NUMERICAL TIME CONSTANT DETERMINATION OF A STEPPER MOTOR BASED ON AN ELECTROMECHANICAL SYSTEM NON-LINEAR MODEL

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Abstract. *Currently, a high percentage of manufacturing operation time are spent in product handling, whether transportation, positioning tasks or both. Electromechanical positioning systems (EPS) are becoming faster and more accurate based on the development of technologies associated with Servomotors and Stepper motors. These last have been gaining land as an economic alternative to Servos. In order to be competitive stepper are increasing its operation velocity and torque. Up to date, several methodologies to carry out transient analysis have been developed for EPS , in order to minimize response time. The majority of them are analytical aproachs which are based on linearization thecniques which disgard important variables such as friction force and electrical time constant In this paper newer and more complete non-linear model of this electromechanical system is presented looking for determination of electrical and mechanical parameters incidence in time constant value. The ordinary differential equations system (ODE's) that governs the transient behavior are solved numerically and compared with linear solutions. Results shows that factors such as friction force and stator electrical time constante affect net system time constan. Finally an adjusted matemathical time constant expresion is obtained by non-linear multivariable regression analysis.*

Keywords *Drive Systems & Power Systems, Electromechanical Model, Stepper motor, Non-linear model, Numerical Solution, Non-linear regression.*

1. Introduction

Modeling an optimization of fast drive electromechanical system has been developed mainly by private companies which applies gained knowledge in products, such as printers, and robotic manipulators. Most recent public work about it was developed by Riba C (1997) who introduces the concept of Transient Power and Double Kinetics Energy Method for electromechanical time optimization. This approach is useful for servomotor drive systems because is based on the assumption that torque delivered by the systems is constant. Maury (1998) introduce the time constant (Ogata, 1987) as a criteria for EPS time minimization and developed linear models for different kinds of motor including Variable Reluctance and Hybrid Step Motors. Mathematical models for time constant determination are based in linearized sets of differential equations, such as Lawrenson and Hughes (1975) two phases model, the four phase model of Alabern (1990) and models used for control purposes, Alin (2001) and Melkote (1997)

2. Model formulation

Figure 1 shows the electro-mechanical system to be modeled in this paper. This EPS is composed by a two phases hybrid stepper motor, a coupling unit (a gearbox, a rack and pinion unit, timing belt and pulley and so on) and a load to move. In this derivation, linear movement of load is assumed, so reflected inertia on rotor is constant.

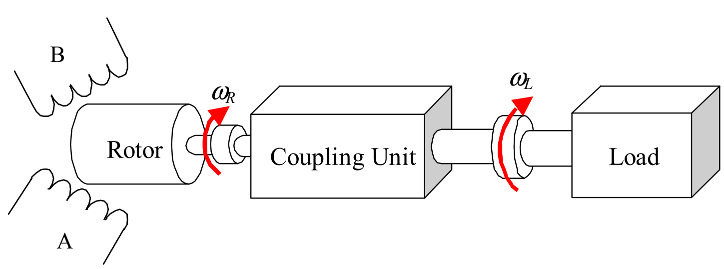


Figure 1. Electromechanical system to be modeled, where ωr is the rotor angular speed, ωL is the load shaft's angular speed.

The mechanical equation that governs rotor motion is obtained applying second Newton Law on rotor axis, obtaining:

$$J_{eq}\frac{d^2\theta}{dt^2}+D_{eq}\frac{d\theta}{dt}+\tau_A+\tau_B+T_f=0 \tag{1}$$

Where:

θ: Is the rotor angle
J_{eq}: Is the equivalent inertia on rotor shaft
D_{eq}: Is the equivalent damping coefficient for viscous and drag forces.
T_f: Is the equivalent friction torque
τ_A, τ_B: Are the magnetic couples generated by the rotor, they can be written as:

$$\begin{aligned}\tau_A &= pn\Phi_M i_A \sin p\theta \\ \tau_B &= pn\Phi_M i_B \sin p(\theta-\lambda)\end{aligned} \tag{2}$$

Where:

p: Is the rotor teeth number
n: Is the number of wounds on stator coils
Φ_M: Is the rotor magnetic flux intensity
λ: Is the angle between phases
i_A, i_B: Is the current on phase A and B.

