

## ANALYSIS OF SHAPE MEMORY BARS USING THE FINITE ELEMENT METHOD

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**Abstract.** Shape memory and pseudoelastic effects are thermo-mechanical phenomena associated with martensitic phase transformations, presented by shape memory alloys. This contribution concerns with the analysis of nonlinear behavior of shape memory bars employing the finite element method. A constitutive equation based on Fremond's theory is considered. The proposed model includes four phases in the formulation: three variants of martensite and an austenitic phase. Different material parameters for austenitic and martensitic phases are concerned. Thermal expansion and plastic strains are included into the formulation and hardening effect is represented by a combination of kinematic and isotropic behaviors. A plastic-phase transformation coupling is incorporated into the model allowing a correct description of the thermo-mechanical behavior of SMAs. Moreover, constitutive equations consider horizontal enlargement of the stress-strain hysteresis loop, allowing better adjustments with experimental data. An iterative numerical process based on operator split technique is developed in order to deal with the nonlinearities of the formulation. Numerical simulations are carried out in order to illustrate the general behavior of shape memory bars under different thermo-mechanical loadings. Results show that the proposed model capture the general behavior of SMAs, allowing the description of bars subjected to non-homogeneous themomechanical loads.

**Key-words:** Shape memory alloys, Finite Element Method, Modeling and simulation

### 1. Introduction

Shape memory alloys (SMAs) have been found in a great number of applications in different fields of sciences and engineering. Self-actuating fasteners (La Cava *et al.*, 2000; van Humbeeck, 1999; Kibirkstis *et al.*, 1997; Borden, 1991), thermally actuator switches and several bioengineering devices are some examples of these applications (Machado & Savi, 2003; Duerig *et al.*, 1999; Lagoudas *et al.*, 1999). Aerospace technology are also using SMAs for solve important problems, in particular those concerning with space savings achieved by self-erectable structures, stabilizing mechanisms, non-explosive release devices and other possibilities (Pacheco & Savi, 1997; Denoyer *et al.*, 2000). Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles (Garner *et al.*, 2001; Webb *et al.*, 2000; Fujita & Toshiyoshi, 1998; Rogers, 1995). Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures (Pietrzakowski, 2000; Birman, 1997; Rogers, 1995). Despite all these applications, the modeling of SMAs is not well established and hence, it is an important task.

This contribution proposes a finite element formulation to deal with shape memory bars. Finite element modeling of SMA structures has been previously addressed by Brinson and Lammering (1993), where a constitutive theory based on Tanaka's model (Tanaka, 1986), and later modified by Brinson (1993), has been employed to describe the SMA behavior. More recently, Auricchio and Taylor (1996) have also proposed a three-dimensional finite element model. Savi *et al.* (1998) discuss an iterative numerical procedure that has been developed to deal with both geometrical and constitutive nonlinearities in the finite element model for adaptive trusses with SMA actuators. Lagoudas *et al.* (1997) consider the thermo-mechanical response of a laminate with SMA strips where the thermo-mechanical response is based on Boyd-Lagoudas' polynomial hardening model (Boyd and Lagoudas, 1996). Kouzak *et al.* (1998) also treats SMA beams using a constitutive equation proposed by Brinson (1993). Trochu & Qian (1997), Masud *et al.* (1997), Bhattacharyya *et al.* (2000), Liu *et al.* (2002) are other contributions in this field. Moreover, dual kriging interpolation has been employed with finite element method (FEM) in order to describe the shape memory behavior (Trochu & Qian, 1997; Trochu & Terriault, 1998; Trochu *et al.*, 1999).

Here, the FEM is employed promoting the spatial discretization of bars using a constitutive equation proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003) to describe the thermo-mechanical behavior of SMAs. This model is

based on Fremond's theory (Fremond, 1987, 1996) and includes four phases in the formulation: three variants of martensite and an austenitic phase. Furthermore, different material parameters for austenitic and martensitic phases are concerned. Thermal expansion and plastic strains are also included into the formulation and hardening effect is represented by a combination of kinematic and isotropic behaviors. A plastic-phase transformation coupling is incorporated into the model allowing a correct description of the thermo-mechanical behavior of SMAs. Moreover, horizontal enlargement of the stress-strain hysteresis loop is considered, allowing better adjustments with experimental data.

An iterative numerical procedure based on operator split technique (Ortiz *et al.*, 1983) is developed in order to deal with the nonlinearities in the formulation. Numerical simulations are carried out showing different behaviors of SMA bars. Results show that the proposed model is able to capture the general behavior of SMAs, including pseudoelastic and shape memory effects and also phase transformations due to thermal expansion.

## 2. Finite Element Formulation

Consider a composite bar reinforced with a SMA actuator subjected to a constant axial load, as show in Figure 1. The actuator is assumed to be significantly thinner than the height of the cross section bar and also, built in such a way to preserve symmetry of the axial load, avoiding flexure loads.

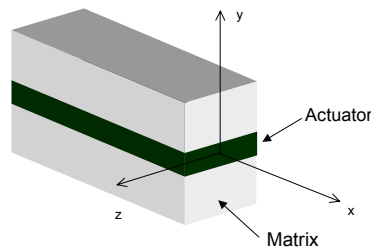


Figure 1 – Composite bar with SMA actuator.

