NONLINEAR ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL INTERACTION

José Marcio Calixto
Escola de Engenharia da UFMG - Av. Contorno 842 - 2° andar - 30110 –060 - Belo Horizonte - MG - Brasil
e-mail: calixto@dees.ufmg.br

Abstract. *In many engineering applications, composite beams of various materials are the most efficient design alternative. The stiffness and strength of composite beams depends on the shear connection behavior between the layers. In some widely used systems, such as in composite steel-concrete beams and in layered wood construction connected with nails, the assumption of rigid interconnection between the layers is highly questionable even under service loads. Consequently there exists a movement or slip between the layers which can affect significantly the overall behavior of these structures. This phenomenum is called partial interaction. In this paper a theory is presented for a two-layered elastic member acting either as a beam or beam column and capable of having slip at the interface of the two layers and displacements perpendicular to the longitudinal axis of the member that are on the order of magnitude of the depth of beam. The force-slip relationship for the interlayer connectors is assumed to be nonlinear. An energy approach is employed in the formulation from which the element stiffness matrices are derived. This element stiffness is incorporated into a finite element analysis package. The Newton-Raphson scheme is used to solve the nonlinear system of algebraic equations. Tests of two-layered wood beams and beam-columns are conducted to verify the model and very good agreement is achieved for deflections, strains and slip between the layers.*

Keywords *Nonlinear analysis, Composite beams, Partial interaction*

1. Introduction

Composite beams are the most efficiente design alternative in many engineering applications and consequently have been used for many years. They are usually composed of two or more different materials attached to each other by a mechanical connector or elastomeric adhesive. Although simply constructed, composite beams are complex to analyze. Their stiffness and strength depend on the shear connection behavior between the layers. In some widely used systems, such as in composite steel-concrete beams and in layered wood construction connected with nails, the assumption of rigid interconnection between the layers is highly questionable even under service loads. Consequently there exists a movement or slip between the layers which can affect significantly the overall behavior of these structures. This phenomenum is called partial interaction. Also for moderate levels of slip, the connectors or nails between the members exhibit nonlinear behavior. These facts have led to an extensive investigation, both experimental and analytical (Ko et al., 1972; Wheat et al. 1983; Kamiya 1988, e Veljkoviv 1996), of composite beams with partial interaction. These studies have documented very well not only the effects of the slip and the nonlinear behavior of the connectors in composite beams. On the other hand, most analytical models developed so far are within the framework of small deflection theory with some including the effects of the nonlinear behavior of the connectors.

In this paper, a theory is presented for a two-layered elastic member acting either as a beam or beam column and capable of having slip at the interface of the two layers and displacements perpendicular to the longitudinal axis of the member that are on the order of magnitude of the depth of beam (Calixto, 1991). The force-slip relationship for the interlayer connectors is assumed to be nonlinear. An energy approach is employed in the formulation from which the element stiffness matrices are derived. This element stiffness is incorporated into a finite element analysis package. The Newton-Raphson scheme is used to solve the nonlinear system of algebraic equations. Tests of two-layered wood beams and beam-columns are conducted to verify the model.

2. General Formulation

2.1. Assumptions

In the analysis of composite beams with partial interaction, the following assumptions are introduced:

1 - the beam, the applied loads, and the deformations lie in a plane; the plane of the loads is a plane of symmetry for the beam;
2 - the beam is assumed to be slender; that is, the length of the beam is much larger than its lateral dimensions;
3 - transverse displacements may be finite while longitudinal displacements are infinitesimal;
4 - only normal strains parallel to the axis of the beam are considered and they vary linearly through the depth of the layers;
5 - at every section of the beam, each layer deflects the same amount and there is no separation between them.
6 - materials are assumed to be linearly elastic except for the connectors which may be nonlinear elastic;
7 - the geometric and elastic properties of each layer are constant along the length, but can be different from one layer to the other;
8 - the shear connection between layers is continuous along the length; that is, discrete deformable connectors are assumed closely spaced with respect to the length of the beam to be replaced by a continuous shear connection;

9 - there is no friction at the interface between the two layers; the interaction between the layers is from the connector load-slip characteristics.

