experimental data.

An iterative numerical procedure based on operator split technique (Ortiz *et al.*, 1983) is developed in order to deal with the nonlinearities in the formulation. Numerical simulations are carried out showing different behaviors of SMA

1bars. Results show that the proposed model is able to capture the general behavior of SMAs, including pseudoelastic and shape memory effects and also phase transformations due to thermal expansion.

2. Finite Element Formulation

Consider a composite beam reinforced with a SMA actuator subjected to a constant axial load, as show in Fig. (1). The actuator is assumed to be significantly thinner than the height of the beam cross section.

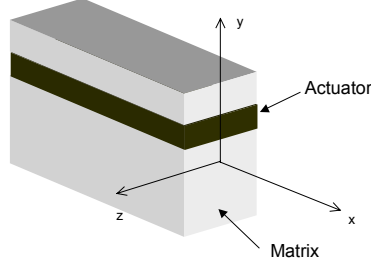


Figure 1 – Composite beam with SMA actuator.

The internal energy increment may be written as follows

$$\delta\Gamma = \int_{V_m} \sigma_m \delta\varepsilon dV + \int_{V_a} \sigma_a \delta\varepsilon dV \quad (1)$$

where V is the volume and the subscripts m and a are associated with matrix and actuator, respectively. An elastic relation is considered for the matrix, $\sigma_m = E_m \varepsilon$, where E_m is the matrix elastic modulus; σ_a is given by constitutive equation proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003). For simplicity, a compact form of the stress-strain relation is here presented,

$$\sigma_a = E_a \varepsilon + A_a \quad (2)$$

where A_a represents nonlinear terms related to phase transformation thermal expansion and plastic behavior,

$$A_a = -E_a \varepsilon^p + (\alpha + E_a \alpha_H) (\beta_2 - \beta_1) - \Omega (T - T_0) \quad (3)$$

ε^p is the plastic strain, β_1 and β_2 are the volumetric fraction of martensitic variants, M^+ and M^- . E_a , α , α_H and Ω are material parameters. A detailed description of the constitutive model may be found in Savi *et al.* (2002) and Baêta-Neves *et al.* (2003), where the evolution equations of all internal variables related to the formulation are presented. Bernoulli-Euler hypothesis is adopted,

$$\varepsilon = u_{,x} - y w_{,xx} \quad (4)$$

where $(\)_{,x} = d(\)/dx$. Now, the principle of virtual work is written as follows

$$\begin{aligned} & (E_m A_m + E_a A_a) \int_L (u_{,x} \delta u_{,x}) dx + (E_m I_m + E_a I_a) \int_L (w_{,xx} \delta w_{,xx}) dx - \\ & - (E_m H_m + E_a H_a) \int_L (u_{,x} \delta w_{,xx} + w_{,xx} \delta u_{,x}) dx + A_a \int_L (A_a \delta u_{,x} - H_a \delta w_{,xx}) dx - \int_L (p \delta u + q \delta w) dx = 0 \end{aligned} \quad (5)$$

where the term A_a is assumed to be constant inside the actuator. Also, A is the cross section area, H and I are the first and second order moment of the cross section, respectively; p is the axial load per length and q is the transversal load per length. The analysis of beams constructed by many SMA actuators must be taken into account considering several actuators.

Spatial discretization is considered by using the finite element method, which establishes the following approximation

$$u(x) = \sum_{j=1}^2 U_j^e \mathcal{G}_j(x) \quad (6)$$

$$w(x) = \sum_{j=1}^4 W_j^e \varphi_j(x)$$

where U_j^e and W_j^e are nodal displacements, $\mathcal{G}_j(x)$ and $\varphi_j(x)$ are respectively, Lagrange and Hermite shape functions, presented below (Reddy, 1984):

$$\mathcal{G}_1 = 1 - \frac{x}{L}, \quad \mathcal{G}_2 = \frac{x}{L} \quad (7)$$

$$\varphi_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}, \quad \varphi_2 = x - \frac{2x^2}{L} + \frac{x^3}{L^2}, \quad \varphi_3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}, \quad \varphi_4 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

From this approximation, Eq.(5) is rewritten as follows,

$$\begin{aligned} & (E_m A_m + E_a A_a) \int_L [\mathbf{B}_u]^T [\mathbf{B}_u] dx \{U^e\} + (E_m I_m + E_a I_a) \int_L [\mathbf{B}_w]^T [\mathbf{B}_w] dx \{U^e\} - \\ & - (E_m H_m + E_a H_a) \int_L ([\mathbf{B}_u]^T [\mathbf{B}_w] + [\mathbf{B}_w]^T [\mathbf{B}_u]) dx \{U^e\} + A_a \int_L (A_a [\mathbf{B}_u]^T - H_a [\mathbf{B}_w]^T) dx - \int_L (p [\mathbf{N}_u]^T + q [\mathbf{N}_w]^T) dx = 0 \end{aligned} \quad (8)$$

