CONTROLLING CHAOS IN A NONLINEAR PENDULUM

Francisco Heitor I. Pereira Pinto
Armando M. Ferreira
Instituto Militar de Engenharia
Departamento de Engenharia Mecânica e de Materiais
Praça General Tibúrcio, 80, Urca
22.290.270 – Rio de Janeiro – RJ
E-Mail: heitor@epq.ime.eb.br, armando@epq.ime.eb.br

Marcelo A. Savi
Universidade Federal do Rio de Janeiro
COPPE - Departamento de Engenharia Mecânica
21.945.970 – Rio de Janeiro – RJ, Caixa Postal 68.503
E-Mail: savi@ufrj.br

Abstract. *Chaotic behavior of dynamical systems offers a rich variety of orbits, which may be controlled by small perturbations in either a specific parameter of the system or a dynamical variable. Therefore, this kind of behavior may be desirable in different applications. Chaos control usually involves two steps. In the first, unstable periodic orbits that are embedded in the chaotic set are identified. After that, a control technique is employed in order to stabilize a desirable orbit. This contribution employs the close-return method to identify unstable periodic orbits (UPO) and a variation of the Ott-Grebogi-Yorke (OGY) technique, called semi-continuous control, to stabilize some UPO. As an application to a mechanical system, a nonlinear pendulum is considered. Based on parameters obtained for a experimental setup, analyses are carried out considering signals that are generated by numerical integration of the mathematical model. Results show that these techniques may be employed to control chaos in mechanical systems.*

Keywords: *Chaos, Control, Time Series, Nonlinear Pendulum.*

1. Introduction

Chaotic behavior has been extensively analyzed from the early sixties when *E. Lorenz* developed studies on the unpredictability of meteorological phenomena. Nowadays, different fields of sciences have special interest in this kind of phenomenon as for example engineering (Moon, 1998; Piccoli & Weber, 1998; Mees & Sparrow, 1987), medicine (Goldberger *et al.*, 1990), ecology (Schaffer, 1985), biology (Hassell *et al.*, 1991) and economy (Peel & Speight, 1994; Aguirre & Aguirre, 1997). As a matter of fact, chaos may occur in many natural processes and the idea that chaotic behavior may be controlled by small perturbations of some physical parameter is making this kind of behavior to be desirable in different applications.

Chaos control is based on the richness of responses and also on the sensitive dependence to initial condition related to a chaotic behavior. Basically, a chaotic attractor has a dense set of unstable periodic orbits, and the system often visits the neighborhood of each one of them. One of these orbits may be the desirable behavior of the system in a particular situation. Moreover, the sensitive dependence to initial condition also implies that the system's evolution may be altered by small perturbations. Therefore, control of chaos may be understood as the use of tiny perturbations for the stabilization of unstable periodic orbits embedded in a chaotic attractor.

It should be pointed out that it is not necessary to have a mathematical model to describe the system dynamics since time series analysis may be employed with this aim. Kantz & Shreiber (1997) say that the most direct link between chaos theory and the real world is the analysis of time series from real systems in terms of nonlinear dynamics. Therefore, all control parameters may be resolved from time series analysis.

Chaos control methods may be classified as discrete or continuous techniques. The first chaos control method has been proposed by Ott *et al.* (1990), nowadays known as the OGY (Ott–Grebogi–Yorke) method. This is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface of section, such as some Poincaré section. On the other hand, continuous methods are exemplified by the so called delayed feedback control (DFC), proposed by Pyragas (1992), which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and the previous state of the system.

Nowadays, there are many variations of the OGY technique that overcome some of the limitations of the original method, as for example: control of orbits with high period (Otani & Jones, 1997 and Hübinger *et al.*, 1994), control by time-delay coordinates (Dressler & Nitsche, 1992; So & Ott, 1995 and Korte *et al.*, 1995), control of unstable periodic orbits with high instability (Hübinger *et al.*, 1994 and Ritz *et al.*, 1997). For more details on chaos control based on OGY method refer to: Chen (2001), Chanfreau & Lyyjynen (1999), Ditto *et al.* (1995), Ditto & Showalter (1997), Dubé & Després (2000), Shinbrot *et al.* (1993), Ogorzalek (1994), Grebogi & Lai (1997), Bayly & Virgin (1994) and Boccaletti *et al.* (2000).

There are reports on some experimental applications of OGY based control methods as in magnetoelastic ribbons (Ditto *et al.*, 1990; In *et al.*, 1995; Hübinger *et al.*, 1994), in nonlinear pendulums (Hübinger *et al.*, 1994; Korte *et al.*, 1995; Starret & Tagg, 1995; Yagasaki & Uozumi, 1997) and in a double pendulum (Christini *et al.*, 1996).

This contribution concerns with the analysis of chaos control in numerical signals obtained from a nonlinear pendulum. The proposed mathematical model is based on an experimental apparatus analyzed by Franca & Savi (2001) and Pinto & Savi (2003). These previous articles consider the chaotic behavior of an experimental nonlinear pendulum, analyzing state space reconstruction, determination of dynamical invariants and prediction. The pendulum has both torsional stiffness, provided by a string-spring device, and viscous damping, provided by a magnetic device. These characteristics make this apparatus distinct from the others reported in the literature. Here, all signals are numerically generated by the integration of the equations of the mathematical model, which uses experimentally identified parameters. The Close-Return (CR) method (Auerbach *et al.*, 1987) is performed to determine unstable periodic orbits embedded in the attractor, and a variation of the OGY technique called SCC (semi-continuous control) method proposed by Hübinger *et al.* (1994) and extended by Korte *et al.* (1995) is considered to perform the control. Results confirm the possibility to use this approach to deal with mechanical systems.

This article is organized as follows. Section 2 describes how the CR method can be used to extract unstable periodic orbits from chaotic data. Section 3 describes the OGY control method and its variation used in this work. Section 4 presents the pendulum model and applies the discussed methods. Section 5 presents some conclusions.

