Vortex-Induced Vibrations Experiments with an Elastically Mounted Cantilevered Cylinder in Water: New results on sub-harmonic resonance and stream-wise added mass

André L. C. Fujarra
University of São Paulo (USP) – Department of Naval Architecture and Ocean Engineering
Av. Prof. Mello Moraes, 2231. , São Paulo, 05508-900, Brazil
afujarra@usp.br

Rodrigo A. Oliveira
University of São Paulo (USP) – Department of Naval Architecture and Ocean Engineering
rodrigo.oliveira@poli.usp.br

Celso P. Pesce
University of São Paulo (USP) – Department of Mechanical Engineering
cppesce@usp.br

Abstract: *In a set of former papers, results on two-degree of freedom Vortex Induced Vibrations (VIV) experiments in water have been reported. A relatively low mass-damping, elastically mounted rigid cylinder has been studied at the Institute for Technological Research of the State of São Paulo (IPT) Towing Tank. Specifically, a cantilevered rigid cylinder was mounted on an elastic (leaf spring) two-degree-of-freedom device. The device acts also as a special mechanical transducer to measure accelerations/forces/displacements in the stream-wise (x) and the cross-wise (y) directions. The present paper aims to bring some additional results, addressing sub-harmonic resonance, coupling in-line and cross-flow vibrations and to present some new experimental results on added mass in the stream-wise direction.*

Keywords*: vortex-induced vibrations; elastically mounted cylinder; two-degrees of freedom; added-mass; towing tank experiments.*

1. Introduction

The added mass concept is, decisively, a very important issue to understand the fundamentals of the fluid-elasticity of cylinders immersed in fluid flow. A lively discussion on this matter took place at the IUTAM Symposium on Bluff Body Wakes and Vortex-Induced Vibrations, BBVIV2, Marseilles, France, as pointed out by Leweke, Bearman and Williamson (2001) in the Preface of the special issue *J of Fluids and Structures*. Vol. 15, No. 3/4, April 2001. Particularly, the interpretation of the occurrence of negative added mass, in the VIV context, has deserved much attention. As a matter of fact, many prominent scientists, as testimonies the pioneer work by Sarpkaya (1963), addressed this subject. In fact Sarpkaya's works discuss this issue and much more; see for instance, Sarpkaya (1978), (1979) and (2001). Regarding the negative added mass, Sarpkaya (2003) points out, in a private communication: "Incidentally, it is because of this reason that I have objected to the use of 1.0 for the added mass coefficient". However, at least to the authors' knowledge, few experimental or numerical results are available in the literature, particularly concerning the added mass in the stream-wise direction.

From a broader perspective and in order to understand the nature of many VIV response behaviors, a (still on going) research project, leaded by University of São Paulo (USP), has been jointly carried out at the Institute for Technological Research of the State of São Paulo (IPT) towing tank. The experiments comprised a cantilevered rigid cylinder elastically mounted in a two-degree of freedom device. Based on the simultaneous measurements of acceleration, displacements and forces, a series of experiments with this device has brought some new results, as those presented in Fujarra et al (2001), Fujarra and Pesce (2002a,b) and Pesce and Fujarra (2002). According to them, the cross-wise added mass variation associated to lift force fluctuations, both functions of the flow velocity, are closely related to the dynamic behaviour at the lock-in range, specially the lower branch VIV response, just after the resonance peak.

Results presented by the authors, in Fujarra and Pesce (2002b), obtained with the same experimental set-up, allowed a better comprehension concerning VIV fundamentals, by re-addressing the phenomenological Van der Pol model proposed in Iwan and Blevins (1974). In that case, cross-wise added mass and lift force, properly considered as functions of the flow velocity, led to a much better emulation of the global synchronisation pattern. Specifically, the modified phenomenological model was able to reproduce the lower branch VIV response behaviour. This branch is the flat response region just after the VIV amplitude curve peak, characteristic of fluid-elastic systems under low values of the mass-damping parameter, $(m^*+C_a)\zeta$; see Khalak and Williamson (1999). Although more extensive work is surely needed to assure this sort of result can be extended towards a certain degree of generality the role played by added mass variation with the velocity has become more and more evident.