Which follows a discrete version of the governing equation for a single element.

$$[K^e] \{U^e\} = \{F^e\} - \{\hat{F}^e\} \quad (9)$$

where $[K^e]$ is the stiffness matrix, $\{U^e\}$ is the displacement vector, $\{F^e\}$ is the force vector and $\{\hat{F}^e\}$ is related to the behavior of the nonlinear shape memory actuator. The definition of these matrixes is given by,

$$[K^e] = [K_u^e] + [K_w^e] + [K_{uw}^e] \quad (10)$$

$$[K_u^e] = (E_m A_m + E_a A_a) \int_L [\mathbf{B}_u]^T [\mathbf{B}_u] dx \quad (11)$$

$$[K_{uw}^e] = -(E_m H_m + E_a H_a) \int_L ([\mathbf{B}_u]^T [\mathbf{B}_w] + [\mathbf{B}_w]^T [\mathbf{B}_u]) dx \quad (12)$$

$$[K_w^e] = (E_m I_m + E_a I_a) \int_L [\mathbf{B}_w]^T [\mathbf{B}_w] dx \quad (13)$$

$$\{F^e\} = \int_L (p [\mathbf{N}_u]^T + q [\mathbf{N}_w]^T) dx \quad (14)$$

$$\{\hat{F}^e\} = A_a \int_L (A_a [\mathbf{B}_u]^T - H_a [\mathbf{B}_w]^T) dx \quad (15)$$

After the assemblage of the global system, an operator split technique (Ortiz *et al.*, 1983) associated with an iterative numerical procedure is applied in order to deal with the nonlinearities in the formulation. At first, the global matrix K_i and the global vector \hat{F}_i are evaluated assuming that neither phase transformation nor plastic strain has taken place, which means that they have the same value of the previous time instant. Under this assumption, displacements U_i are calculated by solving a linear system. In the next step, all variables related to the SMA actuator (strain, stress, volumetric fractions of the phases, etc) are evaluated with the aid of constitutive and evolution equations. Afterwards, the matrix K_i and the vector \hat{F}_i are recalculated. This procedure is repeated to assure a prescribed convergence tolerance.

3. Numerical Simulations

This section considers numerical simulations performed with the proposed formulation. Material properties are the same discussed in Baêta-Neves *et al.* (2003). An intelligent beam of SMA with a square cross section of 15mm side and a 300mm length is analyzed. Thirty layers are considered in order to describe the variables distribution through the height of the beam. It should be pointed out that, the proposed model do not considers geometric nonlinearities. Nevertheless, for the cases studied the proposed formulation presents errors smaller than 10% and it can be employed for a qualitative exploitation of the beam behavior.

At first, a cantilever beam divided in 6 elements is analyzed (Fig. 2). The beam is at a temperature where austenitic phase is stable ($T = 313\text{K} > T_A$ – temperature above which austenite is stable) and is subjected to a vertical load F_y . The characteristics of the thermomechanical loading are shown Fig. (3).

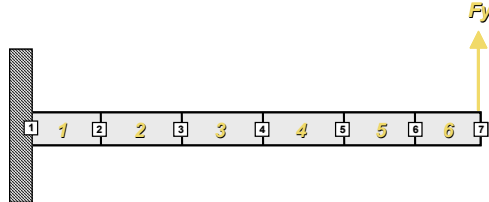


Figure 2. Cantilever beam subjected to a vertical load F_y .

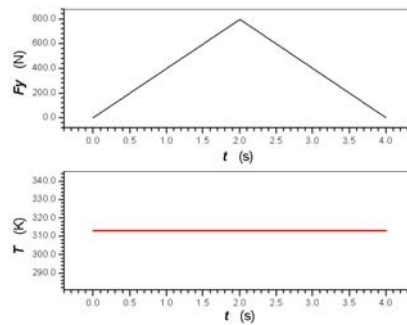


Figure 3. Thermomechanical load.

This loading-unloading process is related to pseudoleasticity. However, different behaviors are expected through the length and through the cross section of the beam. At first, consider the analysis through the length of the beam. A point at the lower surface ($y = -h$) of the beam is analyzed for different axial positions (varying x). Near the free-end, the moment generated by the force F_y does not induce phase transformations. On the other hand, near the clamped end, when $x = L$, complete phase transformation occurs. Figure (4) shows stress-strain curves for different axial position through the length of the beam while Fig. (5) presents volumetric fraction of positive martensitic variant (M^+) and austenite (A).