2. Determination of Unstable Periodic Orbits

A chaotic set has a large number of unstable periodic orbits (UPO) embedded in it. Moreover, the system's trajectories visit the neighborhood of each one of them. The control of chaos can be treated as a two-stage procedure. The first stage is composed by the identification of unstable periodic orbits. This step may be understood as a "learning stage". Since periodic orbits are dynamical invariants, they can be analyzed from time series, exploiting topological invariance (Xu *et al.*, 2002; Gunaratne *et al.*, 1989).

A time series is a sequence of observations of some time variables of the system, and it is usually related to a nonlinear dynamical system whose experimental analysis furnishes a scalar sequence of measurements. Nonlinear analysis involves different tools, including state space reconstruction and determination of dynamical invariants. Lyapunov exponents and attractor dimension are some examples of dynamical invariants that could be used to identify chaotic behavior (Franca & Savi, 2001).

This article considers the close-return method (Auerbach *et al.*, 1987) to the identification of UPOs. The basic idea is to search for a period$-\tau$ UPO in the time series. Let the dynamics of the system be represented by $\{u_i\}_{i=1}^{N}$, a map concerned to a certain surface of section. This state vector may be obtained either by the direct observation of all the states of the system or by state space reconstruction from a scalar time series s_n ($n = 1, \dots N$). The identification of the UPO is based on a search for pairs of points in the time series that satisfy the condition:

$$\left|u_i - u_{i+n}\right|_{i=1}^{(N-\tau)} \leq r_1 , \tag{1}$$

where r_1 is a tolerance value for distinguishing return points. After this analysis, all points that belong to a τ-periodic cycle are grouped together. During the search, the vicinity of a periodic orbit may be visited many times, and in this case it is necessary to distinguish each orbit, remove any cycle permutation and average the orbits extracted for improved estimations. In order to decide if two nearby periodic orbits of period-τ correspond to distinct periodic orbits, the approach presented by Otani & Jones (1997) is employed, who sort each of two UPO with same period in an ascending order to obtain two new sets of points and use them for the comparison. If the distances of all corresponding points of the two new sets are less than the tolerance r_2, then they are grouped together into the same UPO cluster. Otherwise, they are considered to be distinct UPO.

Other different approaches can be employed for the determination of UPO as proposed by Pawelzik & Schuster (1991), Pierson & Moss (1995), So *et al.* (1996), Schmelcher & Diakonos (1997,1998), Diakonos *et al.* (1998), Pingel *et al.* (2000), Davidchack & Lai (1999), Dhamala *et al.* (2000).

Having identified an UPO, one is able to proceed to the next stage and control the chaotic system in order to stabilize it in the desired orbit. In the following section, the OGY control method is described.

3. OGY Control Method

The OGY (Ott *et al.,*1990) approach is described considering a discrete system of the form of a map $\xi_{i+1} = F(\xi_i, p)$, where $p \in \Re$ is an accessible parameter for control. This is equivalent to a parameter dependent map associated to surface of section. Let $\xi_F = F(\xi_F, p_0)$ denote the unstable fixed point on the section corresponding to an orbit in the chaotic attractor that one wants to stabilize. Basically, the control idea is to monitor the system dynamics until the neighborhood of this point is reached. After that, a proper small change in the parameter p causes the next state ξ_{i+1} to fall into the stable direction of the fixed point. In order to find the proper variation in the control parameter, δp, it is considered a linearized version of the dynamical system near the equilibrium point:

$$\delta\xi_{i+1} \cong A\delta\xi_i + w\delta p_i , \tag{2}$$

where $\delta\xi_i = \xi_i - \xi_F$, $\delta p_i = p_i - p_0$, $A = D_\xi F(\xi_F, p_0)$, and $w = \partial F / \partial p(\xi_F, p_0)$. The Jacobian A and the sensitivity vector w can be estimated from a time series using a least-square fit method as described in Auerbach *et al.* (1987) and Otani & Jones (1997).

Considering the fact that the Jacobian matrix A has an unstable eigenvector, e_u, related to eigenvalue λ_u, and also a stable eigenvector, e_s, related to eigenvalue λ_s, defining the local direction of unstable and stable manifolds, respectively. Then, a contravariant basis, f_u and f_s, is defined such that:

$$e_l f_m = \delta_{lm}, \tag{3}$$

where δ_{lm} is the Kronecker delta. The stabilization of the orbit is performed forcing ξ_{i+1} to fall on the local stable manifold of the fixed point, which is represented by the condition:

$$f_u \delta\xi_{i+1} = 0. \tag{4}$$

Applying Eq. (2) in Eq. (4) and using the fact that the Jacobian matrix can be written as $A = \lambda_u\, e_u f_u + \lambda_s\, e_s f_s$, the following condition can be written:

$$f_u (A\delta\xi_i + w\delta p_i) = \lambda_u f_u \delta\xi_i + \delta p_i f_u w = 0. \tag{5}$$

Then, solving Eq. (5) for δp_i yields:

$$\delta p_i = -\lambda_u \frac{f_u \delta\xi_i}{f_u w}, \tag{6}$$

which is the desired control law. Such control is activated if the resulting change in the parameter δp_i is less than the maximal disturbance allowed, $\delta p_{\max}$.

In order to overcome some limitations of the original OGY formulation such as control of orbits with large instability, measured by unstable eigenvalues, and orbits of high period, Hübinger *et al.* (1994) introduced the so called semi-continuous control (SCC) method or local control method, which is described in the following section.

3.1. Semi-Continuous Control Method

The semi-continuous control (SCC) method lies between the continuous and the discrete time control because one can introduce as many intermediate Poincaré sections, viewed as control stations, as it is necessary to achieve stabilization of a desirable UPO. Therefore, the SCC method is based on measuring transition maps of the system. These maps relate the state of the system in one Poincaré section to the next section.

In order to use N control stations per forcing period T, one introduces N equally spaced successive Poincaré sections $\Sigma_n, n = 0, \ldots, (N-1)$. Let $\xi_F^n \in \Sigma_n$ be the intersections of the UPO with Σ_n and $F^{(n,n+1)}$ be the mapping from one control station Σ_n to the next one Σ_{n+1}. Here, the superscript n is used instead of the subscript i of the previous section, to differentiate both methods. Hence, one considers the map

$$\xi_F^{n+1} = F^{(n,n+1)}(\xi_F^n, p^n). \tag{7}$$

As in the OGY method one uses a linear approximation of $F^{(n,n+1)}$ around ξ_F^n and p_0:

$$\delta\xi^{n+1} \cong A^n \delta\xi^n + w^n \delta p^n, \tag{8}$$

where $\delta\xi^{n+1} = \xi^{n+1} - \xi_F^{n+1}$, $\delta p^n = p^n - p_0$, $A^n = D_{\xi^n} P^{(n,n+1)}(\xi_F^n, p_0)$, and $w^n = \dfrac{\partial P^{(n,n+1)}}{\partial p^n}(\xi_F^n, p_0)$.