2.2. Development of the Element Stiffness Matrix

In finite element analysis, a common procedure to derive stiffness matrices is first express the total potential energy of the element in terms of the Lagrangian displacement coordinates. The potential energy can then be differentiated with respect to each degree of freedom to obtain the equilibrium equations. These equations have the element stiffness matrix built in them. This technique is employed next to derive the stiffness matrix for a two-layered composite beam with partial interaction.

The element being developed in this case is one-dimensional and has two nodes. Each node has four degrees of freedom, namely: axial displacement in each layer, a rotation and a translation perpendicular to the longitudinal direction of the element. The element is shown in Fig. (1).

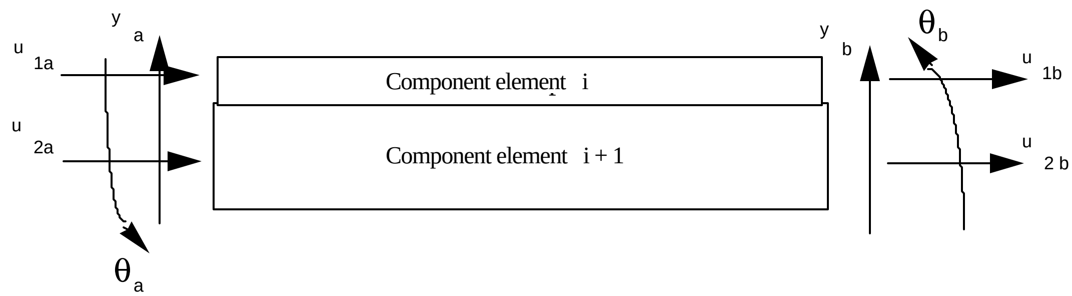

Figure 1. Degrees of Freedom of the Proposed Element.

The nodal displacements in matrix form are:

$$\{d\}^t = \{u_{1a}\ u_{2a}\ y_a\ \Theta_a\ u_{1b}\ u_{2b}\ y_b\ \Theta_b\} \tag{1}$$

Based on the above assumptions, the strain-displacement relationship is given by:

$$e_{xx} = \frac{d\mathbf{u}_i}{dx} - z\frac{d^2\mathbf{w}}{dx^2} + \frac{1}{2}\left(\frac{d\mathbf{w}}{dx}\right)^2, \tag{2}$$

where

u_i = axial displacement at midheight of layer *i*;

w = transversal displacement; and

z = one-half the depth of layer *i*.

The slip can be calculated from the geometry of the deformation between the original and final positions at the interface of the two layers. Consequently the slip is a function of the displacements and can can be expressed as:

$$\Delta = \mathbf{u}_{i+1} - \mathbf{u}_i - \frac{1}{2}\left(h_{i+1} + h_i\right)\frac{d\mathbf{w}}{dx} \quad , \tag{3}$$

where

Δ = slip between layers *i* and *i+1*; and

h_i = depth of layer *i*.

The slip deformations of the connectors must take into consideration the nonlinear behavior of the connectors. Several expressions were developed to describe the nonlinear load-slip curve for commonly used nails. In this study, Foschi's equation (Foschi and Bonac 1977) for nails in single shear is employed. The equation is:

$$F = (P_0 + P_1\Delta)\left[1 - \exp\left(\frac{\mathbf{k}\Delta}{P_0}\right)\right] \quad , \qquad (4)$$

where

- F = force in the nail;
- Δ = slip between the layers (Eq. 3);
- k = nail initial modulus;
- P_0 = intercept of the assymptote; and
- P_1 = nail modulus at high slips.

With the above relationships, the expression for the total potential energy for a two-layered composite beam becomes:

$$U = \sum_{i=1}^{2} \frac{E_iA_i}{2}\int_L \left(\frac{d\mathbf{u}_i}{dx}\right)^2 dx + \sum_{i=1}^{2} \frac{E_iI_i}{2}\int_L \left(\frac{d^2\mathbf{w}}{dx^2}\right)^2 dx + \sum_{i=1}^{2} \frac{E_iA_i}{2}\int_L \left[\left(\frac{d\mathbf{u}_i}{dx}\right)\left(\frac{d\mathbf{w}}{dx}\right)^2\right] dx +$$
$$+ \sum_{i=1}^{2} \frac{E_iA_i}{2}\int_L \left[\frac{1}{4}\left(\frac{d\mathbf{w}}{dx}\right)^4\right] dx + \int_L \frac{Kn}{2s}\left[u_2 - u_1 - \frac{1}{2}(h_1 + h_2)\left(\frac{d\mathbf{w}}{dx}\right)\right]^2 dx - \int_L q\mathbf{w}\,dx$$
$$- \sum_{i=1}^{2} P_i^0.\mathbf{u}_i(0) - \sum_{i=1}^{2} P_i^L.\mathbf{u}_i(L) - V^0.\mathbf{w}(0) - M^0\left(\frac{d\mathbf{w}}{dx}\right)(0) - V^L.\mathbf{w}(L) - M^L.\left(\frac{d\mathbf{w}}{dx}\right)(L)\,. \qquad (5)$$

In the Eq. (5), the first and second terms represent the internal strain energy due to the normal strains in each layer. The third and fourth correspond to the effects of the finite displacements. The energy due to the deformations of the connectors is represented by the fifth term, in which K is the stiffness of the employed connector. The numerical value of this stiffness depends upon the type of the connector and its force-slip relationship. Consequently the term for the strain energy of the connectors is absolutely general and can be applied to any type of connector as long as its force-slip relationship is known. In this study, Foschi's equation (Eq. 4) for nails in single shear is employed The last terms correspond to the work done by the external lateral and axial loads.

For the displacement functions $\mathbf{w}(x)$ and $\mathbf{u}_i(x)$, polynomials were chosen. Since the total energy expression contains only first derivatives of $\mathbf{u}_i(x)$, a linear polynomial was selected. In the case of $\mathbf{w}(x)$, whose second derivative appears in the energy equation, a cubic approximating function is employed. This function ensures continuity of both transverse displacement $\mathbf{w}$ and its first derivative. This way, the relationships between these polynomial functions and the element degrees of freedom are given by:

$$\mathbf{u}_1(x) = \left(1 - \frac{x}{L}\right)u_{1a} + \left(\frac{x}{L}\right)u_{1b} = \{f_1(x)\ 0\ 0\ 0\ f_2(x)\ 0\ 0\ 0\}\{d\} = \{N_1\}^t\{d\} \qquad (6a)$$

$$\mathbf{u}_2(x) = \left(1 - \frac{x}{L}\right)u_{2a} + \left(\frac{x}{L}\right)u_{2b} = \{0\ f_1(x)\ 0\ 0\ 0\ f_2(x)\ 0\ 0\}\{d\} = \{N_2\}^t\{d\} \qquad (6b)$$

$$\mathbf{w}(x) = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)y_a + \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)\boldsymbol{Q}_a + \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right)y_b + \left(-\frac{x^2}{L} + \frac{x^3}{L^2}\right)\boldsymbol{Q}_b =$$

$$= \{0\ 0\ f_3(x)\ f_4(x)\ 0\ 0\ f_5(x)\ f_6(x)\}\{d\} = \{N_3\}^t\{d\}\,. \qquad (6c)$$

Substituting the displacement functions $\mathbf{w}(x)$ and $\mathbf{u}_i(x)$ and its derivatives by the above relationships (Eqs. 6), the expression for the total potential energy, after integration and in matrix form, becomes:

$$U = \{d\}^t\left[\frac{1}{2}\{K_{el}\} + \frac{1}{6}\{K_{G1}\} + \frac{1}{12}\{K_{G2}\} + \frac{1}{2}\{K_{con}\}\right]\{d\} - \{d\}^t\{P\} \quad . \qquad (7)$$

Using the principle of the stationary potential energy, which requires U to have a stationary value at the equilibrium position, we have:

$$\delta U = 0 \quad . \tag{8}$$

Thus

$$\left[\{K_{el}\} + \frac{1}{2}\{K_{G1}\} + \frac{1}{3}\{K_{G2}\} + \{K_{con}\}\right] \{d\} = \{P\}. \tag{9}$$

The above equation represents the equilibrium condition and contains the element stiffness matrix, which is symmetric and has four components. The first one $\{K_{el}\}$ is the so called elastic stiffness matrix, while $\{K_{G1}\}$ and $\{K_{G2}\}$ represent the geometric matrices. The matrix $\{K_{con}\}$ represents the contribution of the interlayer slip deformation to the stiffness matrix.