On the other hand, Computational Fluid Dynamics (CFD) techniques have systematically succeeded only to reproduce the lower branch VIV response. Therefore, by investigating the role added mass variation plays in the VIV amplitude response, it is expected to have some light shed onto the predictability of numerical codes.

It is also expected that taking into account, simultaneously, the stream-wise and the cross-wise dynamics, as proposed by Kim and Perkins (2002), this time by considering the added mass variation, could lead to a better theoretical comprehension of the VIV phenomenon. Kim and Perkins worked out the VIV problem based on multiple-scale analysis, proposing an analytical model with four non-linear coupled oscillators (two-degree of freedom). Such a model has captured hysteresis, unstable branches and eight-shaped oscillations.

In this context, hoping to contribute to clarify some problems encountered either in analytical or numerical predictions, the present work intends to continue the added mass studies, specially concerning the stream-wise direction degree-of-freedom. A further aim would be to propose a general and comprehensive theoretical modeling based on the variable parameters.

2. Previous Results

As mentioned, added mass as function of the reduced velocity has been deeply studied in the last few years. Important results have been published, as those contained in the numerical work by Willden and Graham (2001) or in the experimental work by Vikestad, Vandiver and Larsen (2000). These latter authors investigated a long and rigid cylinder, horizontally assembled to a lightly damped support and free to oscillate only in the cross-wise direction; see Fig. (1)a. In accordance to the numerical studies by Wilden and Graham, by applying a time-domain analysis Vikestad et al observed a monotonically decreasing behavior for the added mass as the reduced velocity increases, with a zero-crossing value close to reduced velocity $V_r \cong 8$. Additionally, their results seem to indicate $C_a \cong -1$ as the asymptotic value at large reduced velocities. It should be highlighted that, as in the present work, reduced velocity has been defined with respect to the first cross-wise natural frequency at null velocity, i.e., $V_r = U/(f_{1y}\, D)$.

(a) (b)

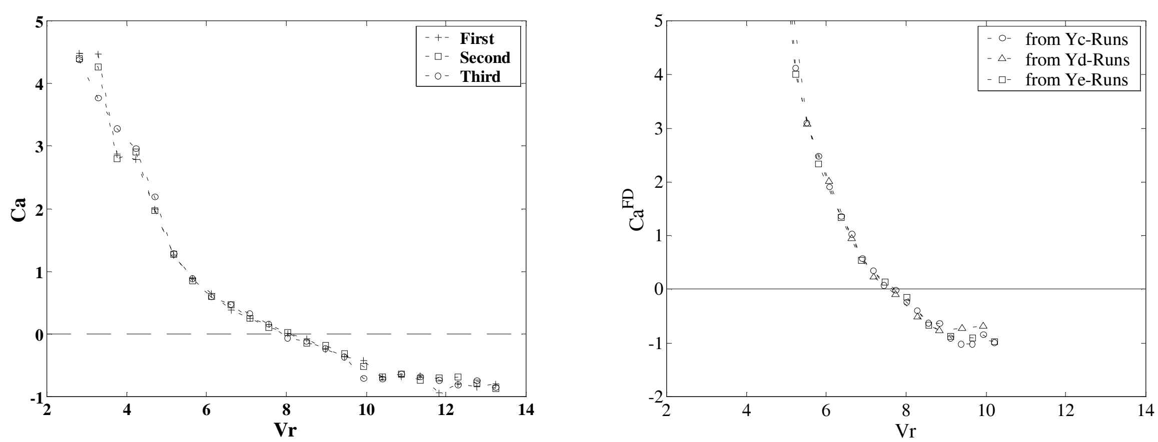

Figure 1. Added mass coefficient versus reduced velocity, concerning the first eigen-frequency in the cross-wise direction. (a) From time domain analysis: Vikestad, Vandiver and Larsen (2000) (b) From frequency domain: Fujarra and Pesce (2002a). In both cases, oscillation in the stream-wise direction is constrained.