Hübinger *et al.* (1994) state the possibility of the eigenvalues of A^n be complex numbers and then they use the fact that the linear mapping A^n deforms a sphere around ξ_F^n into an ellipsoid around ξ_F^{n+1}. Therefore, a singular value decomposition (SVD),

$$A^n = U^n W^n (V^n)^T = \begin{Bmatrix} u_u^n & u_s^n \end{Bmatrix} \begin{bmatrix} \sigma_u^n & 0 \\ 0 & \sigma_s^n \end{bmatrix} \begin{Bmatrix} v_u^n & v_s^n \end{Bmatrix}^T, \tag{9}$$

is employed in order to determine the directions v_u^n and v_s^n in Σ_n which are mapped onto the largest, $\sigma_u^n u_u^n$, and shortest, $\sigma_s^n u_s^n$, semi-axis of the ellipsoid in Σ_{n+1}, respectively. Here, σ_u^n and σ_s^n are the singular values of A^n.

Korte *et al.* (1995) state the control target as being the adjustment of δp^n such that on the map n+1 the direction v_s^{n+1} is obtained, resulting in a maximal shrinking on map n+2. Therefore, it demands $\delta\xi^{n+1} = \alpha v_s^{n+1}$, where $\alpha \in \Re$. Hence, from Eq.8 one has that

$$A^n \delta\xi^n + w^n \delta p^n = \alpha v_s^{n+1}, \tag{10}$$

which is a relation from what α and δp^n can be conveniently chosen. As in the OGY method, all parameters can be extracted from a time series analysis and one must monitor the system dynamics until the neighborhood of any fixed point is reached to make the necessary changes in the control parameter.

In the next section, a nonlinear pendulum is analyzed applying the CR method to search for UPO as described in Section 2 and the SCC-OGY method to perform control of this mechanical device.

4. Nonlinear Pendulum

In this article, a nonlinear pendulum is considered as a mechanical application of the general procedure to control chaos. The motivation of the proposed pendulum is an experimental set up, previously analyzed by Franca & Savi (2001) and Pinto & Savi (2003). In this section, it is presented the mathematical model of a nonlinear pendulum and the corresponding parameters. Numerical simulations of such model are employed in order to obtain time series related to the pendulum response. Systems' parameters are experimentally obtained from an experimental setup. Finally, some unstable periodic orbits are identified and their control simulated.

It is considered the pendulum shown in Figure (1a), which consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). A magnetic device (3) provides an adjustable dissipation of energy. A string-spring device (6) provides torsional stiffness to the pendulum and an electric motor (7) excites the pendulum via the string-spring device. An actuator (5) provides the necessary perturbations to stabilize this system by properly changing the string length.

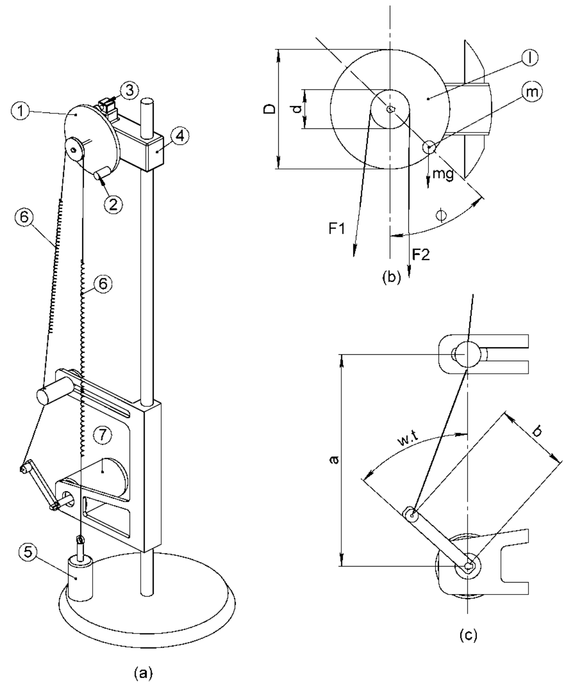


Figure 1. Nonlinear pendulum. (a) Physical Model. (1) Aluminum disc; (2) Lumped mass; (3) Magnetic damping device; (4) Rotary Motion Sensor; (5) Actuator; (6) String-spring device; (7) Electric motor. (b) Parameters and forces on the aluminum disc. (c) Parameters from driving device.

In order to describe the dynamics of this apparatus, a mathematical model is proposed considering Fig.(1). Let F_1 and F_2 be the forces exerted on the rotating masses and given by:

$$F_1 = k(\sqrt{a^2 + b^2 - 2ab\cos(\varpi t)} - (a-b) - \frac{d}{2}\phi), \tag{11}$$

$$F_2 = k(\frac{d}{2}\phi - \Delta l), \tag{12}$$

where ϖ is the forcing frequency, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation arm of the motor, D is the diameter of the aluminum disc and d is the diameter of the driving pulley. The Δl parameter is the length variation in the string provided by the linear actuator (8) shown in Fig. (1a). This parameter is considered as the variation on the accessible parameter for control purposes, being equivalent to δp^n in Eq. (10). Therefore, the equation of motion of pendulum is given by:

$$F_1\frac{d}{2} - F_2\frac{d}{2} - \zeta\dot{\phi} - mg\frac{D}{2}\sin(\phi) = I\ddot{\phi} \tag{13}$$

where I is the inertia of the aluminum disc plus lumped mass, m is the lumped mass and ζ is the viscous dissipation parameter. The pendulum dynamics can also be written:

$$\begin{Bmatrix}\dot{\phi}\\ \ddot{\phi}\end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{kd^2}{2I} & -\frac{\zeta}{I}\end{bmatrix}\begin{Bmatrix}\phi\\ \dot{\phi}\end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{kd}{2I}[\sqrt{a^2+b^2-2ab\cos(\varpi t)} - (a-b) - \Delta l] - \frac{mgD}{2I}\sin(\phi)\end{Bmatrix} \tag{14}$$

In order to determine numerical values for parameters in Eq.(13), it is considered the same setup of Franca & Savi (2001) and Pinto & Savi (2003), presented in Fig. (2).