In the stator coils we obtain the following equations:

$$\begin{aligned}&V-ri_A-L\frac{di_A}{dt}-M\frac{di_B}{dt}+\frac{d}{dt}\left(n\Phi_M\cos p\theta\right)=0 \\ &V-ri_B-L\frac{di_B}{dt}-M\frac{di_A}{dt}+\frac{d}{dt}\left(n\Phi_M\cos p(\theta-\lambda)\right)=0\end{aligned} \tag{3}$$

Where:

V: Is the supplied voltage
r. Is coil resistance
L: Is the coil self inductance
M: Is the mutual inductance between phase A and B

These equations are combined into a set of non-linear differential equations as shows on Eq. (4)

$$\begin{aligned}&J\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+K_T i_A\sin p\theta+K_T i_B\sin p(\theta-\lambda)+T_f=0 \\ &V-ri_A-L\frac{di_A}{dt}-M\frac{di_B}{dt}+\frac{d}{dt}\left(\frac{K_T}{p}\cos p\theta\right)=0 \\ &V-ri_B-L\frac{di_B}{dt}-M\frac{di_A}{dt}+\frac{d}{dt}\left(\frac{K_T}{p}\cos p(\theta-\lambda)\right)=0\end{aligned} \tag{4}$$

Where: $K_T = pn\Phi_M$ is called Motor Torque Constant. It's also assumed that L and M to be independent of θ, and r, J and D to be constant

The main non-linearity of this set of equations are generated by the product of i_A and i_B by the respective functions of θ and the variable behavior of T_f with time.

2.1. Friction Model

In order to account the effect of friction forces in the step response behavior, a simplified Coulomb's based model of friction behavior is developed. Although it's simplicity this model was selected as a first approximation for the friction effect in the EPS. The friction behavior is modeled using the piecewise function of Eq. (5).

$$\begin{cases} T_f = -\sum \tau_{Motor} \text{ if } \omega_R = \wedge T_f \leq T_{f\max} \\ T_f = -T_{f\max} \dfrac{\omega_R}{|\omega_R|} \text{ if } |\omega_R| > 0 \end{cases} \tag{5}$$

Where:

$T_{f\,max}$: Is the maximum friction torque in order to begin the motion

3. Model Parameters and simulation

In order to solve the set of non-linear differential equations showed on eq. (4) numerical computation is carried out. A simulation graphical language (MATLAB Simulink ™) is used to develop such solutions using a Runge-Kutta integration method with a variable step solver (ODE45). The initial conditions are established as follows:

$$\begin{cases} \theta = \dfrac{d\theta}{dt} = \dfrac{d^2\theta}{dt^2} = 0 \\ i_A = 0, i_B = 0 \\ V_A = V_R, V_B = 0 \end{cases}$$

Where V_R is the rated voltage for an specific motor.

Simulations were carried out using motor parameters obtained from step motor catalogs, four different motors were selected. Table (1). shows main parameters.

Table 1. Motor parameters used for simulation

PARAMETERS	NEMA 23 PACSI	NEMA 34 PACSI	NEMA 23 IMS	106 MM. SANYO
Holding Torque (Nm)	0.71	4.16	1.69	19
Detent Torque (Nm)	0.066	0.18	0.069	-
Rotor Inertia (kgm^2)	1.20E-05	1.40E-04	4.60E-05	2.20E-03
Phase Resistance (ohms)	0.46	0.36	1.5	0.63
Phase Inductance (mH)	0.7	1.2	5.4	8
Rated Current (Amps)	4	6.1	2.4	6
Step Angle (Deg)	1.8	1.8	1.8	1.8

4. Results and discussion

A single step response analysis was performed in order to establish the influence of several mechanical and electrical variables on the System Time Constant.(τ_{EM}) The variables selected for this investigation were:

- Electrical Time Constant
- System Friction Force
- Equivalent Damping Factor
- Equivalent Inertia

System Time Constant is not measured directly from simulation. The direct measured variable is the Settling Time (T_s), which is the time to reach an oscillation amplitude less than a percentage of final value (in this case 3%). Settling Time is easier to read from an oscillatory graph and is directly related to τ_{EM} by Eq. (6).

$$T_s = 3.51\tau_{EM} \tag{6}$$

4.1. Electrical Time Constant

Electrical Time Constant (T_e=L/r) in the stator windings determines how quickly winding current reach its final value. This is a very important parameter if we consider that motor torque is directly related to winding current and that the model considered in this paper includes current variation within time.