Almost concomitantly, a set of experiments was carried out at IPT towing tank, with a cantilevered rigid cylinder elastically mounted in a two-degree of freedom device, with no end plate. Alternatively to the time-domain approach, a frequency-domain analysis was applied to evaluate cross-wise added-mass variation with respect to reduced velocity, Fujarra and Pesce (2002a). By considering the oscillatory system described by the dynamic equilibrium equation:

$$m\ddot{y}+c\dot{y}+F_S(t)=F_H(t). \tag{1}$$

The total transverse hydrodynamic forces can be indirectly evaluated through:

$$F_H(t)=\Im^{-1}[m\Im[\ddot{y}]-\Im[F_S]], \text{ where:} \tag{2}$$

$\Im[\ ]$ stands for the Fast-Fourier Transform and $\Im^{-1}[\ ]$ for the inverse operation, S for structural and H for hydrodynamic.

Neglecting damping terms, since they are considered to be small enough, the following approximate relationship is obtained:

$$\frac{\Im[F_H]}{\Im[\ddot{y}]}\approx -m_a(\omega)+\frac{ic_v(\omega)}{\omega}, \tag{3}$$

leading to an experimental method to determine the added mass coefficient:

$$C_a^{FD} = C_a^{FD}(\omega) = \frac{-4\, real\left\{\dfrac{\Im[F_H]}{\Im[\ddot{y}]}\right\}}{\rho \pi D^2}. \quad (4)$$

According to this procedure, added mass coefficients are calculated for each reduced velocity as a function of the dominant response frequency, obtained from all acquired time histories.

Despite the differences in experimental setups, which are probably responsible for some discrepancies in added-mass results, an interesting similarity does exist. As can be observed in Fig. (1)b, the same monotonically decreasing trend is confirmed, as well as the same zero-cross point, showing the consistency of the frequency-domain analysis. Notice that results at low reduced velocity were not reported, a that time, so that no discussion concerning the unitary value attained by the added mass coefficient at $V_r = 0$ could be done.

By using the same procedure, the main goal of the present work is to go further into a more comprehensive analysis relating to the added mass variation, now taking also into account the stream-wise direction. Moreover, with an additional frequency-domain analysis for the two-degree of freedom experiment, it intends to clarify the subject regarding the added mass at zero-velocity. Before going ahead, the experimental set-up is presented as well as some results of response amplitudes and frequencies.

3. The Experimental Setup

Free vortex-induced vibration tests were conducted at Institute for Technological Research of the State of São Paulo (IPT) towing tank, a facility 220m long, 6.6m wide and 4.5m deep.

A two-degree of freedom device was built, based on three horizontal rigid plates, assembled by two pairs of leaf springs. Therefore, as shown in Fig. (2), the device is composed by two cells, the upper one corresponding to the stream-wise (x) vibrations (associated to drag loads) and the lower one related to the cross-wise (y) vibrations (lift loads). The device is not only an elastic support, also working as a transducer to measure forces/displacements, by means of strain-gages installed in each pair of leaf springs. A 2D accelerometer is also mounted on the lower horizontal rigid plate.

Sets of experiments were carried out, corresponding to a velocity range that fully excites the first cross-wise natural mode. For the sake of redundancy and safety, all measurements were repeated at least twice.

It seems important to highlight that, as built, the device can have constrained any of the two degree of freedom.

A rigid cylinder with external diameter of 38.10mm and an immersed span length of 1143mm was clamped to the lower plate and the arrangement of Fig. (2) was fixed to the towing tank carriage structure. A complete experimental characterization of this device is presented in Fujarra et al (2001) and Pesce and Fujarra (2002).