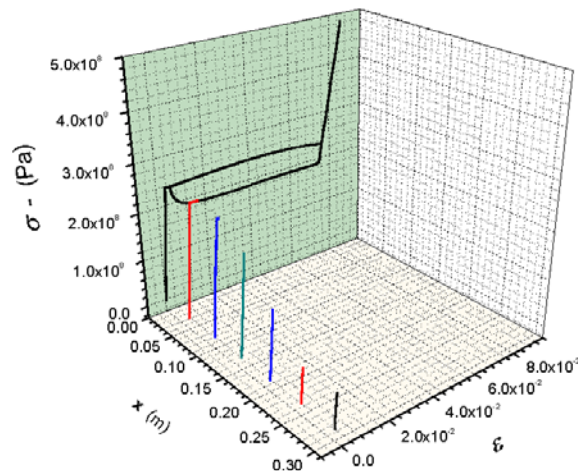


Figure 4. Evolution of stress-strain curves through the length of the beam.

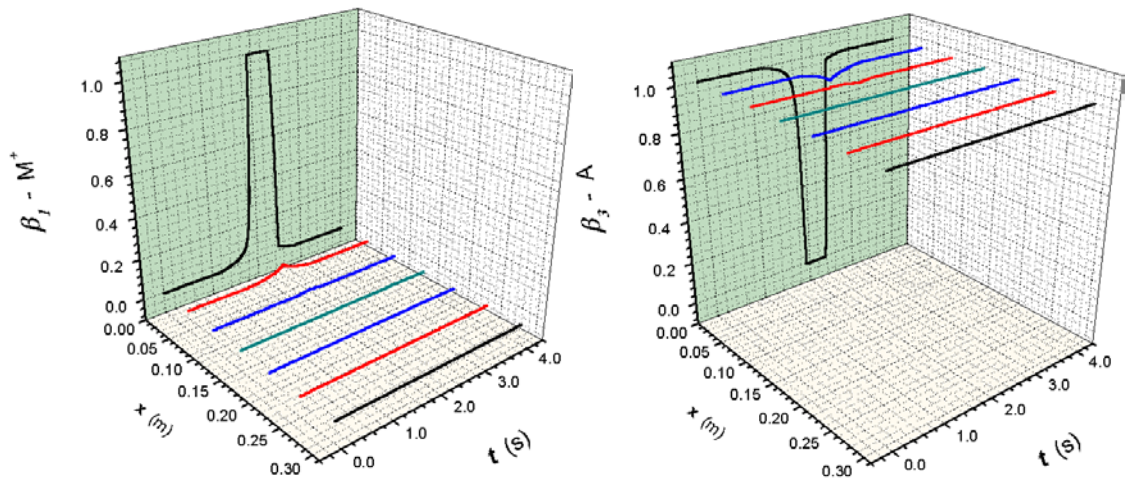


Figure 5. Evolution of volumetric fractions through the length of the beam.

At this point, an analysis of the clamped end is carried out. At this cross section, incomplete phase transformation may occur depending on its position. Figure (6) presents the stress-strain distribution through the height of the beam. A complete comprehension of phase transformation can be obtained analyzing volumetric fraction of each phase. Figures (7-9) show these volumetric fractions, respectively for positive martensitic variant (M+), negative martensite (M-) and austenite. Notice the incomplete transformation and also the formation of the positive variant in lower layers and negative on the upper, according to tensile or compressive stress, respectively.