The internal energy increment may be written as follows

$$\delta\Gamma = \int_{V_m} \sigma_m \delta\varepsilon dV + \int_{V_a} \sigma_a \delta\varepsilon dV \quad (1)$$

where  $V$  is the volume and the subscripts  $m$  and  $a$  are associated with matrix and actuator, respectively. An elastic relation is considered for the matrix,  $\sigma_m = E_m \varepsilon$ , where  $E_m$  is the matrix elastic modulus;  $\sigma_a$  is given by constitutive equation proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003). For simplicity, a compact form of the stress-strain relation is here presented,

$$\sigma_a = E_a \varepsilon + A_a \quad (2)$$

where  $A_a$  represents nonlinear terms related to phase transformation and plastic behavior,

$$A_a = -E_a \varepsilon^p + (\alpha + E_a \alpha_H) (\beta_2 - \beta_1) - \Omega (T - T_0) \quad (3)$$

$\varepsilon^p$  is the plastic strain,  $\beta_1$  and  $\beta_2$  are the volumetric fraction of martensitic variants,  $M^+$  and  $M^-$ .  $E_a$ ,  $\alpha$ ,  $\alpha_H$  and  $\Omega$  are material parameters. A detailed description of the constitutive model may be found in Savi *et al.* (2002) and Baêta-Neves *et al.* (2003), where the evolution equations of all internal variables related to the formulation are presented. Kinematics equation, similar to infinitesimal strains hypothesis, is adopted,

$$\varepsilon = u_{,x} \quad (4)$$

where  $( )_{,x} = d( )/dx$ . Now, it is possible to consider the principle of virtual work as follows, since the term  $A_a$  is assumed to be constant in the actuator and in the length of the bar.

$$(E_m A_m + E_a A_a) \int_L (u_{,x} \delta u_{,x}) dx + A_a \int_L (A_a \delta u_{,x}) dx - \int_L (p \delta u) dx = 0 \quad (5)$$

where  $A$  is the cross section area and  $p$  is the axial load per length. This model allows one to analyze bars constructed only by SMA considering several actuators.

Spatial discretization is considered by using the finite element method, which establishes the following approximation

$$u(x) = \sum_{j=1}^2 U_j^e \mathcal{G}_j(x) \quad (6)$$

where  $U_j^e$  are nodal displacements and  $\mathcal{G}_j(x)$  are Lagrange shape functions, presented bellow (Reddy, 1984):

$$\mathcal{G}_1 = 1 - \frac{x}{L} \quad \mathcal{G}_2 = \frac{x}{L} \quad (7)$$

From this approximation, Eq.(5) is rewritten as follows,

$$(E_m A_m + E_a A_a) \int_L [\mathbf{B}_u]^T [\mathbf{B}_u] dx \{U^e\} + A_a \int_L (A_a [\mathbf{B}_u]^T) dx \{U^e\} - \int_L (p [\mathbf{N}_u]^T) dx = 0 \quad (8)$$

Which follows a discrete version of the governing equation.

$$[K^e] \{U^e\} = \{F^e\} - \{\hat{F}^e\} \quad (9)$$

where  $[K^e]$  is the stiffness matrix,  $\{U^e\}$  is the displacement vector,  $\{F^e\}$  is the force vector and  $\{\hat{F}^e\}$  is related to the behavior of the nonlinear shape memory actuator. The definition of these matrixes is given by,

$$[K^e] = (E_m A_m + E_a A_a) \int_L [\mathbf{B}_u]^T [\mathbf{B}_u] dx \quad (10)$$

$$\{F^e\} = \int_L (p [\mathbf{N}_u]^T) dx \quad (11)$$

$$\{\hat{F}^e\} = A_a \int_L (A_a [\mathbf{B}_u]^T) dx \quad (12)$$

After the construction of the global system, an operator split technique (Ortiz *et al.*, 1983) associated with an iterative numerical procedure is applied in order to deal with the nonlinearities in the formulation. At first, the global vector  $\hat{F}_i$  is evaluated assuming that neither phase transformation nor plastic strain has taken place, which means that it has the same value of the previous time instant. Under this assumption, displacements  $U_i$  are calculated by solving a linear system. In the next step, all variables related to the SMA actuator (strain, stress, volumetric fractions of the phases, etc) are evaluated with the aid of constitutive and evolution equations. Afterwards, the matrix  $K_i$  and the vector  $\hat{F}_i$  is recalculated. This procedure is repeated to assure a prescribed convergence tolerance.

### 3. Numerical Simulations

This section considers numerical simulations performed with the proposed formulation. Material properties are the same discussed in Baêta-Neves *et al.* (2003). A bar of SMA material with a square cross section of 10mm side and 100mm length is analyzed.

At first, a homogeneous thermo-mechanical load process is considered, allowing a comparison between the FEM formulation with results obtained by Baêta-Neves *et al.* (2003) simulations. These comparisons are used as a validation of the proposed finite element model. Therefore, consider a SMA bar with four elements (Figure 2), subjected to an axial load  $F_x$ . Two different effects are treated: pseudoelastic and shape memory.

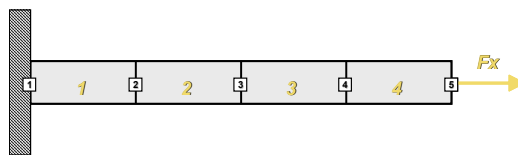


Figure 2 - SMA bar subjected to homogeneous axial load process.

Pseudoelastic effect is now in focus regarding a SMA specimen subjected to an isothermal mechanical loading performed at  $T = 313\text{K}$  ( $T > T_A$ ). Figure 3 shows this load process, the stress-strain curve and the evolution of volumetric fractions of phases. Notice that simulations of FEM and Baêta-Neves *et al.* (2003) are in agreement. All characteristics of the constitutive model are captured by the FEM analysis. During loading process, the specimen experiences phase transformations from austenitic phase  $A$ , to positive martensitic variant  $M^+$ . Afterwards, during unloading process, the reverse transformation is induced.