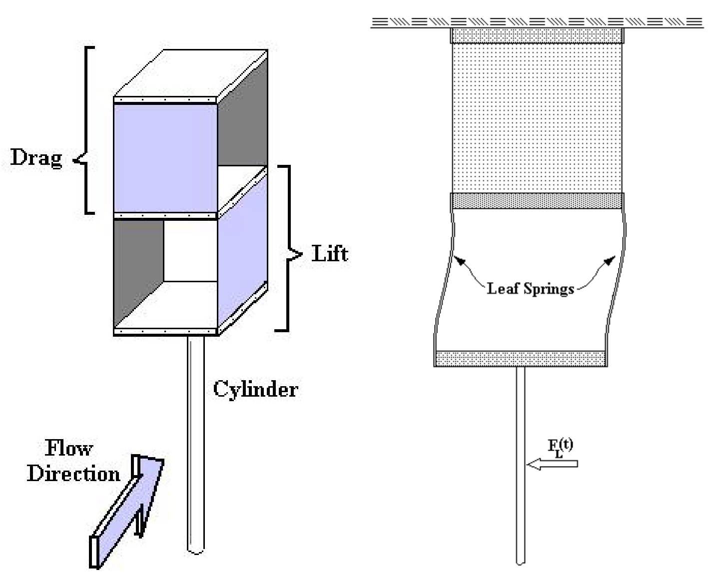

Figure 2. Sketch of the elastic support: a device with two degrees-of-freedom (x and y) and low structural damping.

In a former investigation with the cylinder free to oscillate in both directions, response amplitudes and frequencies were obtained by means of standard frequency domain analysis applied to acceleration signals. In Fig. (3)a, a typical slightly damped VIV response can be observed in the cross-wise direction (■ and ●). The simultaneous excitation of the stream-wise oscillations (□ and ○) in the range $6 \leq V_r \leq 8$ denotes a better synchronization. In other words, this range is a broad region with higher amplitude responses. This can also be inferred from the respective non-dimensional frequency responses, seen in Fig. (3)b. The non-dimensional frequencies plotted are the dominant ones, obtained from spectral analysis, normalized with respect to the first cross-wise natural frequency at null velocity.

In terms of the dominant frequency, a better correlation near the lock-in peak is related to a coupling with the first order sub-harmonic ($f_{dx} \cong 2.\ f_{dy}$). On the other hand, outside the peak region, a coupling with the second order sub-harmonic ($f_{dx} \cong 4.\ f_{dy}$) is observed. As well known, even in the range $6 \leq V_r \leq 8$ several non dominant frequencies of oscillation coexist, particularly those which are four times the first cross-wise natural frequency; see Fig. (4). It will be seen next that such a sub-harmonic resonance behavior is closely related to the added mass variation.

(a) (b)