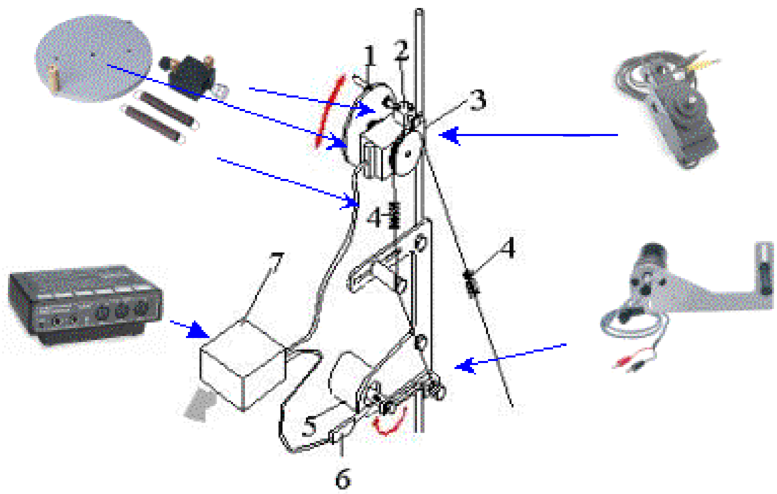


Figure 2. Experimental apparatus of the nonlinear pendulum: (1) Disc with concentrated mass; (2) Magnetic damping device; (3) Rotary motion sensor: PASCO CI-6538; (4) Springs; (5) DC Motor: PASCO ME-8750; (6) Magnetic transducer: TEKTRONIX; (7) Science workshop interface: PASCO CI-6760.

In this setup, the motor (5), *PASCO ME-8750*, has the following characteristics: 12V DC, 0.3-3Hz and 0-0.3A. The signal measurement is done with the aid of two transducers: a rotary motion sensor (3), *PASCO encoder CI-6538*, which has 1440 orifices and a precision of 0.25^0 and a magnetic transducer (6), employed to generate a frequency signal associated with the angular phase of the motor, which is used to construct the Poincaré map of the signal. The apparatus is connected with an A/D interface (7), *Science Workshop Interface 500* (*CI-6760*) where the sampling frequency varies from 2Hz to 20kHz. The interface samples the signal 8 times for frequencies below 100Hz and a single time for higher sampling rates. Furthermore, this interface does not have any anti-aliasing filters. A 0-18V variable output AC-DC adapter provides power supply. Table (1) shows the parameters of Eq.(14) that were measured on the experimental setup and adopted here.

Table. 1 – Experimental values of parameters.

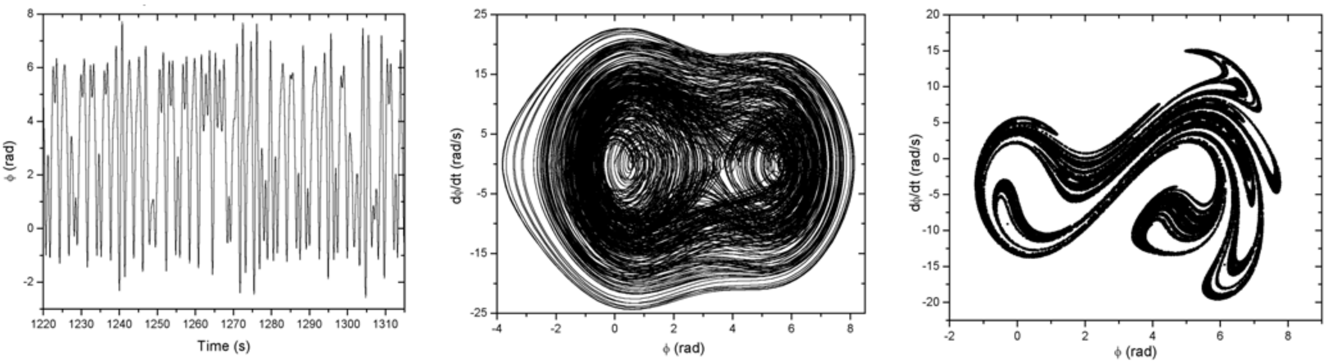
Parameter	a (m)	b (m)	d (m)	D (m)	I (kgm^4)	k (N/m)	m (kg)
Value	1.6 x 10^{-2}	6.0 x 10^{-2}	2.9 x 10^{-2}	9.2 x 10^{-2}	1.876 x 10^{-4}	4.736	1.6 x 10^{-2}

Values of the adjustable parameters ϖ and ζ were tuned to generate chaotic behavior in agreement to the experimental work done by Franca & Savi (2001). The Δl parameter has a null value for the system without control action. Hence, these parameters are as shown in Tab. (2).

Table. 2 –Values of adjustable parameters.

Parameter	ϖ (rad/s)	ζ (kg.m^2/s)	Δl (m)
Value	5.15	5.575 x 10^{-5}	0

After applying these parameters in the model described by Eq. (14), that is without control action, a fourth-order Runge-Kutta scheme with time step equal to $(2\pi/100\varpi)$ was employed to perform numerical simulations. From such data, it can be confirmed that the system presents a chaotic motion. Lyapunov exponents, calculated by the algorithm proposed by Wolf *et al.* (1985), assure this conclusion showing one positive value: $\lambda = \{+18.73, -5.64\}$. Figure (3) shows temporal evolution, phase space and a strange attractor on the Poincaré section.



(a) (b) (c)

Figure 3. Chaotic motion. (a) Temporal evolution in 90 seconds. (b) Phase space. (c) Strange attractor.