Simulation where carried out varying T_e from zero to approximately ten times its original value (T_{e0}). Settling Time was measured and normalized by Settling Time when $T_e = T_{e0}$.

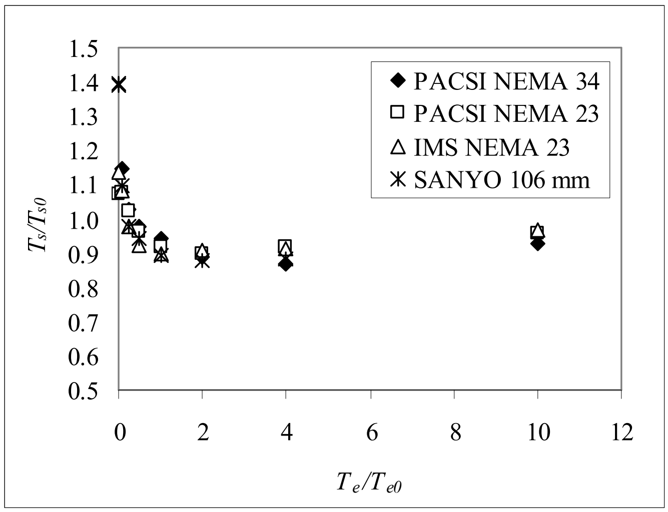


Figure 2. Effect of Electrical Time Constant on Normalized Settling Time

According to data results showed in Fig (2) the behavior of T_s with T_e has three phases. The first one, when T_e reaches zero, there are T_s increment with T_e forming a pulse with stabilizes until T_e/T_{e0} reaches 0.1. The second phase is for T_e/T_m (where T_m is the mechanical time constant defined as $T_m = 2J_{eq}/D_{eq}$) between 0.1 and 2, this is characterized by potential reduction of T_s, the final phase is for T_e/T_{e0} greater than two, in which T_s increases slightly with T_e. Figure (3) is used to explain the three phases behavior, in this figure a one step response is plotted in terms of angle deviation and current rise with time for three different T_e/T_{e0} values, each one correspond with a different phase mentioned above.

Figure (3) shows that T_s is affected by two main phenomena. One of them is the first peak amplitude which increase with current slope and its effect on T_s is to reduce it in the way first peak is shallower. The second one is the time to reach first peak value, which increase for greater values of T_e and tend to increment Settling time. These effects are opposite and T_s is a superposition of both. Finally the initial T_s increment observed in stage one, has been originated by current ripple showed in black curve, which increase first peak amplitude.

The behavior above described is modeled by Eq.(7) which is adjusted for T_e/T_{e0} between 0.1 and 2 because is the interval that contains the practical variations of T_e.