The element stiffness matrix shown in Eq. (9) is a secant stiffness since it operates on the total displacements and not on incremental ones. To find an incremental form we need to differentiate this equation one more time with respect to each nodal displacement. Since $\{K_{G1}\}$ and $\{K_{G2}\}$ are, respectively, linear and quadratic functions of the displacements, it follows:

$$\left\lfloor\{K_{el}\} + \{K_{G1}\} + \{K_{G2}\} + \{K_{con}\}\right\rfloor \{\Delta d\} = \{\Delta P\}. \tag{10}$$

The above equation is incremental and the coefficients of $\{\Delta d\}$ represent the tangent stiffness matrix.

2.3. The Finite Element Model

The element stiffness was incorporated into a finite element analysis package. The program is capable of analyzing beams and beam-columns subjected to axial and lateral loads and includes the effects of finite displacements as well as non-linear behavior for the connectors. Since fixed-end actions for a two-layered beam with partial interaction have yet to be derived, loads can only be applied at the nodes.

The equilibrium equations (Eq. 9 and 10) are nonlinear since the stiffness matrices are function of the displacements and slips. As a consequence, these equations must be solved iteratively. The method chosen is the Newton-Raphson scheme since it converges quickly for this type of nonlinearities.

3. Experimental Program

Nailed two-layered wood beams and beam-columns were constructed and tested to verify the finite element model. Deflections, strains and slips at several locations along the length of the beams were monitored and recorded for comparison with the theory (Calixto, 1991). Each beam was composed of a joist nailed to a single layer of sheathing. Since the primary objective of the theory is to describe the behavior of layered beams in the finite displacement range, the joists were placed flat; thus bending occurred through their minor axis and finite displacements could be obtained for a medium length beam under moderate loads.

The beams were tested in a simply supported condition with a span of 2140 mm and subjected to a single concentrated load at midspan. The joists in the beams had dimensions of 140 x 37 mm and a modulus of elasticity, determined experimentally, of 15870 MPa. The size of the sheathing was 200 x 18 mm and had an axial and flexural modulus of elasticity equal to 5060 MPa and 9110 MPa respectively.

In the beam-column testing, the axial loads were applied only to the studs. Hence the sheathing was built 25 mm shorter than the studs so as to prevent the loading plates from touching the ends of the plywood. The axial loads were applied eccentrically in relation to the centerline of the stud; thus each beam-column was subjected to an axial force plus a bending moment at the ends. The beam-columns had a span of 1520 mm. The studs were 140 x 38 mm in size and had a modulus of elasticity of 9566 MPa. The sheathing had dimensions of 200 x 18 mm with an axial and bending modulus of elasticity of to 4518 MPa and 8332 MPa respectively.

The connectors employed in both cases were 6d bright common nails spaced every 180 mm. Being an important factor on the behavior of layered beams with partial interaction, the load-slip curve of the 6d bright common nails was carefully determined through a series of simple tests from which the parameters employed in Foschi's equation (Foschi and Bonac 1977) were determined . An average of each one of the parameters was then calculated from which the values for k, P_1, and P_0 were 4492 N/mm, 173 N/mm and 467 N respectively.

4. Comparison of the Finite Element Analysis with Experiments

The load versus midspan deflection relationship for beam 1 is given in Fig. (2). In this case, good agreement is still obtained for magnitudes of deflections substantially larger than the depth of the members since the depth of the joists for the beam was 38 mm.

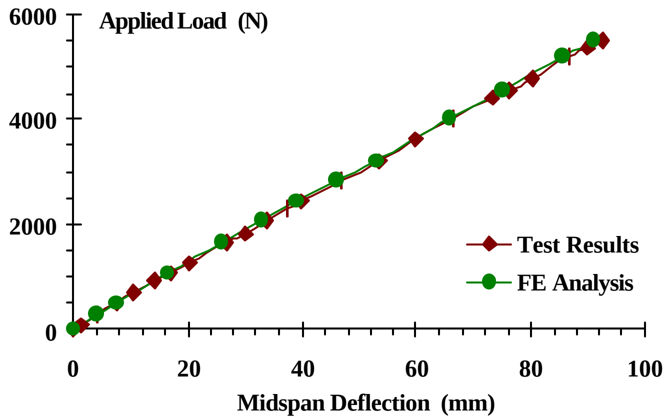

Figure 2. Load versus Midspan Deflection for Beam 1

Figure (3) depicts the comparative results of the midspan strain for beam 1 at the bottom fiber of the joist. Good comparison is achieved as long as the strains remain in the elastic region. For strains above the elastic limit (4000 micro), the finite element analysis predicts a stiffer behavior.