As pointed out in Section 2, the first stage of the control strategy is the identification of UPOs embedded in the chaotic attractor. The close-return method (Auerbach *et al.*, 1987) is employed after dividing the coordinates ϕ and $\dot{\phi}$ by a factor 9 and 18, respectively. The value of the tolerance r_1 is chosen to be 0.003 and r_2 is set to be ten times r_1. Table (3) shows the number of UPOs found up to period-15.

Table 3. Unstable periodic orbits up to period 15 found.

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
UPOs found	-	1	7	1	-	3	5	3	1	2	6	6	1	4	5

Figure (4) presents a strange attractor of the motion showing points in the surface of section corresponding to the UPOs to be stabilized. The SCC method is applied considering three intermediate sections (named intermediate Poincaré section #2, #3, #4) (Fig. (5)). Therefore, a total of four maps per forcing period are considered.

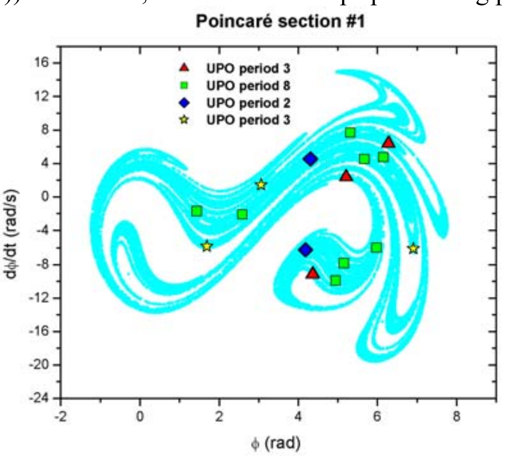


Figure 4. Strange attractor showing UPO.

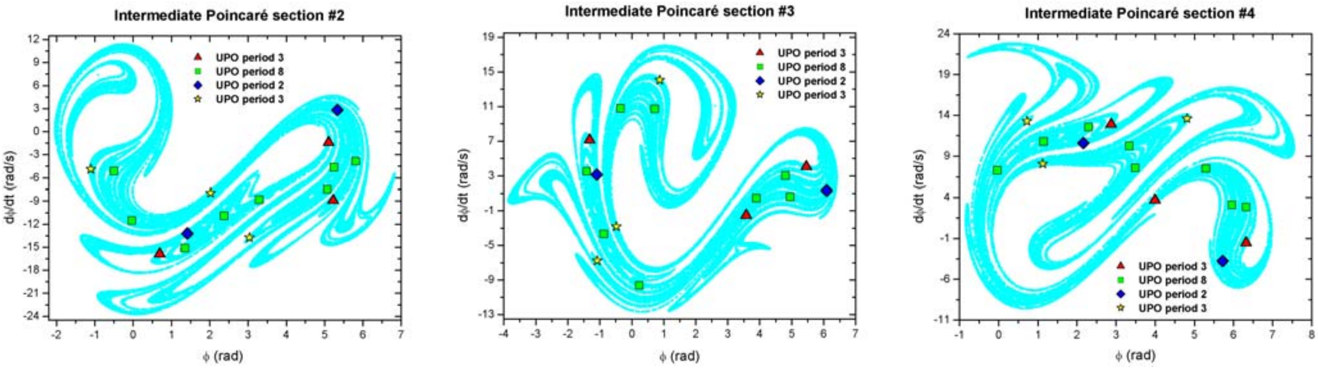
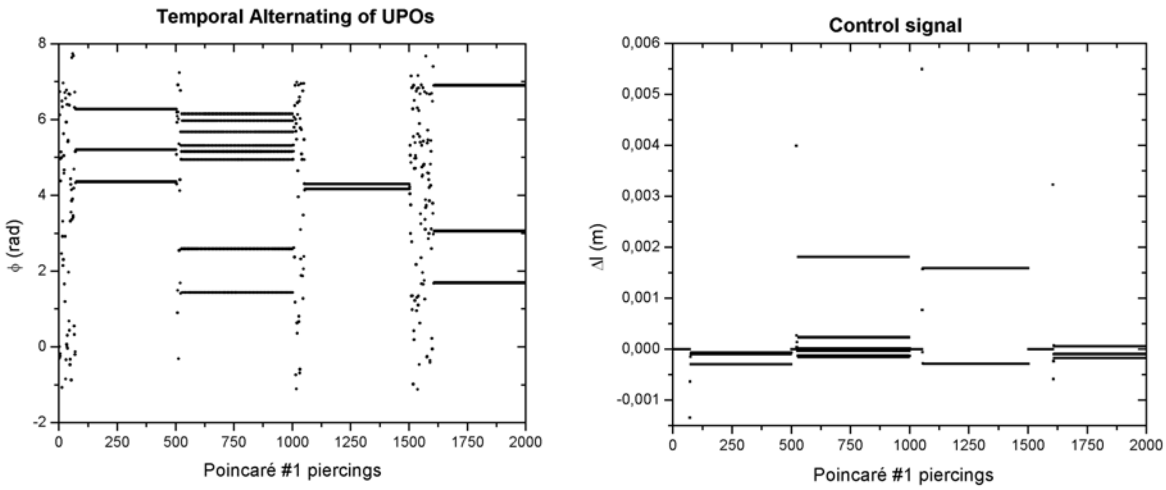


Figure 5. Strange attractor in intermediate sections showing UPO.

After the identification of the UPOs embedded in the Poincaré section #1, the piercing of the same UPOs in the other three Poincaré sections is determined. Then, the local dynamics expressed by the Jacobian matrix and the vector sensitivity of the transition maps in a neighborhood of the fixed points are determined using the least–square fit method (Auerbach *et al.*, 1987 and Otani & Jones, 1997). After that, the SVD technique is employed for determining the stable and unstable directions near the next fixed point. The sensitivity vectors are determined allowing the trajectories to come close to a fixed point and then one perturbs the parameters by the maximum allowable value. In this case, a perturbation in Δl of $\pm 2x10^{-2}\ m$ is performed, fitting the resulting deviations $[\delta\xi^{n+1}(\Delta l) - A^n \delta\xi^n]/\Delta l$ from the next piercing by the least square procedure. After that, SCC method is employed to stabilize unstable periodic orbits and the parameter changes are calculated from Eq. (10).