$$\frac{T_s}{T_{s0}} = 0.927\left(\frac{T_e}{T_{e0}}\right)^{-0.0552} \tag{7}$$

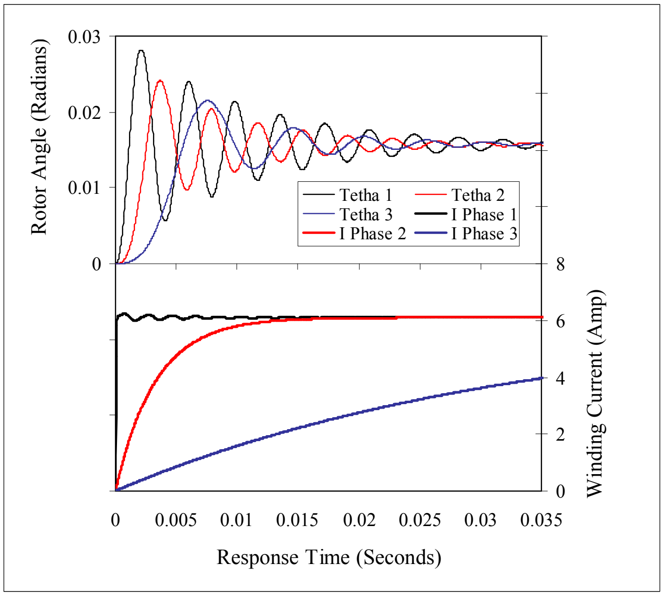
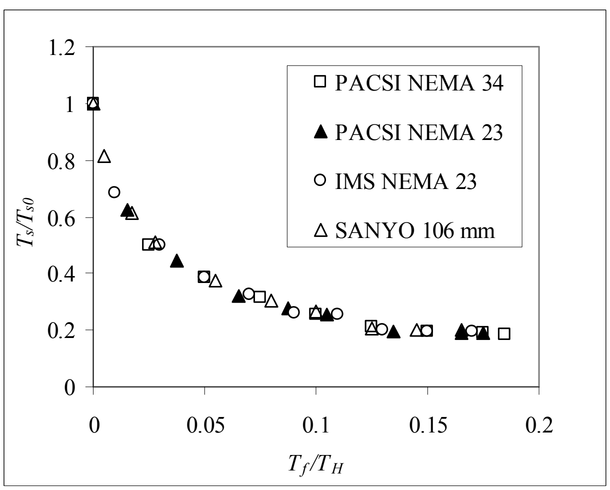


Figure 3. Pulse response (above) and current rise behavior (below) comparison for three different electrical time constant. T_e/T_{e0}, = 0.1, black curve; T_e/T_{e0}, = 1, red curve; T_e/T_{e0}, = 4, blue curve.

4.2. Friction force

Friction force effect on Settling Time was evaluated varying friction torque as a percentage of Motor Holding Torque (T_H). Result in the four motors considered in this paper were normalized with respect to initial Settling Time (T_{s0}) giving out the behavior in Fig (4).

Results shows that Friction force effect on T_S is strongly affected by variations in T_e as shows in Fig(5) (Left) which is obtained for $T_e/T_{e0} = 1$



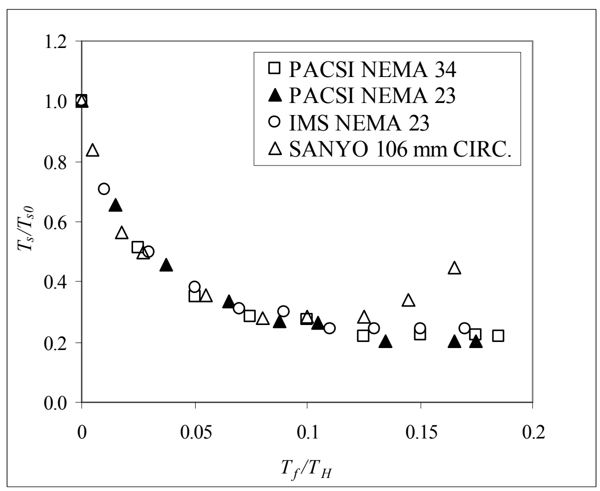


Figure 4. Friction force effect on settling time for $T_e/T_{e0} = 0$ (Left curve) and for . $T_e/T_{e0} = 1$ (right curve) Variables are normalized in order to exclude motor size into results.

These Variations are due to the fact that higher Electrical Time Constant represents a time delay between voltage application and the beginning of rotor motion as shows in Fig (5) where this effect is normalized by T_e/T_m.. But in the other hand, higher values in Te represents a lower initial acceleration rate which is caused by low current values in the initial phase of motion. This phenomena, combined with high friction torques tend to increase Ts again, as shown in Fig (5) (Right) for T_f/T_H up to 0.1.

An undesired effect of friction force is a final rotor deviation from the demanded position, which is considered as an positioning error. Final values are randomly distributed, Fig (5) (Left), but maximum error seems to follow a linear relationship with T_f/T_H. This appreciation limits the percentage of Friction Torque tolerated by a systems in terms of the maximum error allowed in the application.