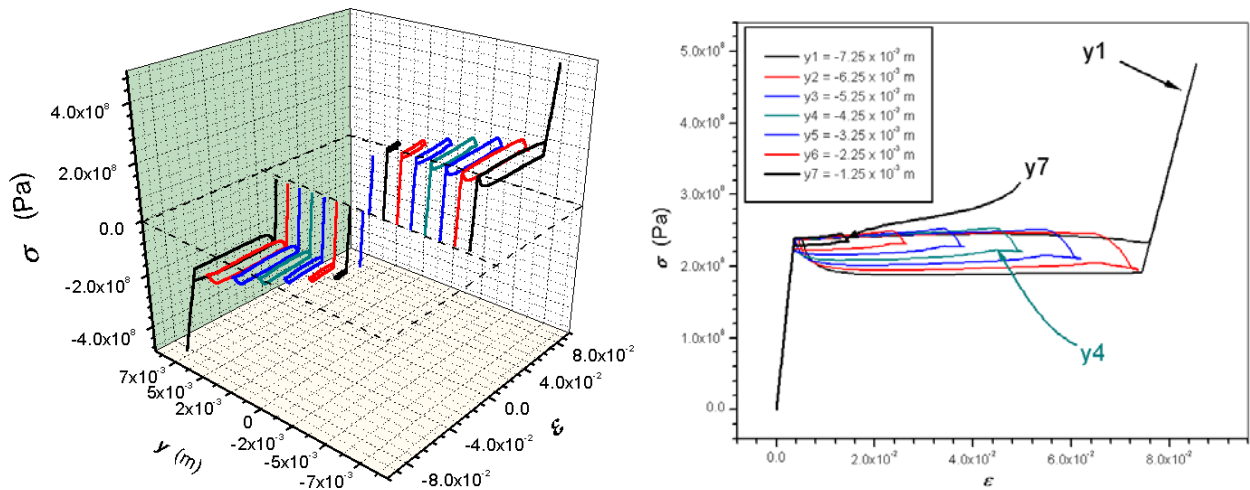


Figure 6. Stress-strain curves through the height of the cross section.

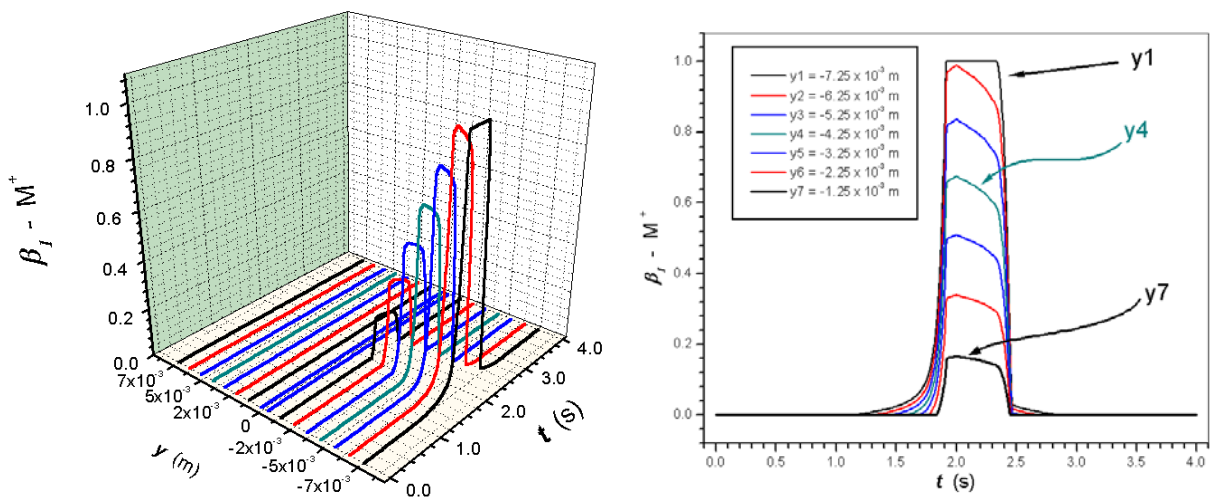


Figure 7. Volumetric fraction of positive martensitic variant (M+) through the height of the cross section.

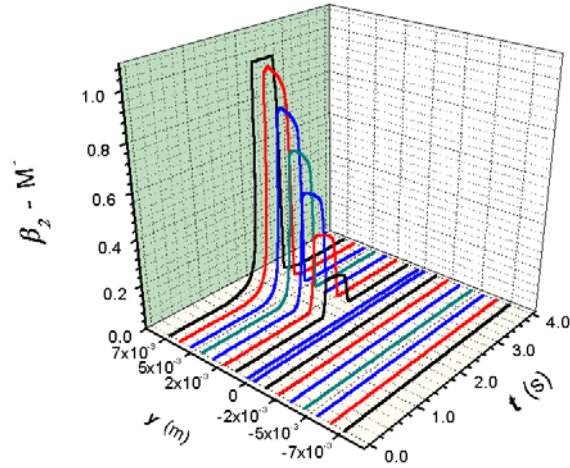


Figure 8. Volumetric fraction of negative martensitic variant (M-) through the height of the cross section.