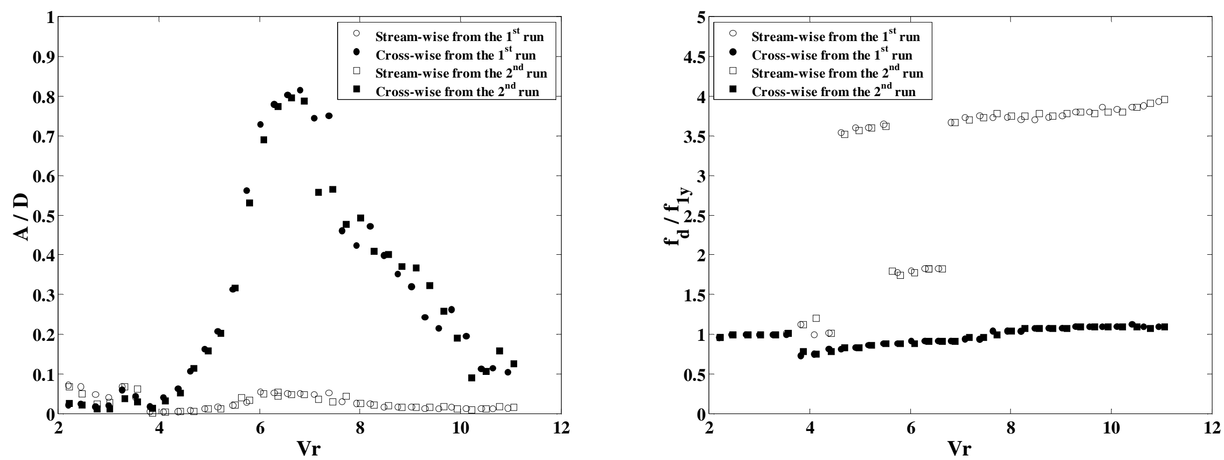

Figure 3. Response of a rigid cylinder elastically supported with two-degrees of freedom (x and y) and low structural damping, Pesce and Fujarra (2002). (a) Non-dimensional amplitudes in the cross-wise direction (■ and ●) and in the stream-wise direction (□ and ○). (b) Respective non-dimensional dominant frequency responses.

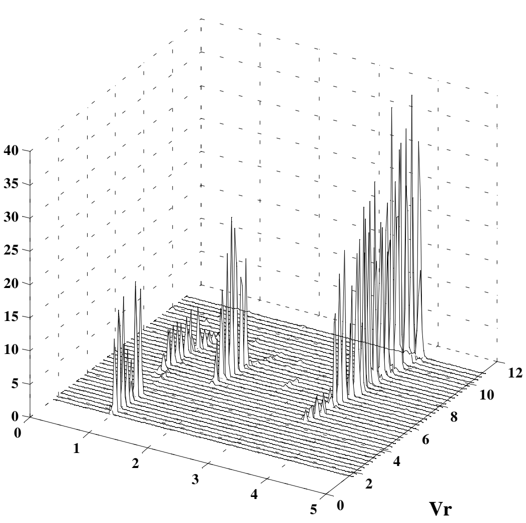

Figure 4. Waterfall of x-acceleration power spectra (stream-wise) shows the coexistence of different sub-harmonic orders at the lock-in peak.

4. Results on Cross-wise and Stream-wise Added Mass

As a first investigation, applying the same frequency-domain analysis to the experiments with the rigid cylinder free to oscillate in both directions, one can achieve the results shown in Fig. (5). It is possible to realize a decreasing on the added mass variation, similar to that presented by the cylinder oscillating only in the cross-wise direction (y).

Moreover, the asymptotic value $C_a \cong -1$ for higher reduced velocities and, approximately the same zero-crossing point, are also observed.

However, in the case of simultaneous oscillations (x and y) an inflection behavior is verified in the cross-wise added mass curve, near the lock-in peak (upper branch seen in Fig. (3)a, $6 \leq V_r \leq 8$), exactly where the stream-wise oscillations are more pronouncedly coupled. Although the stream-wise oscillations present low amplitudes, they are responsible to change the added mass decreasing, possibly due to a higher span-wise correlation in vortex shedding.

Although analysis has been carried out for $V_r > 2$, one can infer, from Fig. (5), another important point: the asymptotic value attained as reduced velocity goes to zero is $C_a(V_r \to 0) \cong 1$. This result indicates the analysis reliability as it is in concordance with theory. As the lock-in region is entered, there is an abrupt change in the added mass coefficient value, associated to a rapid phase shift between inertial and driven forces. To the authors' knowledge, none experiment on the VIV phenomenon has reported such results, even with the cylinder free to oscillate only in the cross-wise direction. Also to be emphasized is the asymptotic value of -1, at high reduced velocities, earlier observed by Vikestad et al (2000) and by Willden and Graham (2001).