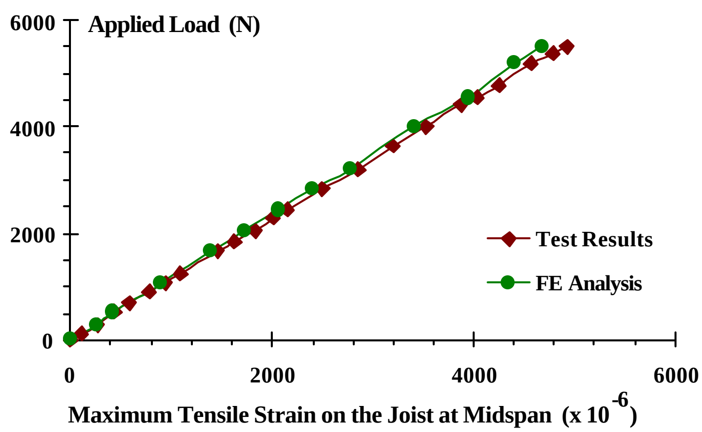

Figure 3. Load versus Maximum Tensile Strain on the Joist at Midspan for Beam 1

The axial load versus midspan deflection relationship for beam-column 1 is presented in Fig. (4). The second order effects shown by the test results are very well predicted by the finite element model. The maximum measured midspan deflections are on the order of the magnitude of the depth of the studs used in the beam-column.

Figure (5) presents a plot of the axial load versus the midspan strain on the stud at the interface sheathing-stud. The test results show a reversal in the sign of this strain from compression to tension as the load increases. The finite element analysis predicts very well not only the values of this strain but also the reversal in the sign of the strain.

The results for the maximum compressive strain on stud at midspan of beam-column 1 is presented in Fig. (6). Good correlation with the finite element model is once again achieved. This indicates that the hypothesis of linear elastic behavior for layered beam components is valid up to deformations of 4000 microstrains.

.

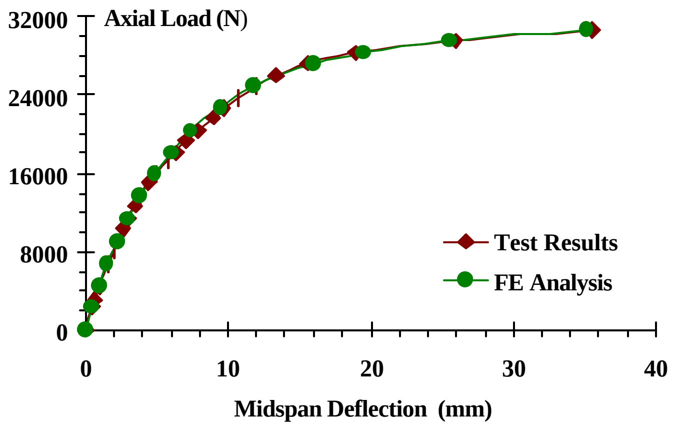

Figure 4. Load versus Midspan Deflection for Beam-Column 1

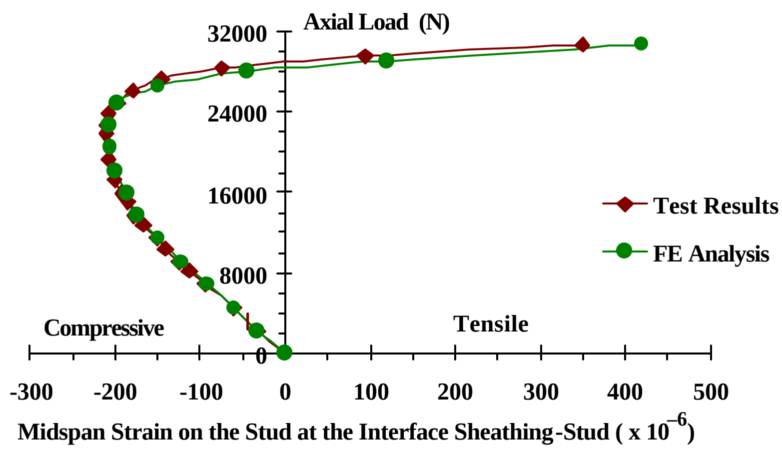

Figure 5. Axial Load versus Midspan Strain on the Stud at the Interface Sheathing-Stud for Beam-Column 1