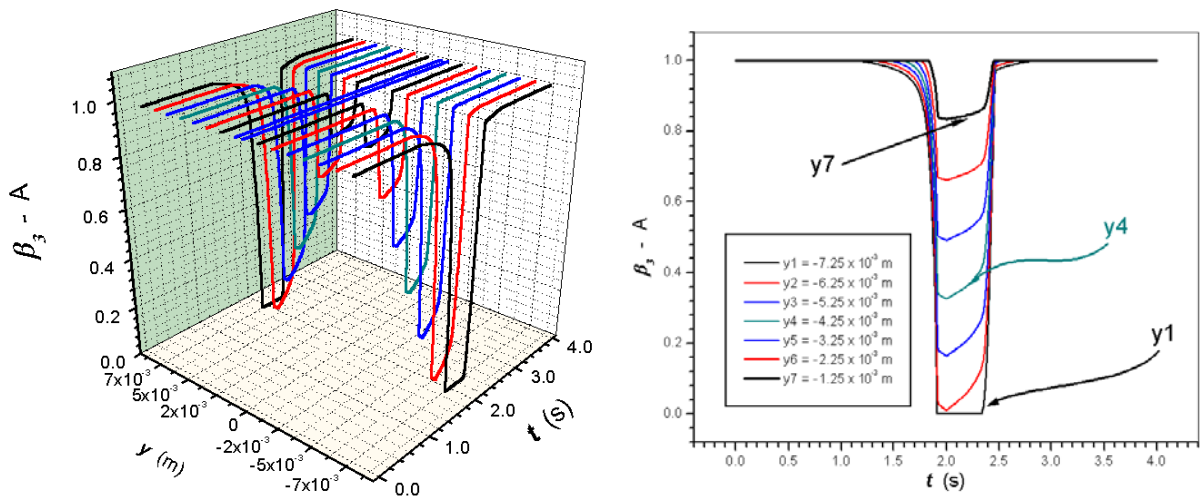


Figure 9. Volumetric fraction of austenite (A) through the height of the cross section.

The forthcoming analysis considers the evolution of some variables through the height of the cross section area. Figure (10) presents two different instants for stress and strain distributions, while Fig. (11) shows the volumetric fractions distributions of each phase for the same time instants.

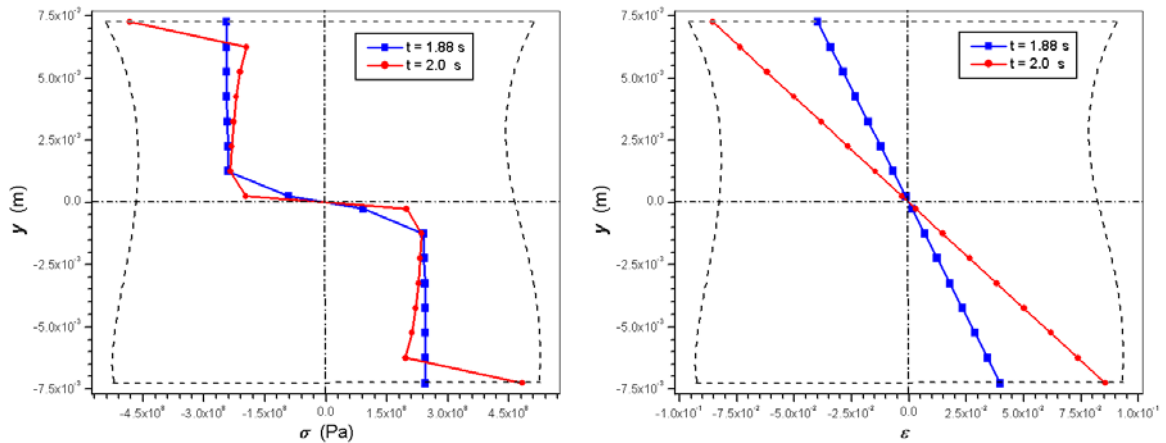


Figure 10. Evolution of stress and strain through the height of the cross section area.