In order to explore the possibilities of alternating the stabilized orbits with small changes in the control parameter we perform a simulation that aims the stabilization of a period–3 UPO in the first 500 forcing periods, of a period–8 UPO between 500 and 1000 forcing periods, of a period–2 UPO between 1000 and 1500 forcing periods and of a period–3 UPO, different from the first one, between 1500 and 2000 forcing periods. Figure (6) shows the system's dynamics in the Poincaré section #1 during the actuation.



(a) (b)

Figure 6. Response under control. (a) Temporal alternating of UPOs in Poincaré section #1. (b) Control signal.

It is important to note the different times needed for the system to achieve the desired stabilization on a particular UPO. This happens because one must wait until the trajectory comes close enough to a control point to perform the necessary perturbation. It should be pointed out that, as expected, results show that unstable orbits are stabilized with small variations of control parameter, less than 2mm in this case.

More details on the orbits stabilized due to SCC method are presented in Figures (7-10). In all cases, as the target orbit changes, one notes short transients on the temporal evolution of Δl followed by tiny periodic perturbations, as well as good results regarding to keeping the system in the desired orbit.

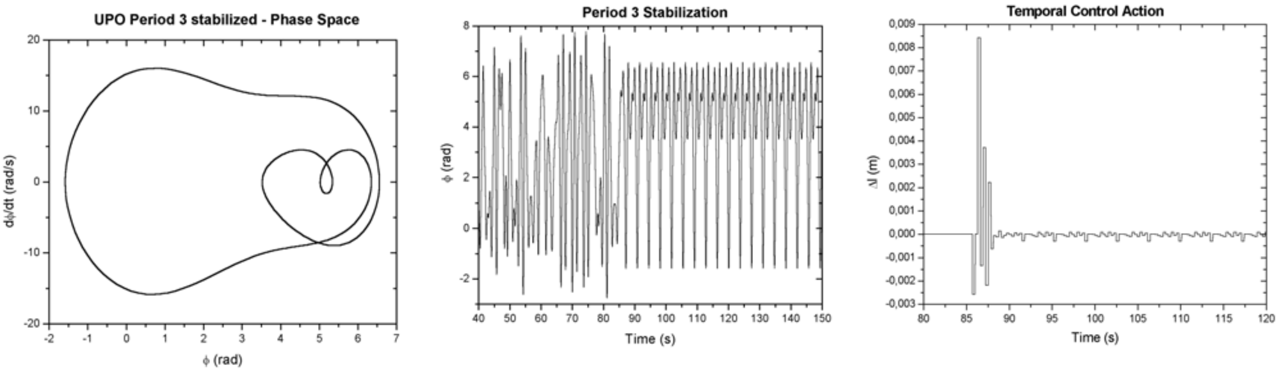


Figure 7. UPO period−3 stabilized.

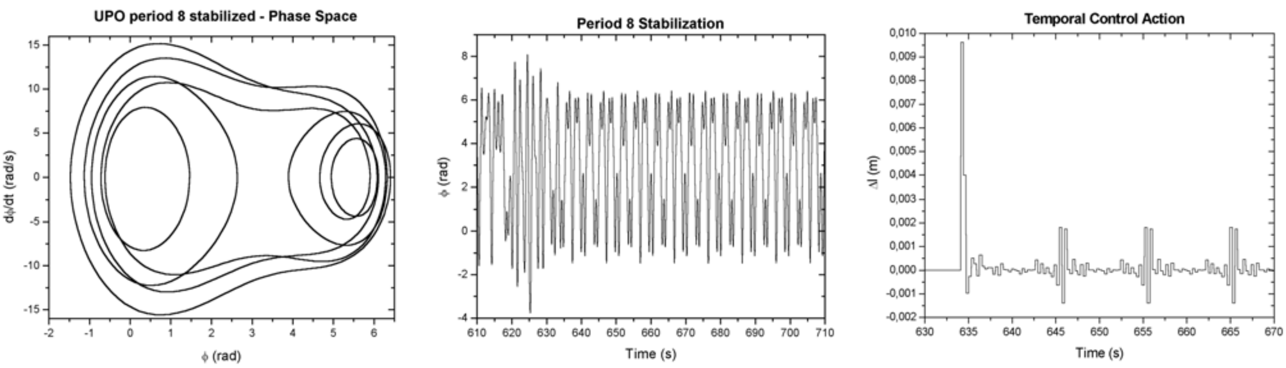


Figure 8. UPO period−8 stabilized.

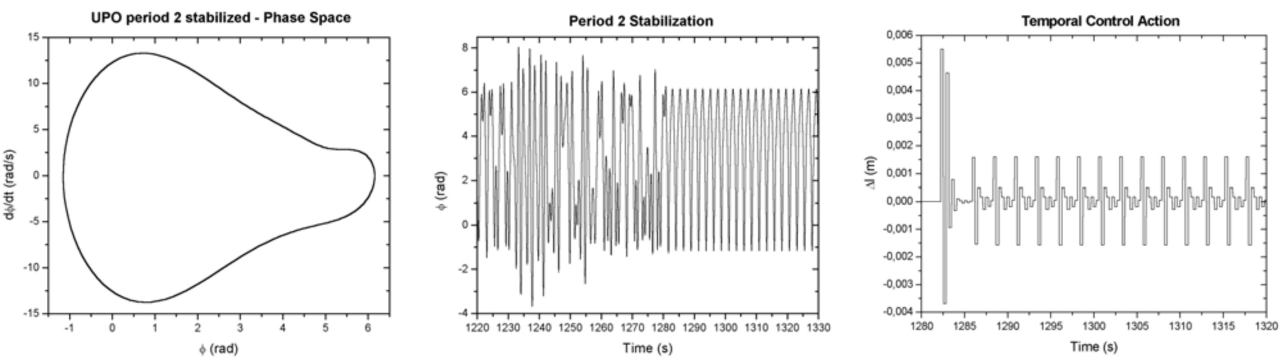


Figure 9. UPO period−2 stabilized.