The following model is used to fit data obtained:

$$\frac{T_s}{T_{s0}} = 0.0575\left(\frac{T_f}{T_H} + 0.0105\right)^{-0.7452} + 0.3519\left(\frac{T_m}{T_e}\right)\left(\frac{T_f}{T_H}\right) \quad (8)$$

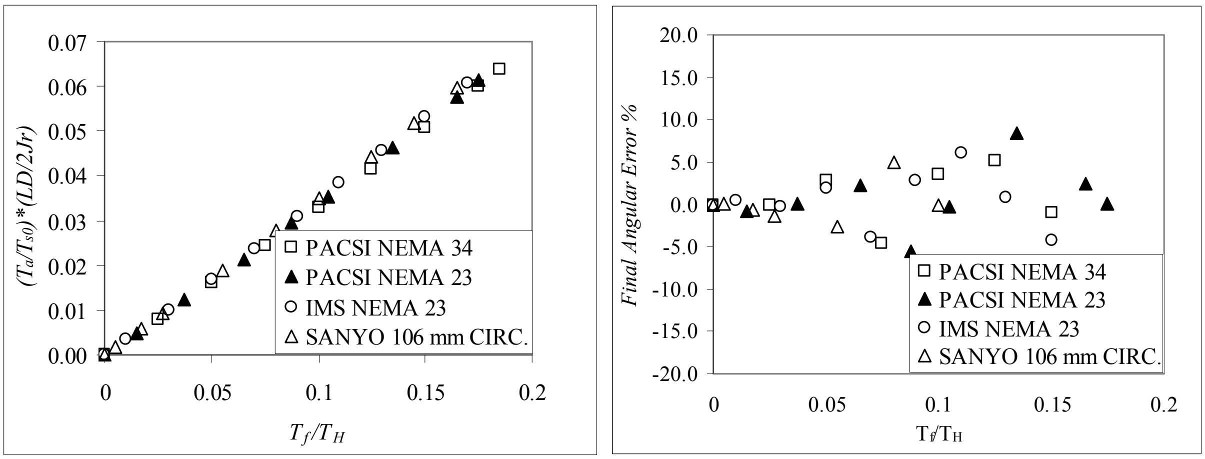


Figure 5. Friction force ratio effect on starting time (Ta) normalized by inertial ratio T_e/T_m (Left). Final angle error percentage versus Friction force ratio(Right)

4.3. Equivalent Damping Ratio

This factor accounts for different forces that are proportional to rotor speed, such as, air drag, viscous forces on bearings, eddy currents and so on. The damping coefficient, Eq. (9) is used to evaluate all these factors. Settling time is measures for ξ between 0.2 and 1 typical values of this parameters are 0.4 to 0.7.

$$\xi = \frac{D_{eq}}{2\sqrt{J_{eq} K_t i_R}} \quad (9)$$

Where i_R is the rated winding current.

Ts is normalized by the Settling Time for ξ = 0.7, although relationship between normalized variables is independent from this selection. Figure (6) shows results for the four motors investigated.

Additional simulations reveal that effect of this parameter on T_s is not influenced by electrical time constant and the following mathematical model gives a good adjust for the data

$$\frac{T_s}{T_{s0}} = 0.6856\xi^{-1.0256} \quad (10)$$

Equation. (10) represents an inverse relationship as outlined by Maury (1997) in their time constant equation.

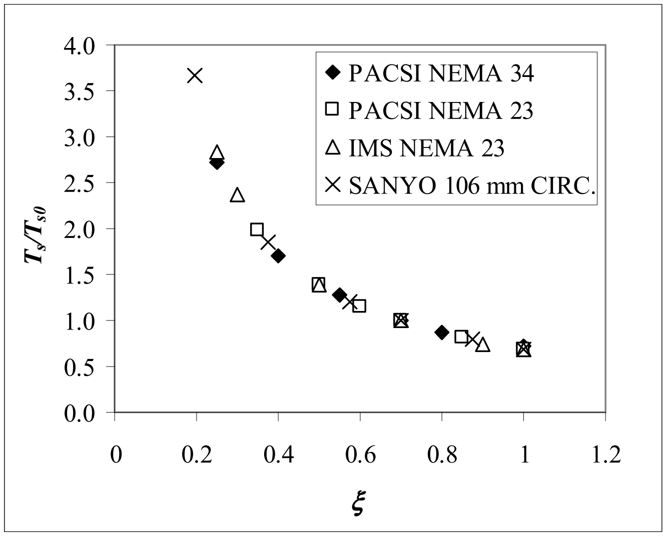


Figure 6. Damping factor effect on Settling time. T_s is normalized by T_{s0} when $\xi = 0.7$ as reference value.