The shape memory effect is now focused regarding a thermo-mechanical loading depicted in Figure 4. At first, a constant temperature  $T = 263\text{K}$  ( $T < T_M$ ) is considered, where the martensitic phase is stable. After mechanical loading-unloading process, the specimen presents a residual strain that can be eliminated by a subsequent thermal loading (Figure 4). Notice that the stress-strain curve represents the shape memory effect. Again, FEM and Baêta-Neves *et al.* (2003) results are in agreement except for small variations on the evolution of volumetric fractions of phases. This small discrepancy is due to convergence criteria employed on both models.

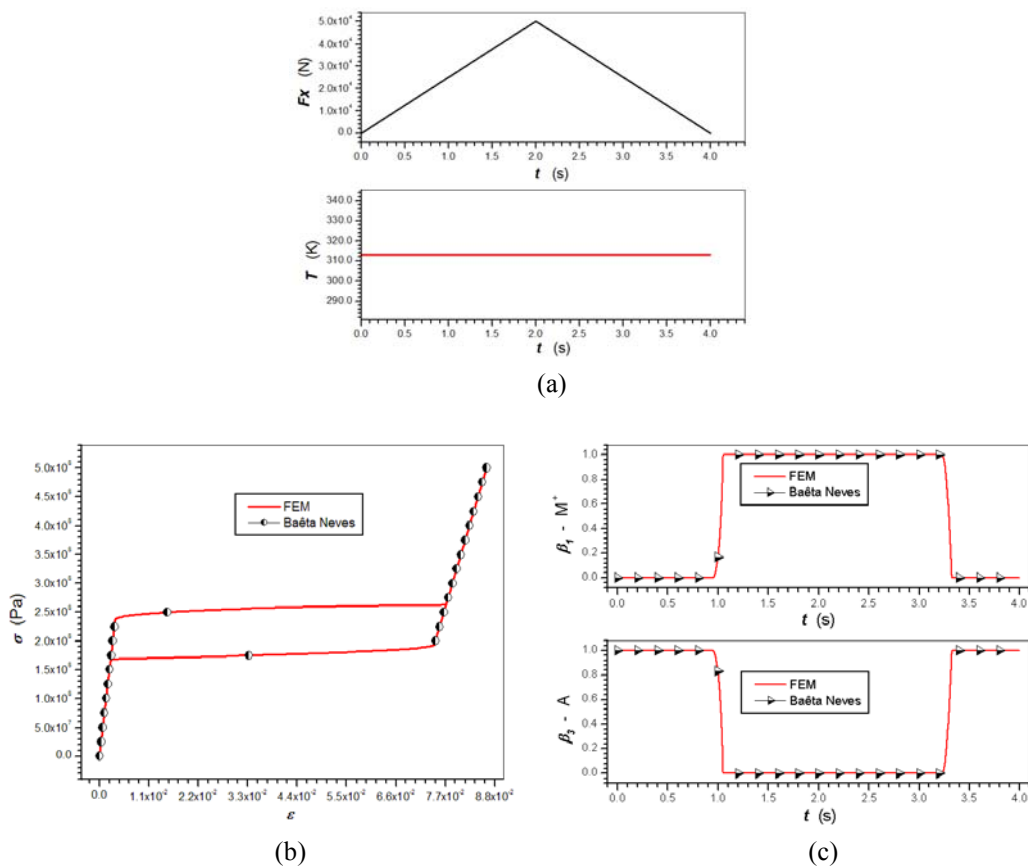


Figure 3 – Pseudoelasticity. (a) Thermo-mechanical loading. (b) Stress-strain curve. (c) Volumetric fraction of phases.

The thermal expansion effect is now considered regarding a thermal loading depicted in Figure 5, with the specimen free of stress. The response of the material under this loading process presents thermal expansion and its phase transformations. Notice the hysteretic characteristics of phase transformation in strain-temperature curve. Again, FEM and Baêta-Neves *et al.* (2003) results are in agreement.