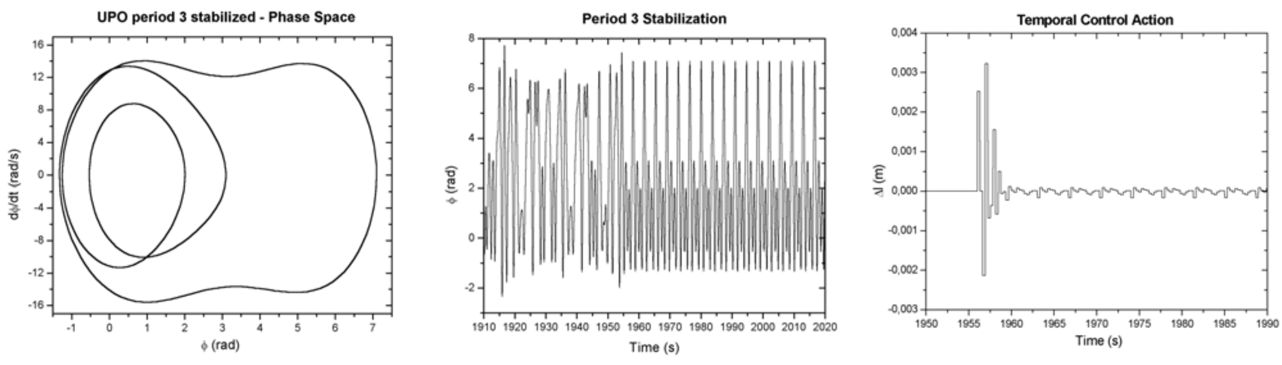


Figure 10. UPO period−3 stabilized.

5. Conclusions

This contribution discusses the control of chaos in signals obtained from a nonlinear pendulum, based on an experimental apparatus previously analyzed by Franca & Savi (2001) and Pinto & Savi (2003). All signals are generated by numerical integration of the equations of motion, but the parameters used were experimentally evaluated. In the first stage of the control process, the close-return method is employed to identify unstable periodic orbits. After that, a variation of the OGY technique, the semi-continuous control method, is considered to stabilize desirable unstable orbits. Moreover, least-square fit method is employed to estimate Jacobian matrixes and sensitivity vectors. Singular value decomposition is employed to estimate directions of unstable and stable manifolds in the vicinity of control points. The general procedure here discussed is applied to a chaotic signal of a nonlinear pendulum. After identification of unstable orbits, some of them are stabilized by the proposed techniques. Results confirm the possibility of using such approach to control chaotic behavior in mechanical systems.

6. Acknowledgments

The authors acknowledge the support of the Brazilian Research Council (CNPq) and of the Brazilian Army.

7. References

Aguirre, L. A. & Aguirre, A., 1997, "A Tutorial Introduction to Nonlinear Dynamics in Economics", *Nova Economia*, v.7, n.2, pp.9-47.

Auerbach, D., Cvitanovic, P., Eckmann, J. -P., Gunaratne, G. & Procaccia, I., 1987, "Exploring Chaotic Motion Through Periodic Orbits", *Physical Review Letters*, v. 58, n. 23, pp. 2387-2389.

Bayly, P. V. & Virgin, L. N., 1994, "Practical Considerations in the Control of Chaos", *Physical Review E*, v. 50, n. 1, pp. 604-607.

Boccaletti, S., Grebogi, C., Lai, Y. -C., Mancini, H. & Maza, D., 2000, "The Control of Chaos: Theory and Applications", *Physics Reports*, n. 329, pp. 103-197.

Chanfreau, P. & Lyyjynen, H., 1999, "Viewing the Efficiency of Chaos Control", *Journal of Nonlinear Mathematical Physics*, v. 6, n. 3, pp. 314-331.

Chen, G., 2001, "Introduction to Chaos Control and Anti-control", In: Chapter 6, in *Advanced Topics in Nonlinear Control Systems*, ed. by T. P. Leung and H.-S. Qin, World Scientific Pub. Co., Singapore, pp.193-245.

Christini, D. J., Collins, J. J. & Linsay, P. S., 1996, "Experimental Control of High-dimensional Chaos: The Driven Double Pendulum", *Physical Review E*, v. 54, n. 5, pp. 4824-4827.

Davidchack, R. L. & Lai, Y. -C., 1999, "Efficient Algorithm for Detecting Unstable Periodic Orbits in Chaotic Systems", *Physical Review E*, v. 60, n.5, pp. 6172-6175.

Korte, R. J. de, Schouten, J. C. & van den Bleek, C. M. V., 1995, "Experimental Control of a Chaotic Pendulum with Unknown Dynamics Using Delay Coordinates", *Physical Review E*, v. 52, n. 4, pp. 3358-3365.

Dhamala, M., Lai, Y. –C & Kostelich, E. J., 2000, "Detecting Unstable Periodic Orbits from Transient Chaotic Time Series", *Physical Review E*, v. 61,n. 6, pp. 6485-6489.

Diakonos, F. K., Schmelcher, P. & Biham, O., 1998, "Systematic Computation of the Least Unstable Periodic Orbits in Chaotic Attractors", *Physical Review Letters*, v.81, n. 20, pp. 4349-4352.

Ditto, W. L., Rauseo, S. N. & Spano, M. L., 1990 "Experimental Control of Chaos", *Physical Review Letters*, v. 65, n. 26, pp. 3211-3214.

Ditto, W. L., Spano, M. L. & Lindner, J. F., 1995, "Techniques for the Control of Chaos", *Physica D*, v.86, pp. 198-211.

Ditto, W. L. & Showalter, K., 1997, "Introduction: Control and Synchronization of Chaos", *Chaos*, v. 7, n. 4, pp. 509-511.

Dressler, U. & Nitsche, G., 1992, "Controlling Chaos Using Time Delay Coordinates", *Physical Review Letters*, v. 68, n. 1, pp. 1-4.

Dubé, L. J. & Després, P., 2000, "The Control of Dynamical Systems – Recovering Order from Chaos" -, In: *The Physics of Electronic and Atomic Collisions*, ed. Y. Itikawa, AIP Conference Proceedings 500, pp. 551-570;

Franca, L. F. P. & Savi, M. A., 2001, "Distinguishing Periodic and Chaotic Time Series Obtained from an Experimental Nonlinear Pendulum", *Nonlinear Dynamics*, v. 26, pp. 253-271.