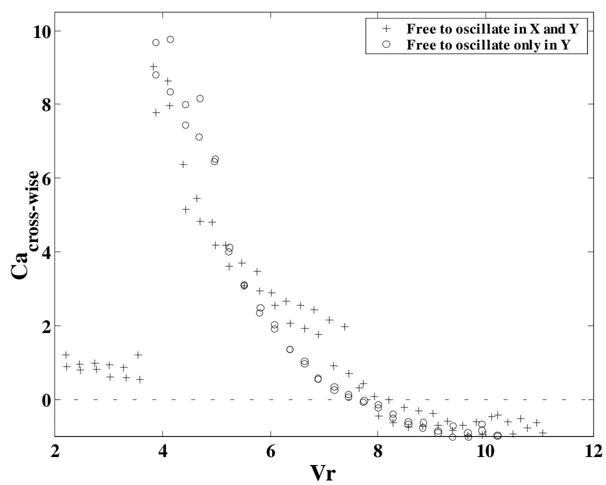

Figure 5. Cross-wise added mass versus reduced velocity. (+) Rigid cylinder is free to oscillate in both directions (x and y). (o) Rigid cylinder is free to oscillate only in cross-wise direction (y).

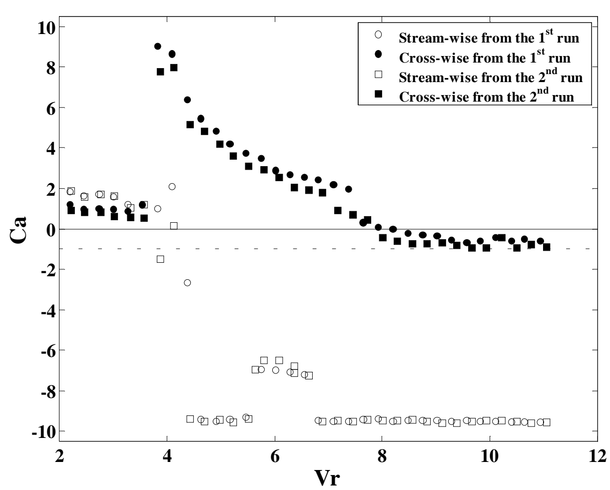

Figure 6. Stream-wise and cross-wise added mass versus reduced velocity concerning the first eigen frequency in the cross-wise direction. Rigid cylinder is free to oscillate in both directions (x and y).

Based on the same set of runs, Fig. (6) compares the added mass coefficients in both directions (x and y). Therefore, (■ , ●) are related to the cross-wise oscillations and (□ , ○) to the stream-wise ones. It is noticeable that the stream-wise added mass coefficient starts from a value equal to 2, for $V_r \cong 2$ and, as $V_r \cong 4$ (beginning of the stream/cross-wise synchronization), rapidly decreases to $C_a \cong -9.5$. Around $V_r \cong 5.5$ (beginning of the lock in peak region), the added mass coefficient jumps to $C_a \cong -7$ and, at $V_r \cong 7.0$ (where the lock in peak starts to decrease), it returns to the lower level. Note that $C_a \cong 2$ is not the asymptotic value at null velocity for the stream-wise added mass coefficient. In fact, observing Fig. (3)a, at $V_r \cong 2$, the amplitude of the motion in the stream-wise direction is of the same order of magnitude attained in the lock-in peak region. Contrarily, the cross-wise amplitude is very small, almost negligible at $V_r \cong 2$.

At the upper branch of amplitude response, a different pattern of wake is expected to appear, as pointed out in Govardham and Williamson (2000). The jump in the stream-wise added mass, accompanied by the inflection observed in the cross-wise added mass curve, is therefore another experimental evidence of the existence of distinct wake patterns at the upper-branch (peak region) and at the lower-branch (almost flat region); Fig. (3)a. For reduced velocity values greater than $V_r \cong 7.0$ the added mass in the stream wise direction is invariant, returning to the previous level $C_a \cong -9.$, until the end of the VIV response.

Moreover, observing Fig. (6) and (3)b it may be inferred that the changing on the added mass behaviour inside the upper branch $(5.5 < V_r < 7.5)$ is closely related to the change observed in sub-harmonic resonance coupling. In fact, sub-harmonic resonance coupling jumps from the second to the first order and back to second order, as reduced velocity passes the upper branch. It should be remembered that drag pulsates twice as fast as the lift force does, so that a double frequency is associated to the first order sub-harmonic resonance coupling. For the second-order sub-harmonic the stream-wise oscillation frequency is four times the cross-wise one.