4.4. Equivalent Inertia

Equivalent inertia is a very important factor, because for given motor and an inertial load it can be modified varying transmission ratio in order to optimize response time, Riba (1987). Data were normalized by rotor inertia and varied between 1 and 4, since the minimum inertia is the rotor inertia, the best inertial matching according with Riba C is obtained when reflected load inertial is equal to rotor inertia ($J_{eq}/J_0 = 2$). Results, Fig (7), exhibit almost a linear relationship which is consistent with Maury (1997) Linear model. The Eq. (11) adjust the data with a R^2 value of 0.9993.

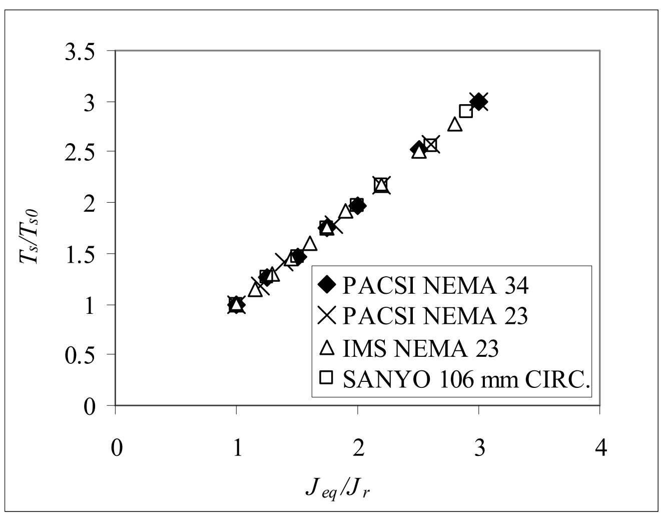


Figure 7. Equivalent inertia reflected in rotor effect on settling time. Variables are normalized.

$$\frac{T_s}{T_{s0}} = 0.9984\left(\frac{J}{J_0}\right)^{0.9989} \quad (11)$$

Result obtained for individual parameter influence on T_s are limited in terms of utilization but very useful because that shows the part of the model consistent with linear model. Also simplifies the multivariable analysis by variables reduction and the general mathematical model is derived from the individual observation

4.5. General Mathematical Model for Settling Time

Based on individual analysis observations, a general mathematical model for Ts was built. This model includes the variables that are not consistent with linear model.

$$\frac{T_s}{T_m} = c_1\left(c_2\left(\frac{T_f}{T_H}+c_3\right)^{c_4} + c_5\frac{T_m}{T_e}\frac{T_f}{T_H}\right)\left(\frac{T_e}{T_m}\right)^{c6}\left(\frac{K_t}{K_{t0}}\right)^{c7}$$

Where: $c_1,...c_7$ are model constants

Simulations were carried out following a three level factorial 3^k experimental scheme. T_s is now normalized by T_m in order to make a comparison with linear model. Results are shown in Table (2)

Table 2. Variable ranges and T_s obtained from simulation under the 27 simulation carried out in the PACSI NEMA 34 stepper motor model