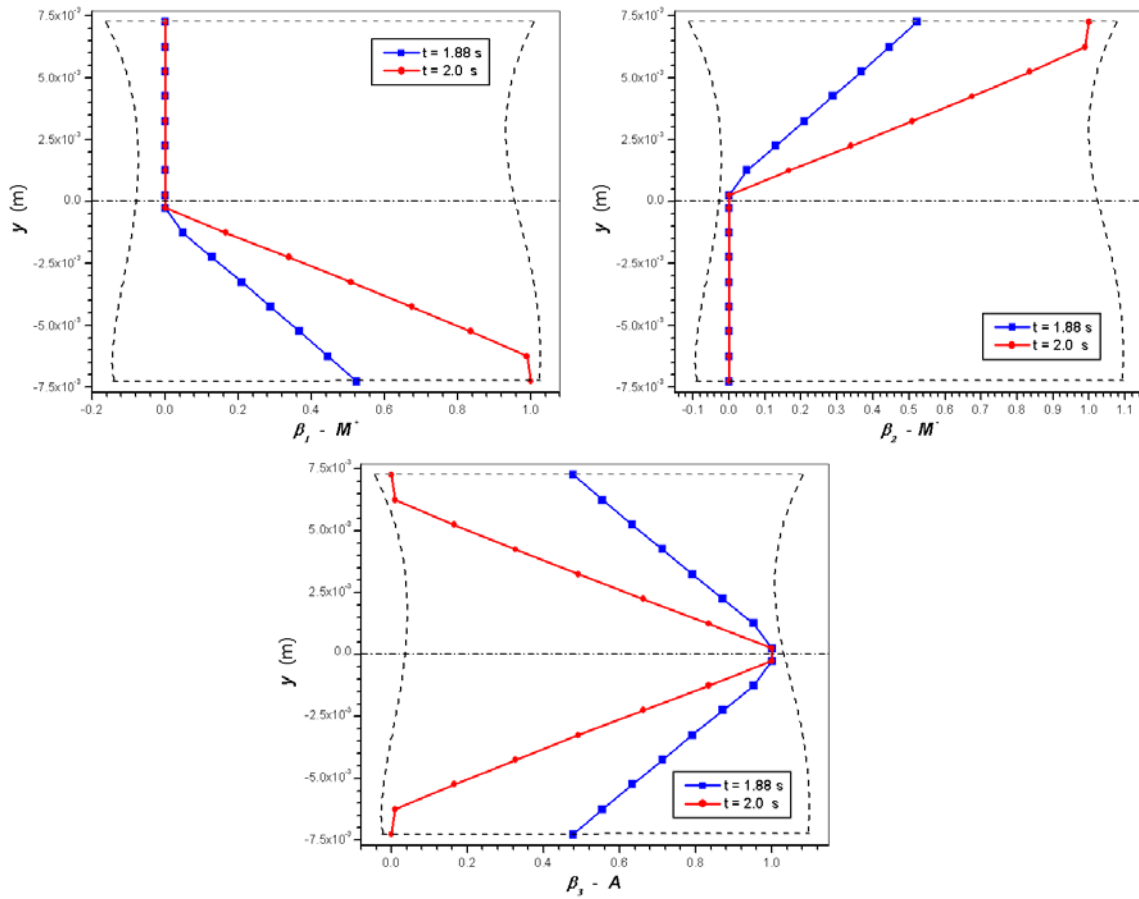


Figure 11. Evolution of volumetric fractions through the height of the cross section area.

A lower temperature ($T = 263\text{K} < T_M$ – temperature below which martensite is stable) is now in focus. Consider a thermomechanical loading process similar to the one presented in Figure (3) where the maximum load is 25N. Figure (12) presents stress-strain curves for the clamped end of the beam. Notice incomplete phase transformation similar to the previous case and also the tendency to eliminate residual strain. Figures (13) and (14) show this behavior.

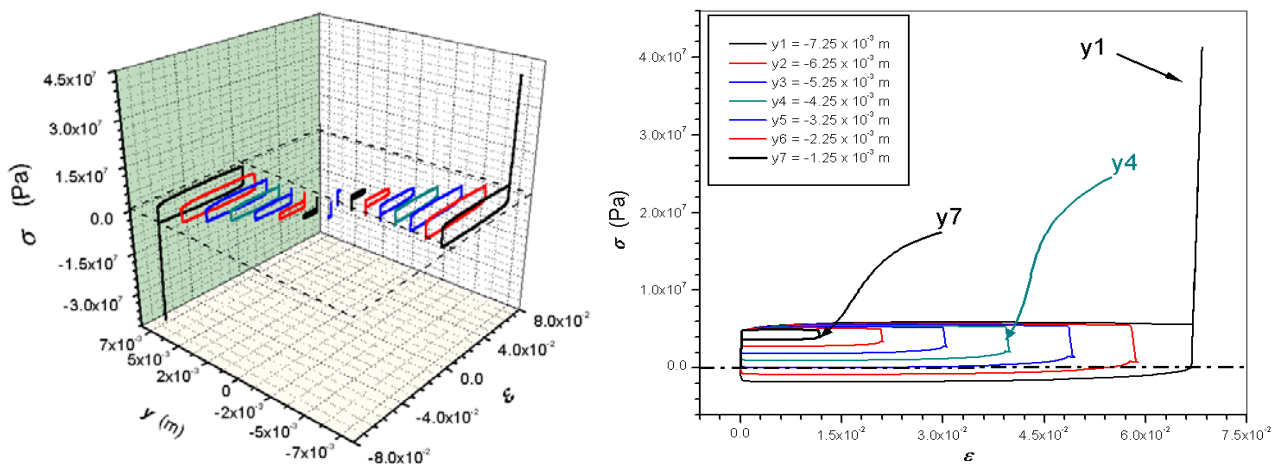


Figure 12. Stress-strain curves through the height of the cross section.