Goldberger, A., Ringney, D. & West, B., 1990, "Chaos and Fractal in Human Physiology", *Scientific American*, v.262, n.2, pp.34-41.

Grebogi, C. & Lai, Y. -C., 1997, "Controlling Chaotic Dynamical Systems", *Systems & Control Letters*, v. 31, pp. 307-312.

Gunaratne, G., Linsay, P. S. & Vinson, M. J., 1989, "Chaos Beyond Onset: A Comparison of Theory and Experiment", *Physical Review Letters*, v. 63, n. 1, pp. 1-4.

Hassel, M. P., Comins, H. N. & May, R. M., 1991, "Spatial Structure and Chaos in Insect Population Dynamics", *Nature*, v.353, pp.255-258.

Hübinger, B., Doerner, R., Martienssen, W., Herdering, M., Pitka, R. & Dressler, U., 1994, "Controlling Chaos Experimentally in Systems Exhibiting Large Effective Lyapunov Exponents", *Physical Review E*, v. 50, n. 2, pp. 932-948.

In, V., Ditto, W. L. & Spano, M. L., 1995, "Adaptative Control and Tracking of Chaos in a Magnetoelastic Ribbon", *Physical Review E*, v. 51, n.4 part A, pp. R2689-R2692.

Kantz, H. & Schreiber, T., 1997, "*Nonlinear Time Series Analysis*", Cambridge University Press, United Kingdom.

Mees, A. I. & Sparrow, C., 1987, "Some Tools for Analyzing Chaos", *Proceedings of the IEEE*, v.75, n.8, pp.1058-1070.

Moon, F. C., 1998, "*Dynamics and Chaos in Manufacturing Processes*", John Wiley & Sons.

Ogorzalek, M., 1994, "Chaos Control: How to Avoid Chaos or Take Advantage of It", *Journal of the Franklin Institute*, v. 331B, n. 6, pp. 681-704.

Otani, M. & Jones, A. J., 1997, "*Guiding Chaotic Orbits*", 130, Research Report, Imperial College of Science Technology and Medicine, London.

Ott, E., Grebogi, C. & Yorke, J. A., 1990, "Controlling Chaos", *Physical Review Letters*, v. 64, n. 11, pp. 1196-1199.

Pawelzik, K. & Schuster, H. G., 1991, "Unstable Periodic Orbits and Prediction", *Physical Review A*, v. 43, n. 4, pp.1808-1812.

Peel, D. A. & Speight, A. E. H., 1994, "Hysteresis and Cyclical Variability in Real Wages, Output and Unemployment - Empirical-Evidence from Nonlinear Methods for the United-States", *International Journal Systems Science*, v.25, n.5, pp.943-965.

Piccoli, H. C. & Weber, H. I., 1998, "Experimental Observation of Chaotic Motion in a Rotor with Rubbing", *Nonlinear Dynamics*, v.16, n.1, pp.55-70.

Pinto, E. G. F. & Savi, M. A., 2003, "Nonlinear Prediction of Time Series Obtained from an Experimental Pendulum", *Current Topics in Acoustical Research – Research Trends*, to appear.

Pierson, D. & Moss, F., 1995 "Detecting Periodic Unstable Points in Noisy Chaotic and Limit Cycle Attractor with Application to Biology", *Physical Review Letters*, v. 75, n. 11, pp. 2124-2127.

Pingel, D., Schmelcher, P., Diakonos, F. K. & Biham, O., 2000, "Theory and Applications of the Systematic Detection of Unstable Periodic Orbits in Dynamical Systems", *Physical Review E*, v. 62, n. 2, pp. 2119-2134, August 2000.

Pyragas, K., 1992, "Continuous Control of Chaos by Self-controlling Feedback", *Physics Letters A*, v. 170, pp. 421-428.

Ritz, T., Schweinsberg, A. S. Z., Dressler, U., Doerner, R., Hübinger, B. & Martienssen, W., 1997, "Chaos Control with Adjustable Control Times", *Chaos, Solitons & Fractals*, v. 8, n. 9, pp. 1559-1576.

Schaffer, W. M., 1985, "Order and Chaos in Ecological Systems", *Ecology*, v.66, n.1, pp.93-106.

Schmelcher, P. & Diakonos, F. K., 1997, "Detecting Unstable Periodic Orbits of Chaotic Dynamical Systems", *Physical Review Letters*, v.78, n.25, pp. 4733-4736, 23 June 1997.

Schmelcher, P. & Diakonos, F. K., 1998, "General Approach to the Localization of Unstable Periodic Orbits in Chaotic Dynamical Systems", *Physical Review E*, v.57, n.3, pp. 2739-2746.

Shinbrot, T., Grebogi, C., Ott, E. & Yorke, J. A., 1993 "Using Small Perturbations to Control Chaos", *Nature*, v. 363, pp. 411-417.

So, P. & Ott, E., 1995, "Controlling Chaos Using Time Delay Coordinates via Stabilization of Periodic Orbits", *Physical Review E*, v.51, n. 4, pp. 2955-2962.

So, P., Ott, E., Schiff, S., Kaplan, D. T., Sauer, T. & Grebogi, C., 1996, "Detecting Unstable Periodic Orbits in Chaotic Experimental Data", *Physical Review Letters*, v. 76, n.25, pp.4705-4708.

Starret, J. & Tagg, R., 1995, "Control of a Chaotic Parametrically Driven Pendulum", *Physical Review Letters*, v. 74, n. 11, pp. 1974-1977.

Xu, D., Zhigang, L., Bishop, S. R. & Galvanetto, U., 2002, "Estimation of Periodic-like Motions of Chaotic Evolutions Using Detected Unstable Periodic Patterns", *Pattern Recognition Letters*, v. 23, pp. 245-252.

Yagasaki, K. & Uozumi, T., 1997, "Controlling Chaos in a Pendulum Subjected to Feedforward and Feedback Control", *International Journal of Bifurcation and Chaos*, v. 7, n. 12, pp. 2827-2835.

Wolf, A., Swift, J. B., Swinney, H. L. & Vastano, J. A., 1985, "Determining Lyapunov Exponents from a Time Series", *Physica D*, v.16, pp.285-317.