	Input Variables			Response			Input Variables			Response	
Sim.	T_f/T_H	T_e/T_m	K_t/K_{t0}	T_s	T_s/T_m	Sim.	T_f/T_H	T_e/T_m	K_t/K_{t0}	T_s	T_s/T_m
1	0.00	0.10	0.50	0.0277	3.344	15	0.03	0.60	1.50	0.0144	1.738
2	0.00	0.10	1.00	0.029	3.501	16	0.03	1.10	0.50	0.0122	1.473
3	0.00	0.10	1.50	0.0299	3.609	17	0.03	1.10	1.00	0.013	1.569
4	0.00	0.60	0.50	0.0251	3.030	18	0.03	1.10	1.50	0.0131	1.581
5	0.00	0.60	1.00	0.0258	3.114	19	0.05	0.10	0.50	8.41E-03	1.015
6	0.00	0.60	1.50	0.026	3.139	20	0.05	0.10	1.00	0.0112	1.352
7	0.00	1.10	0.50	0.0249	3.006	21	0.05	0.10	1.50	0.0123	1.485
8	0.00	1.10	1.00	0.0261	3.151	22	0.05	0.60	0.50	8.33E-03	1.006
9	0.00	1.10	1.50	0.0254	3.066	23	0.05	0.60	1.00	9.92E-03	1.197
10	0.03	0.10	0.50	0.0131	1.581	24	0.05	0.60	1.50	0.0101	1.219
11	0.03	0.10	1.00	0.0149	1.799	25	0.05	1.10	0.50	9.80E-03	1.183
12	0.03	0.10	1.50	0.0166	2.004	26	0.05	1.10	1.00	9.57E-03	1.155
13	0.03	0.60	0.50	0.0123	1.485	27	0.05	1.10	1.50	9.94E-03	1.200
14	0.03	0.60	1.00	0.0135	1.630						

The coefficients are obtained by non-linear regression least squares analysis using the iterative Gauss-Newton method included in MATLAB software with Levenberg-Marquardt modifications for global convergence.

$$\frac{T_s}{T_m} = 0.0431\left(4.7059\left(\frac{T_f}{T_H}+0.0134\right)^{-0.6299} - 0.4630\frac{T_f}{T_H}\frac{T_m}{T_e}\right)\left(\frac{T_e}{T_m}\right)^{-0.0570}\left(\frac{K_t}{K_{t0}}\right)^{0.0776} \quad (12)$$

With a R^2 value of 0.991 which represents a good fit. This equation is valid for the interval used for variables in simulation. Extrapolation is not recommended.

The behavior of T_s/T_m in terms of variable couples are shown in Fig (8) to Fig (10). Graphical analysis reveal the most relevant variable is T_f, followed by T_e.

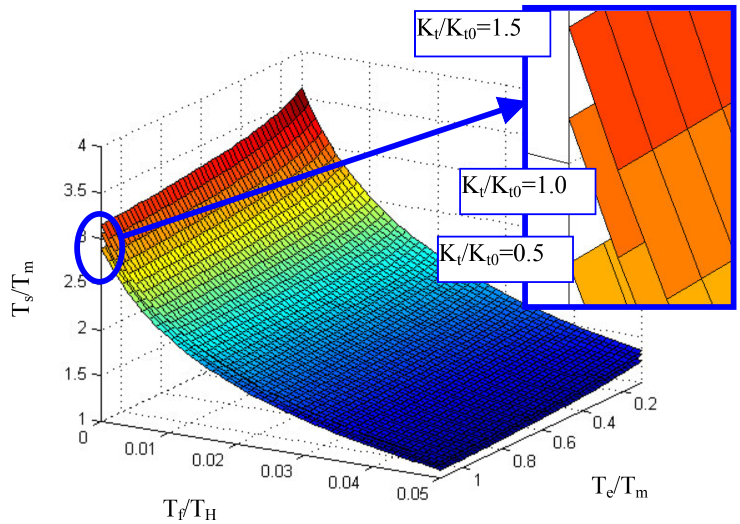


Figure 8. Graphical representation of T_s normalized by Tm for K_t/K_{t0} = 0.5,1 and 1.5 as shows in the detail surfaces are slightly superimposed