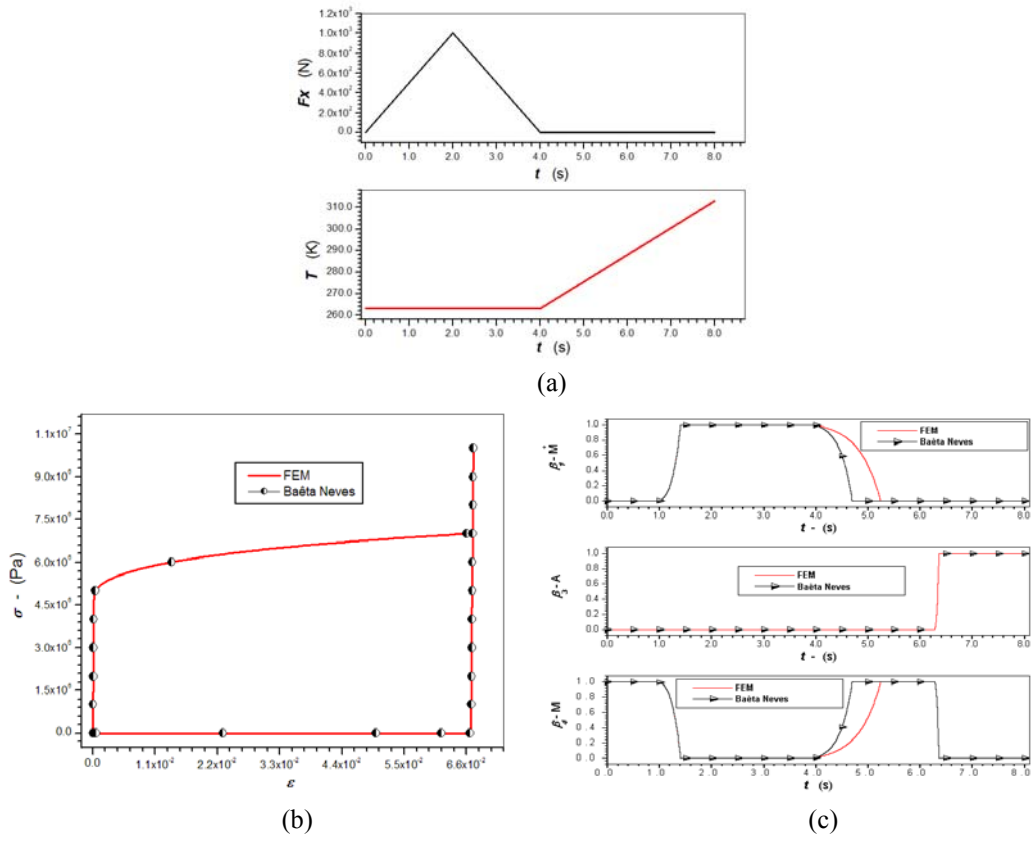


Figure 4 – Shape memory effect. (a) Thermo-mechanical loading. (b) Stress-strain curve. (c) Volumetric fraction of phases.

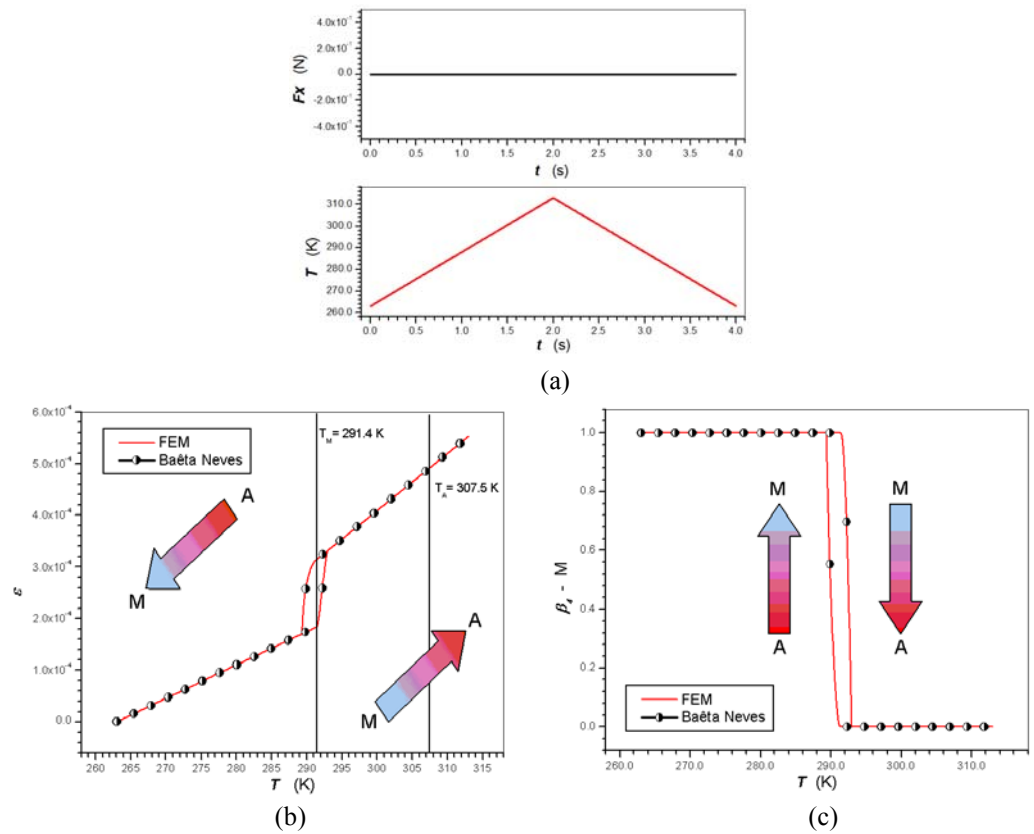


Figure 5 – Thermal expansion. (a) Thermo-mechanical loading. (b) Strain-temperature curve. (c) Volumetric fraction of phases.

At this point, a situation where an axial load is applied at the midpoint of the bar clamped at both ends is discussed (Figure 6). The evolution of this loading is similar to the one presented in Figure 3. Nevertheless, it is clear that its distribution through the bar is different. Figure 7 shows stress-strain curves and the evolution of volumetric fractions of each phase for different elements. In elements 1 and 2 there are phase transformations, causing pseudoelastic effect related to martensitic variant  $M^+$ . Different behavior is observed in elements 3 and 4, where variant  $M^-$  is induced. Figure 7, bottom, shows a schematic picture of phase distribution during loading-unloading process.

The thermo-mechanical loading process depicted in Figure 4 is now considered in order to analyze the shape memory effect in a bar subjected to an axial load applied at the midpoint clamped at both ends. Figure 8 shows stress-strain curves and the evolution of volumetric fractions of each phase for different elements. On the left side of the bar (elements 1 and 2), positive variant is induced ( $M^+$ ), while negative variant is induced at the right side (elements 3 and 4) ( $M^-$ ). Figure 8, bottom, shows schematic pictures of phase distribution during loading-unloading process.

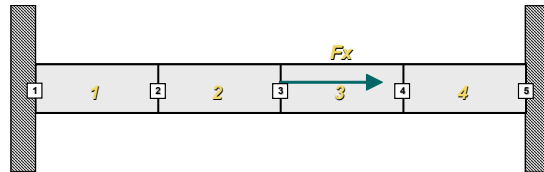


Figure 6 – Bar subjected to an axial load at the midpoint and restricted at both ends.