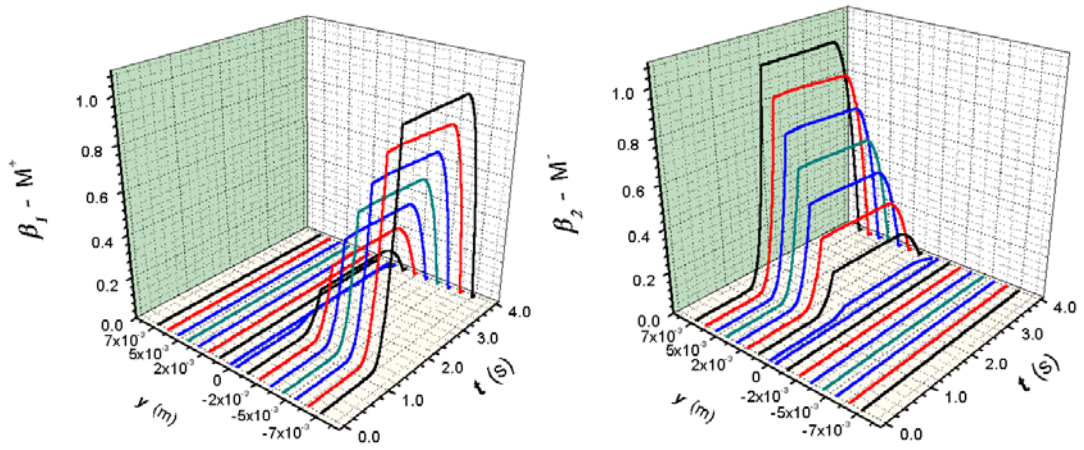


Figure 13. Volumetric fraction of positive (M+) and negative (M-) martensitic variants through the height of the cross section.

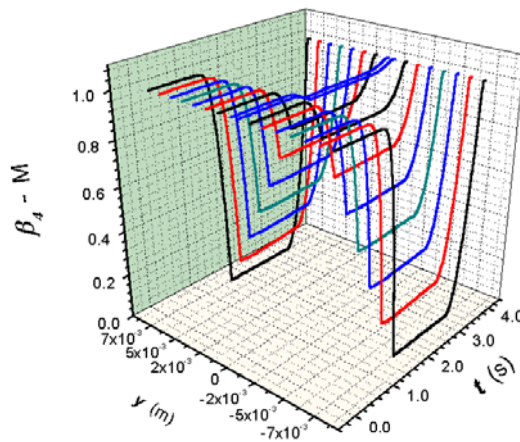


Figure 14. Volumetric fraction of detwinned martensite (M) through the height of the cross section.

At this point, the evolution of some variables through the height of the cross section area is considered. Figure (15) presents different instants for stress and strain distributions, while Fig. (16) shows the volumetric fractions distribution of each phase for the same time instants.

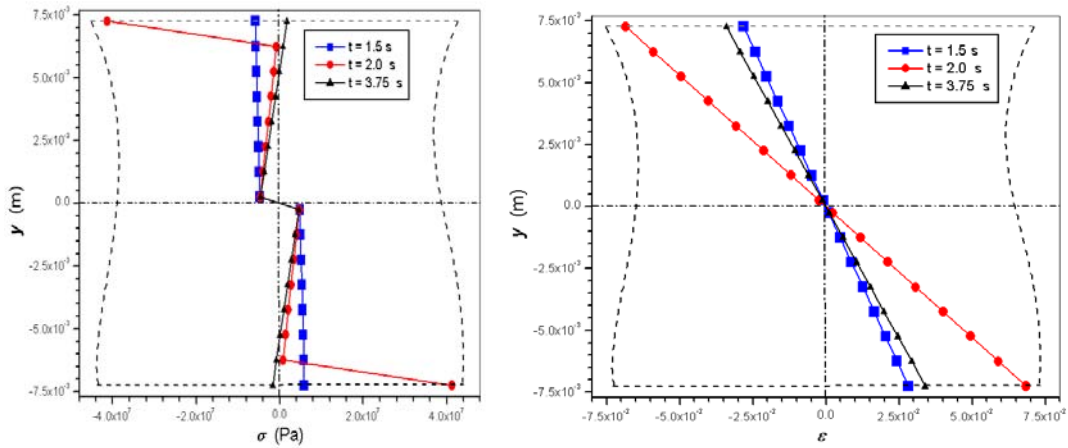


Figure 15. Evolution of stress and strain through the height of the cross section area.