Further evidences on the correlation between the jump on the stream-wise added mass and the sub-harmonic resonance coupling mode can be seen from the analysis of a set of experiments were cross-wise oscillations were constrained. Figure 7 shows the stream-wise added mass for this set of "y-constrained" runs. The added-mass coefficient value at $V_r \cong 2$ is also 2 and an abrupt variation still exists at $V_r \cong 4$, the point where the lock-in usually starts. Nevertheless, there is no more jumps at $5.5 < V_r < 7.5$, as the stream-wise oscillation is now restrained. The stream-wise added mass value is invariant ($C_a \cong -10$), a bit lower than in the two-degree of freedom experiment.

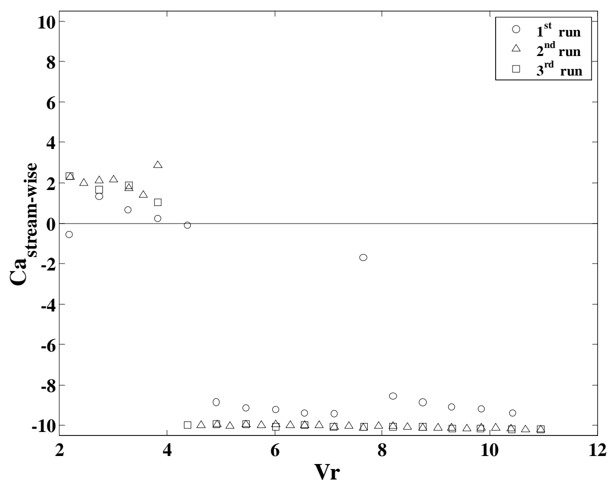

Figure 7. Stream-wise added mass versus reduced velocity. Rigid cylinder is free to oscillate only in stream-wise direction (x).

5. Conclusions

This work presented some new experimental results on added-mass variation with respect to reduced velocity. Specifically, the stream-wise added mass variability was addressed and showed to be in close correlation with the sub-harmonic resonance coupling, considering stream and cross-wise oscillations.

Besides, the cross-wise added mass variation was re-addressed, and some new features were obtained. One of crucial importance, for consistency sake, is the recovering of the asymptotic value of 1 at low velocity. Another one is the clear correlation between synchronization and the shift in relative phase between inertial and driven forces, as both stream and cross-wise added mass values jump at the beginning of the lock-in region. A third point to be emphasized is the asymptotic value of -1 for the cross-wise added mass coefficient, at high reduced velocities, earlier observed by Vikestad et al (2000) and by Willden and Graham (2001).

As a summary, this work, as a partial report of an on-going research project, was aimed to contribute to clarify some problems encountered either in analytical or numerical predictions. A further aim would be to propose a general and comprehensive theoretical model based on variable hydrodynamic parameters. Well succeed results, concerning cross-wise oscillations, were obtained by Fujarra (2002) (see also Fujarra and Pesce (2003)), modifying older phenomenological models to emulate the lower branch amplitude response, by including added mass variability with reduced velocity. From those and the results here presented, it is now expected that taking into account, simultaneously, the stream-wise and the cross-wise dynamics, as proposed by Kim and Perkins (2002), this time by considering the added mass variation, phenomenological models can be improved much more. Finally, as already observed, by investigating the role added mass variation plays in the VIV amplitude response, it is also expected that some light may be shed onto the predictability of numerical codes.

Acknowledgements

We are indebted to FAPESP, the State of São Paulo Research Foundation, for the financial support, through graduate/undergraduate scholarships (processes no. 02/04534-5 and no. 02/08886-3, first and second author respectively) within a Research Project Grant on VIV (process no. 01/00054-6). Third author acknowledges a CNPq Research Grant, no. 304062-85.

7. References

Fujarra. A.L.C., 2002, "Experimental and Analytical Stuides on Vortex Induced Vibrationsof Flexible and Rigid Cylinders" (in portuguese). Doctoral Thesis, Escola Politécnica, University of S. Paulo, 234 p.