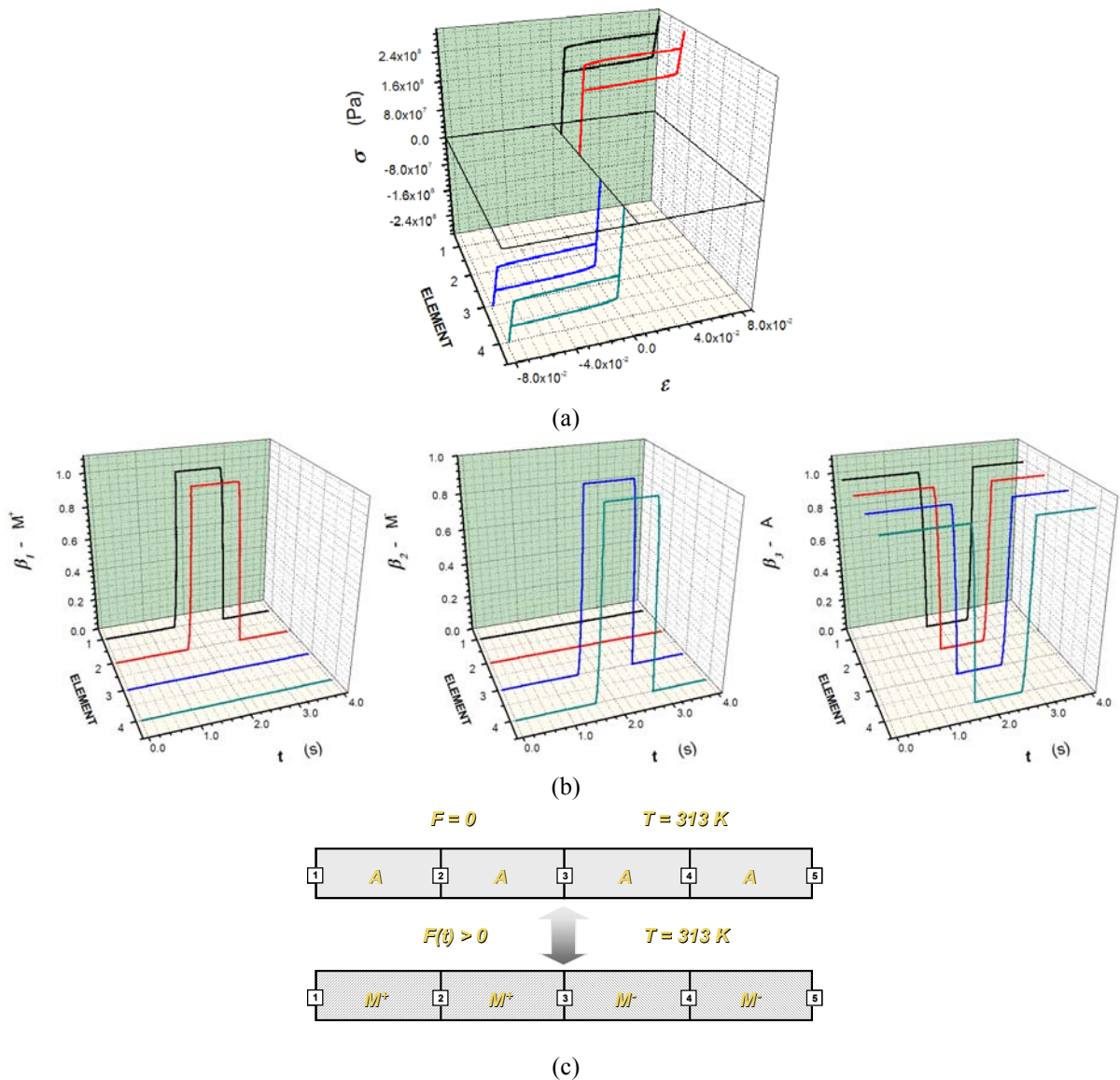


Figure 7 – Pseudoelastic effect for a bar subjected to an axial load at the midpoint and restricted at both ends. (a) Stress-strain curves. (b) Volumetric fraction of phases. (c) Schematic representation of phase distribution.