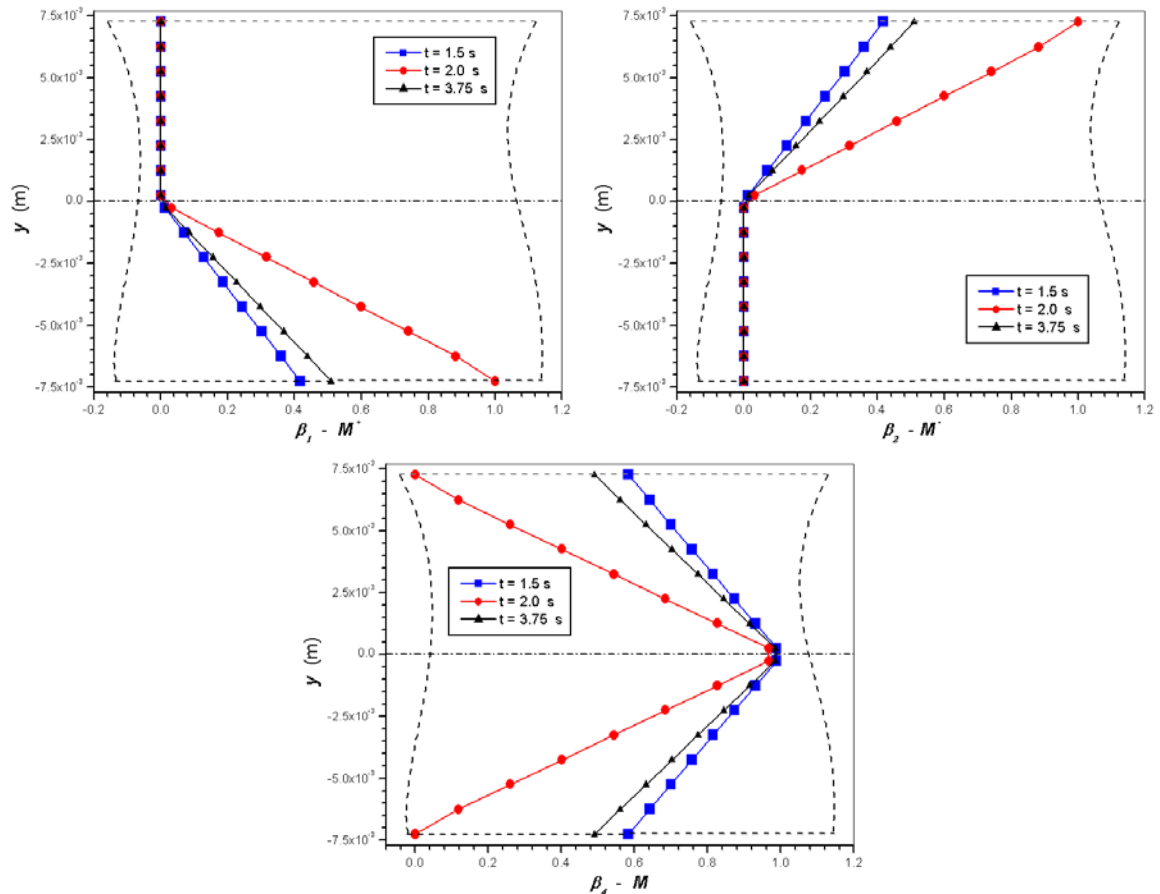


Figure 16. Evolution of volumetric fractions through the height of the cross section area.

4. Conclusions

A nonlinear finite element method is proposed in order to analyze intelligent beams with shape memory actuators. A constitutive model proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003) is used to describe the thermo-mechanical behavior of SMAs. The model considers thermal expansion and plastic strains with hardening. Bernoulli-Euler kinematics hypothesis is adopted. Even though geometrical nonlinearities are not included in the formulation, errors introduced by this hypothesis are inferior to 10% for the cases studied, allowing a qualitative exploitation of the phenomenon. An iterative numerical procedure based on operator split technique is developed in order to deal with nonlinearities of the formulation. Numerical simulations show a complex behavior related to this kind of problem presenting nonhomogeneous phase transformation either through the length or through the height of the beam.

5. Acknowledgements

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