Fujarra, A.L.C., Menezes, M.R. and Pesce, CP, 2001 "Investigation on Vortex-Induced Vibration in Water: Further Analysis and a new Measurement Device for 2D Experiments", 16th Brazilian Congress of Mechanical Engineering, 26-30 November, paper TRB1384.

Fujarra, A.L.C. and Pesce, C.P., 2002, "Experiments on VIV Added Mass with Elastically Mounted Cylinder in Water", 21st International Offshore Mechanics and Arctic Engineering Conference, OMAE, Norway, 23-28 June.

Fujarra, A.L.C. and Pesce, C.P., 2002, "Added Mass variation and Van der Pol Models Applied to Vortex-induced Vibrations", ASME International Congress on Mechanical Engineering, Proceedings of 5th International Symposium on Fluid-Structure Interactions, Aero-elasticity, Flow-Induced Vibrations and Noise, Vol. 1, New Orleans, 17-22 November.

Fujarra, A.L.C. and Pesce, C.P., "Added Mass Variation and Van der Pol Models Applied to Vortex-Induced Vibrations". Journal of Fluids and Structures (revised, expanded and submitted in 2003).

Govardhan, R. and Williamson, C. H. K., 2000, "Modes of vortex formation and frequency response of a freely vibrating cylinder", Journal of Fluid Mechanics, Vol. 420, p. 85-130.

Iwan, W. D. and Blevins, R. D., 1974, "A Model for Vortex Induced Oscillation of Structures". ASME - Journal of Applied Mechanics, Vol. 41, p. 581-586, September, 1974.

Khalak, A. and WIlliamson, C.H.K., 1997, "Investigation of Relative Effects Mass and Damping in Vortex-induced Vibrations of a Circular Cylinder", Jounal of Wind Engineering and Ind. Aerodynamics, Vol. 69-71, p. 341-350.

Kim, W.J. and Perkins, N., 2002, "Two-dimensional vortex-induced vibration of cable suspensions", Journal of Fluids and Structures, Vol. 16: (2), p. 229-245, February 2002.

Leweke, T., Bearman, P.W. and Williamson, C.H.K, 2001, "Preface of the special issue on IUTAM Symposium on Bluff Body Wakes and Vortex-Induced Vibrations, BBVIV2, Marseilles, France", Journal of Fluids and Structures. Vol. 15, No. 3/4, April 2001.

Pesce, C.P. and Fujarra, A.L.C., 2002, "Vortex Induced Vibrations Experiments with an Elastically Mounted Cylinder in Water", 12th International Offshore and Polar Engineering Conference, ISOPE, Kita-Kyushu, Japan, Vol. 2, p. 502-509, 26-30 May.

Sarpkaya, T., 1963, "Lift, Drag and added Mss Coefficiets for a Circular Cylinder Immersed in a Time-Dependent Flow", Journal of Applied Mechanics, Vol. 30, p.13-18.

Sarpkaya, T., 1978, "Fluid Forces on Oscillating Cylinders", Journal of Waterway, etc. Div., ASCE, WW4, Vol. 104, pp. 275-290.

Sarpkaya, T., 1979, "Vortex Induced Oscillations- A Selective Review", Journal of Applied Mechanics, ASME, Vol. 46, p. 241-258.

Sarpkaya, T., 2001, "On the Force Decompositions of Lighthill and Morison", Journal of Fluids and Structures, Vol. 15, No. 2, pp. 227-233, February 2001.

Sarpkaya, T., 2003, "Private Communication".

Vikestad, K., Vandiver, J.K. and Larsen, C.M., 2000, "Added Mass and Oscillation Frequency for a Circular Cylinder Subjectec to Vortex-Induced Vibrations and Extenal Disturbance", Journal of Fluids and Structures, Vol. 14, p.1071-1088.

Willden, R.H.J. and Graham, J.M.R., 2001, "Numerical Prediction of VIV on Long Flexible Circular Cylinder", Journal of Fluids and Structures, Vol. 15, p.659-669.