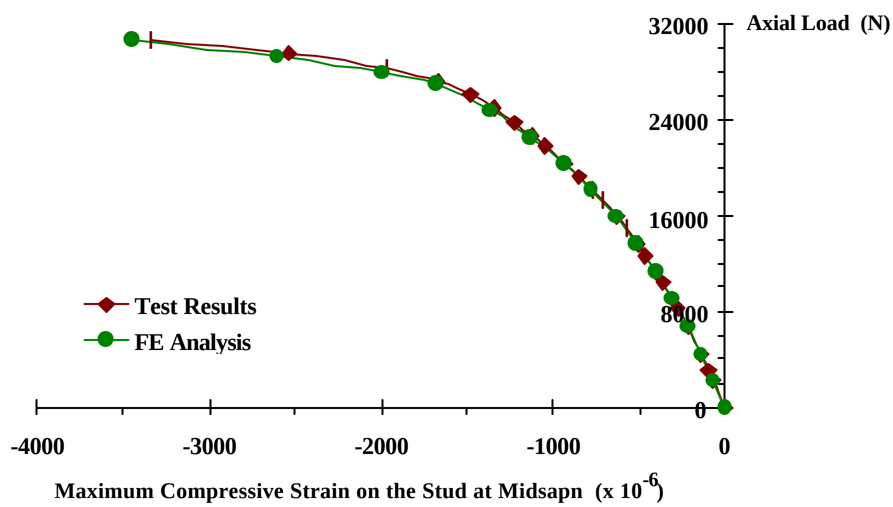

Figure 6. Axial Load versus Midspan Strain on the Stud at the Interface Sheathing-Stud for Beam-Column 1

5. Concluding Remarks

A consistent model for the analysis of two-layered beams with partial interaction and including the effects of finite displacements and nonlinear behavior for the connectors has been presented. By finite displacements it is meant displacements in the order of magnitude of the depth of the members. An energy formulation is applied from which the element stiffness matrices were derived. The element stiffness was incorporated into a finite element analysis package. Since the equilibrium algebraic equations are nonlinear, the Newton-Raphson scheme is used to solve them. Tests of two-layered wood beams and beam-columns are conducted to verify the model and very good agreement is achieved for deflections, strains and slips between the layers.

6. References

Calixto, J. M. F., 1991, "A Geometrically Nonlinear One-Dimensional Element for Analyzing Layered Systems with Interlayer Slip", Doctoral Thesis, University of Texas at Austin, 234 p.

Foschi, R. O., e Bonac, T., 1977, "Load-Slip Characteristics for Connections with Common Nails", Wood Science, Vol. 9, No. 3, pp 118-123.

Kamiya, F., 1988, "Buckling of Sheathed Walls: Nonlinear Analysis", ASCE - Journal of Structural Engineering Vol. 114, No. 3, pp. 625-641.

Ko, M. F., Goodman, J. R., Vanderbilt, M. D., Criswell, M. E., e Bodig, J., 1972, "Layered Beam Systems with Interlayer Slip", Structural Research Report No. 8, Civil Engineering Department, Colorado State University, Fort Collins, Colorado, 125 p.

Veljkoviv M., 1996, "Behaviour and Resistance of Composite Slabs – Experiments and Finite Element Analysis", Doctoral Thesis, Lulea University of Technology, Lulea, Sweden, 201 p.

Wheat, D. L., Vanderbilt, M. D., e Goodman, J. R., 1983, "Wood Floors with Nonlinear Nail Stiffness", ASCE - Journal of Structural Engineering, Vol. 109, No. 5, pp. 1290 – 1302.

7. Copyright Notice