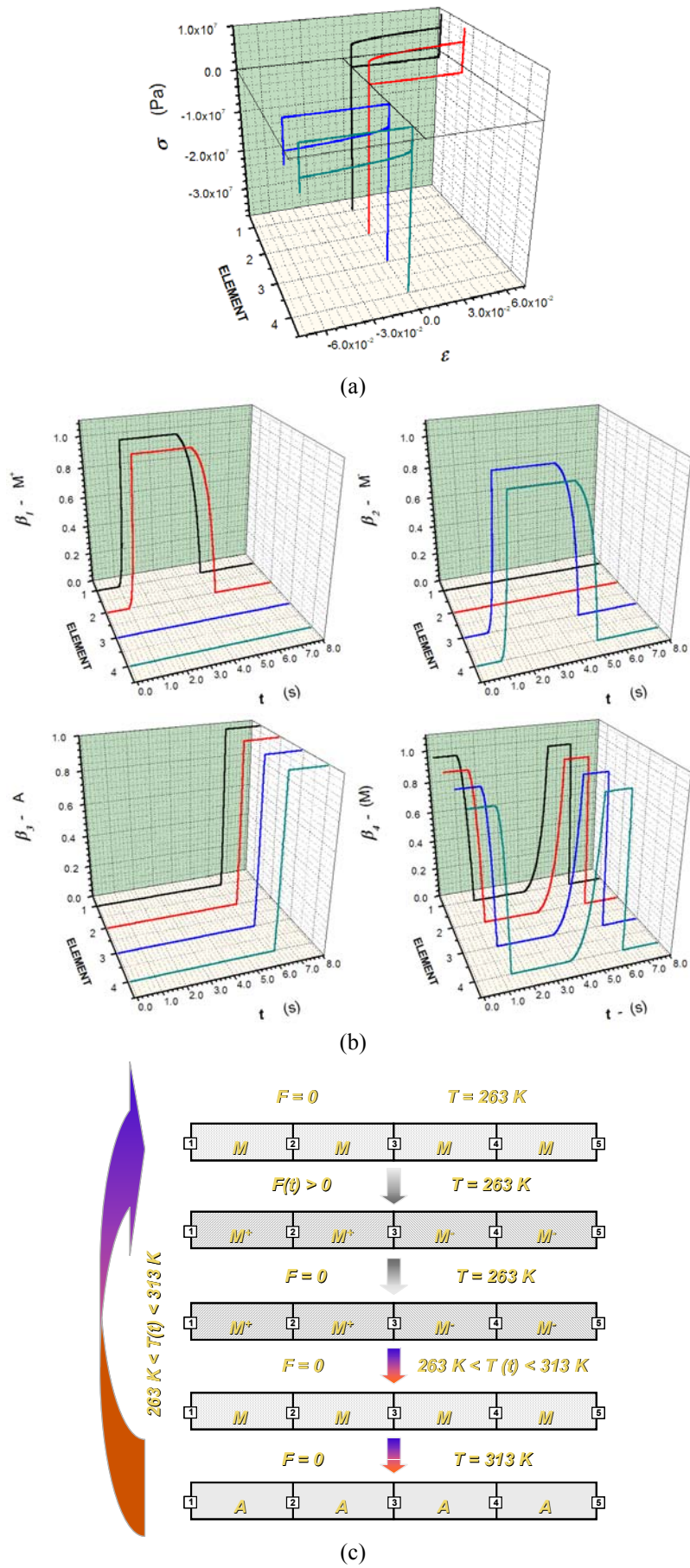


Figure 8 – Shape memory effect for a bar subjected to an axial load at the midpoint and restricted at both ends. (a) Stress-strain curves. (b) Volumetric fraction of phases. (c) Schematic representation of phase distribution.

A bar with a non-homogeneous temperature distribution is of concern. Discretization is done considering 20 elements (Figure 9). Figure 10 shows the thermo-mechanical loading process, stress-strain curve, volumetric fractions of phases and plastic strains time histories. The loading process begins with a thermal loading that promotes non-homogeneous temperature distribution through the length of the bar. This induces a situation where austenitic phase ( $A$ ) and twinned martensite ( $M$ ) are distributed through the bar. Afterwards, a mechanical load is applied. The loading process induces the formation of positive martensitic variant ( $M^+$ ). Since temperature distribution is non-homogeneous, different behavior is expected through the bar. Regions with low temperatures present lower values of critical stresses where phase transformations starts. Moreover, yield limit is also smaller and the load level causes plastification. On the other hand, on regions with higher temperatures, phase transformation starts for higher stress levels and plastification do not occur. The subsequent unloading process shows regions with pseudoelastic, partial pseudoelastic and shape memory effects, depending on this position. Moreover, some regions present plastic strains related to all thermo-mechanical process.

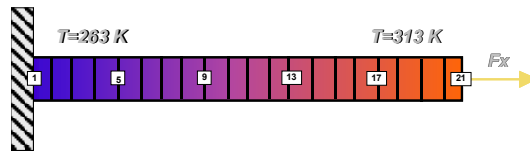


Figure 9 – Bar subjected to non-homogeneous temperature distribution.