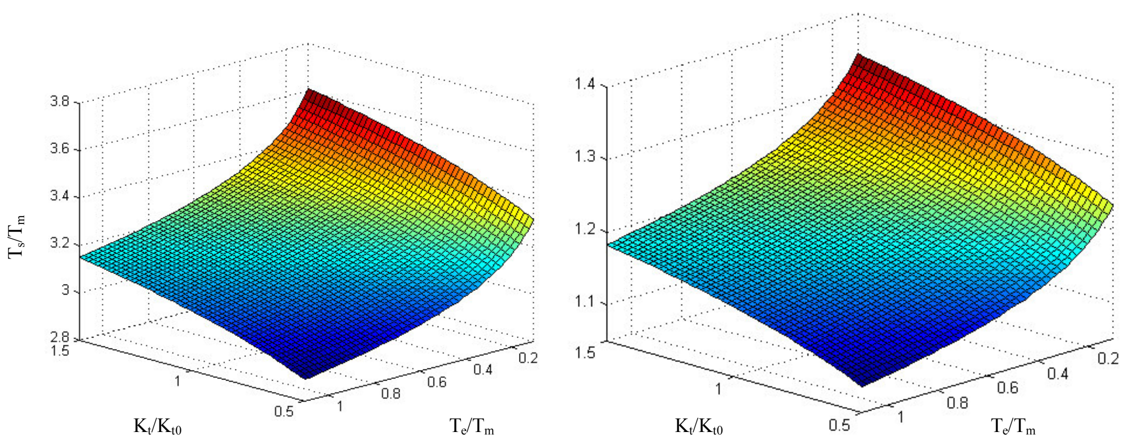


Figure 9. T_s/T_m versus Normalized Electrical Constant (T_e/T_m) and Torque constant (K_t/K_{t0}) for $T_f/T_H = 0$ (Left) and $T_f/T_H = 0.05$ (Right)

6. Conclusions and future work

From the variable analysis obtained from-the model simulations results, we can conclude the following:

Friction force in a small percentage (no more than 5% of holding torque) have a great influence in the Time constant reduction. Only 2.5 % of friction can reduce System Time Constant in a 50%. Higher values of friction are non-representative in T_s reduction but very harmful in terms of positional error increase.

Electrical Time Constant has not only an important influence on overall time constant, also interacts with friction force reducing its convenient effects. Increasing T_e until two times its original value, tend to reduce overall time constant.

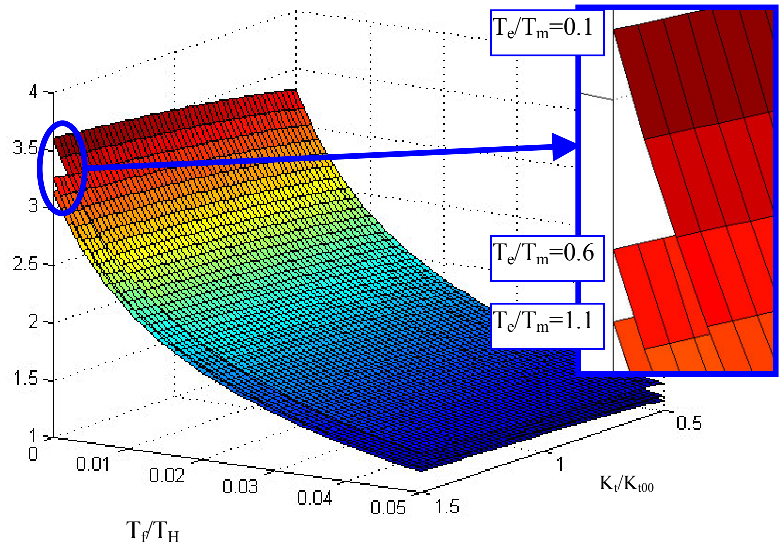


Figure 10. T_s/T_m versus Normalized Friction Torque (T_f/T_H) and Normalized Torque Constant (K_t/K_{t0}) for three values of Normalized Electrical Time Constant (T_e/T_m), as shows in the detail.

Damping factor and equivalent inertial load effect on T_s don not interact with T_e effect. And are consistent with linear models. Developed by earlier authors investigation such as Maury (1997)

The model presented in this paper is not only useful to evaluate single step response, it can be used to simulate the influence of friction and electrical time constant in the motor dynamics characteristics. For out coming research in this area we will include: to develop a more complex friction models in the analysis and the experimental validation of the model, the dynamics characterization under different operating modes such as full step and half-step. And the EPS response when different velocity profiles are applied.

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