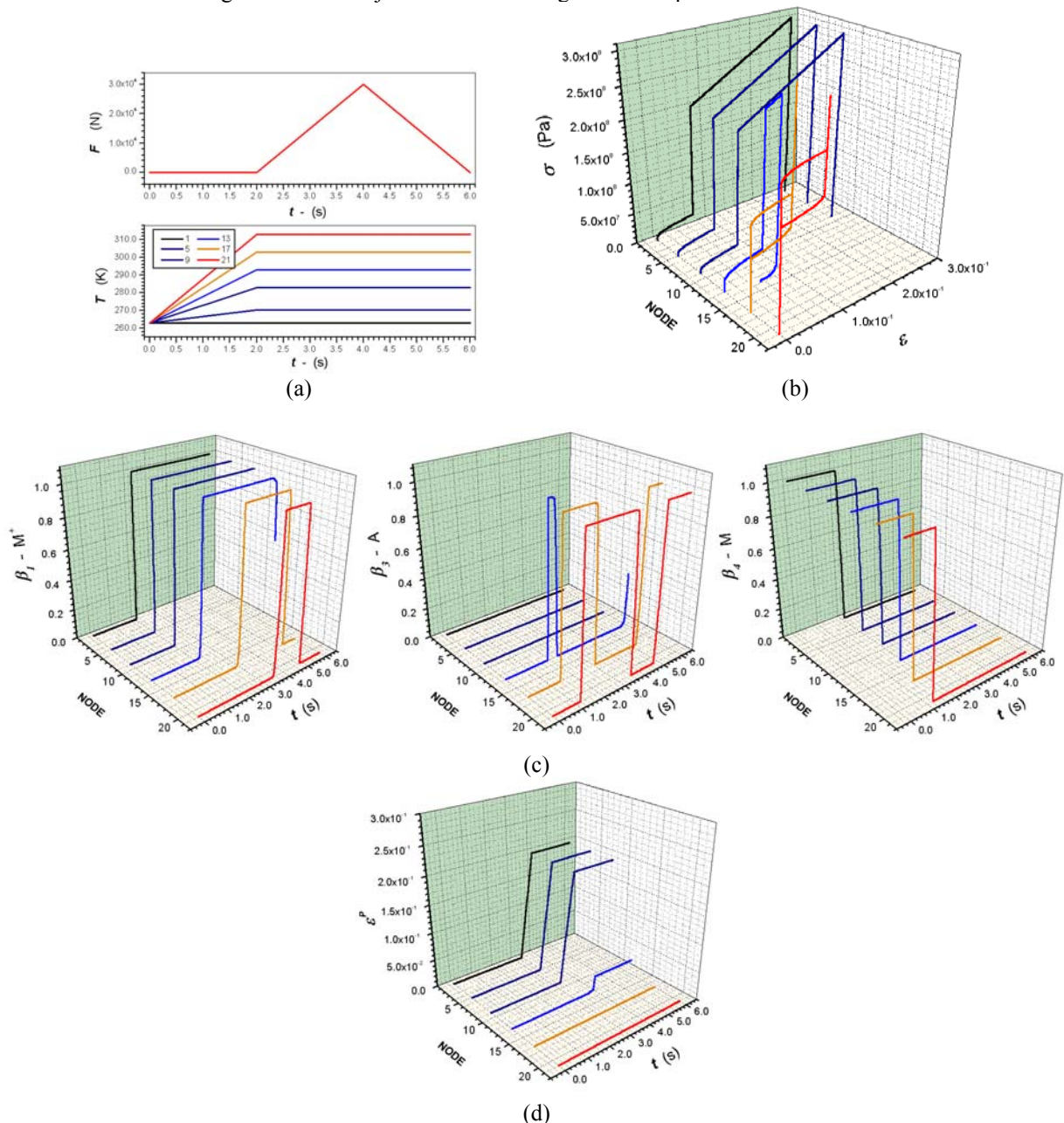


Figure 10 – Response of the bar under non-homogeneous temperature distribution. (a) Thermo-mechanical loading. (b) Stress-strain curves. (c) Volumetric fraction of phases. (d) Plastic strain.



## 4. Conclusions

This article presents a nonlinear finite element analysis of shape memory bars. A constitutive model proposed by Savi *et al.* (2002) and Baêta-Neves *et al.* (2003) is used to describe the thermo-mechanical behavior of SMAs. The model considers thermal expansion and plastic strains with hardening. An iterative numerical procedure based on operator split technique is developed in order to deal with nonlinearities of the formulation. Numerical simulations show that results from FEM capture the general behavior of the constitutive equation due to Baêta-Neves *et al.* (2003). Moreover, other simulations show how non-homogeneous loadings can produce interesting behaviors in shape memory bars. These results indicate that the response of SMA devices subjected to non-homogeneous loadings can be very complex being of special interest to be investigated.

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## 6. References

- Auricchio, F. & Taylor, R.L., 1996, "Shape Memory Alloy Superelastic Behavior: 3D Finite Element Simulations", *Proceedings of the 3<sup>rd</sup> International Conference on Intelligent Materials*, June 3-5, Lyon.
- Baêta-Neves, A.P., Savi, M.A. & Pacheco, P.M.C.L., 2003, "Horizontal Enlargement of the Stress-Strain Loop on a Thermo-Plastic-Phase Transformation Coupled Model for Shape Memory Alloys", *Proceedings of COBEM 2003 - 17<sup>th</sup> International Congress of Mechanical Engineering*, São Paulo, Brazil.
- Bhattacharyya, A., Faulkner, M.G. & Amalraj, J.J., 2000, "Finite Element Modeling of Cyclic Thermal Response of Shape Memory Alloy Wires with Variable Material Properties", *Computational Materials Science*, v.17, pp.93-104.
- Birman, V., 1997, "Review of Mechanics of Shape Memory Alloy Structures", *Applied Mechanics Review*, v.50, pp.629-645.
- Borden, T., 1991, "Shape Memory Alloys: Forming a Tight Fit", *Mechanical Engineering*, pp.66-72.
- Boyd, J.G. & Lagoudas, D.C., 1996, "A Thermodynamic Constitutive Model for the Shape Memory Materials, Part I: The Monolithic Shape Memory Alloys", *International Journal of Plasticity*, v.12, n.86, pp.805-842.
- Brinson, L.C., 1993, "One-dimensional Constitutive Behavior of Shape Memory Alloys: Thermomechanical Derivation with Non-constant Material Functions and Redefined Martensite Internal Variable", *Journal of Intelligent Material Systems and Structures*, v.4, pp.229-242.
- Brinson, L.C. & Lammering, R., 1993, "Finite Element Analysis of The Behavior of Shape Memory Alloys and Their Applications", *International Journal of Solids and Structures*, v.30, n.23, pp. 3261-3280.
- Denoyer, K.K., Scott Erwin, R. & Rory Ninneman, R., 2000, "Advanced Smart Structures Flight Experiments for Precision Spacecraft", *Acta Astronautica*, v.47, pp.389-397.
- Duerig, T., Pelton, A. & Stöckel, D., 1999, "An Overview of Nitinol Medical Applications", *Materials Science and Engineering A*, v.273-275, pp.149-160.
- Fremont, M., 1987, "Matériaux à Mémoire de Forme", *C R Acad Sc Paris*, Tome 34, s.II, No.7, pp.239-244.
- Fremont, M., 1996, "Shape Memory Alloy: A Thermomechanical Macroscopic Theory", CISM courses and lectures, Springer Verlag.
- Fujita, H. & Toshiyoshi, H., 1998, "Micro Actuators and Their Applications", *Microelectronics Journal*, v.29, pp.637-640.
- Garner, L.J., Wilson, L.N., Lagoudas, D.C. & Rediniotis, O.K., 2001, "Development of a Shape Memory Alloy Actuated Biomimetic Vehicle", *Smart Materials & Structures*, v.9, n.5, pp.673-683.
- Kibirkstis, E., Liaudinskas, R., Pauliukaitis, D. & Vaitasius, K., 1997, "Mechanisms with Shape Memory Alloy", *Journal de Physique IV*, C5, pp.633-636.
- Kouzak, Z., Levy Neto, F. & Savi, M.A., 1998, "Finite Element Model for Composite Beams using SMA Fibers", *Proceedings of CEM NNE 98 - V Congresso de Engenharia Mecânica Norte e Nordeste*, Fortaleza – Brazil, 27-30 October, v.II, pp.112/119.
- La Cava, C.A.P.L., Silva, E.P., Machado, L.G., Pacheco, P.M.C.L. & Savi, M.A., 2000, "Modeling of a Shape Memory Preload Device for Bolted Joints", *Proceedings of CONEM 2000 – Congresso Nacional de Engenharia Mecânica*, Natal-RN, Brazil (in portuguese).
- Lagoudas, D.C., Rediniotis, O.K. & Khan, M.M., 1999, "Applications of Shape Memory Alloys to Bioengineering and Biomedical Technology", *Proceedings of 4th International Workshop on Mathematical Methods in Scattering Theory and Biomedical Technology*, October 1999, Perdika, Greece.
- Lagoudas, D.C., Moorthy, D., Qidwai, M.A. & Reddy, J.N., 1997, "Modeling of the Thermomechanical Response of Active Laminates with SMA Strips Using the Layerwise Finite Element Method", *Journal of Intelligent Material Systems and Structures*, v.8, pp.476-488.
- Liu, K.M., Kitipornchai, Ng, T.Y. & Zou, G.P., 2002, "Multi-dimensional Superelastic Behavior of Shape Memory Alloys via Nonlinear Finite Element Method", *Engineering Structures*, v.24, pp.51-57.
- Machado, L.G. & Savi, M.A. (2003), "Medical Applications of Shape Memory Alloys", *Brazilian Journal of Medical and Biological Research*, v.36, n.6, pp.683-691.

- Masud, A., Panahandeh, M. & Aurichio, F., 1997, "A Finite-Strain Finite Element Model for the Pseudoelastic Behavior of Shape Memory Alloys", *Computer Methods in Applied Mechanics and Engineering*, v.148, pp.23-37.
- Ortiz, M., Pinsky, P.M. & Taylor, R.L., 1983, "Operator Split Methods for the Numerical Solution of the Elastoplastic Dynamic Problem", *Computer Methods in Applied Mechanics and Engineering*, v.39, pp.137-157.
- Pacheco, P.M.C.L. & Savi, M.A., 1997, "A Non-Explosive Release Device for Aerospace Applications using Shape Memory Alloys", Proceedings of *COBEM-97, XIV Congresso Brasileiro de Engenharia Mecânica*, Bauru, São Paulo, November.
- Pietrzakowski, M., 2000, "Natural Frequency Modification of Thermally Activated Composite Plates", *Mec. Ind.*, v.1, pp.313-320.
- Reddy, J.N., 1984, *An Introduction to the Finite Element Method*, McGraw-Hill.
- Rogers, C.A., 1995, "Intelligent Materials", *Scientific American*, September, pp.122-127.
- Savi, M. A., Paiva, A., Baêta-Neves, A. P. & Pacheco, P. M. C. L., 2002, "Phenomenological Modeling and Numerical Simulation of Shape Memory Alloys: A Thermo-Plastic-Phase Transformation Coupled Model", *Journal of Intelligent Material Systems and Structures*, v.3, n.5, pp.261-273.
- Savi, M.A., Braga, A.M.B., Alves, J.A.P. & Almeida, C.A., 1998, "Finite Element Model for Trusses with Shape Memory Alloy Actuators", *EUROMECH 373 Colloquium - Modeling and Control of Adaptive Mechanical Structures*, Magdeburg, 11-13 March.
- Tanaka, K., 1986, "A Thermomechanical Sketch of Shape Memory Effect: One-dimensional Tensile Behavior", *Res. Mech.*, v.18, pp.251-263.
- Trochu, F. & Qian, Y-Y., 1997, "Nonlinear Finite Element Simulation of Superelastic Shape Memory Alloy Parts", *Computers and Structures*, v.62, n.5, pp.799-810.
- Trochu, F. & Terriault, P., 1998, "Nonlinear Modelling of Hystereitic Material Laws by Dual Kriging and Application", *Computer Methods in Applied Mechanics and Engineering*, v. 151, pp.545-558.
- Trochu, F., SacéPé, N., Volkov, O. & Turenne, S., 1999, "Characterization of NiTi Shape Memory Alloys Using Dual Kriging Interpolation", *Materials Science & Engineering*, A273-275, pp.395-399.
- van Humbeeck, J., 1999, "Non-medical Applications of Shape Memory Alloys", *Materials Science and Engineering A*, v.273-275, pp.134-148.
- Webb, G., Wilson, L., Lagoudas, D.C. & Rediniotis, O., 2000, "Adaptive Control of Shape Memory Alloy Actuators for Underwater Biomimetic Applications", *AIAA Journal*, v.38, n.2, pp.325-334.