DEVELOPMENT OF A THERMAL IDENTIFICATION TECHNIQUE USING TIME AND FREQUENCY DOMAIN

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Abstract. *This paper describes the development of a thermal identification technique to be applied in different materials (conductor and non-conductor). The technique, which estimates simultaneously the thermal diffusivity,α, and the thermal conductivity, λ, is based on signal treatment in both time and frequency domains of a same data. In the frequency domain, a dynamic model is defined by combining two heat flux and two superficial temperature. The thermal diffusivity is, then, obtained by identifying the delay of the frequency response of the model performed numerically. Once the thermal diffusivity is known the same data can be used to obtain the thermal conductivity in the time domain. In this case a least square function of measured and calculated temperature is used. The independent process for the thermal properties identification assures the obtained values of α and λ to be unique. One novelty of this technique is the numerical dynamic model that allows one or two-dimensional treatments and also the use of optimal sensor locations. The new technique is presented by simulating two different materials: a stainless steel (AISI304) and a polymer (PVC).*

Keywords. *Thermophysical properties estimation, Heat conduction, Optimization, Experimental Techniques, Inverse Problems.*

1. Introduction

The thermal property knowledge is essential to predict the behavior of any thermal system. Several examples can be given in this way: obtaining the temperature at interface of the tool-ship in a cutting machining, the thermal efficiency in a welding process, the isolate capacity of a coating tool, or a moving source in a phase change process. Each one of these problems could not be carried out without knowing the values of the thermal conductivity or thermal diffusivity.

Several methods can be found in literature about obtaining these properties. It can be cited the hot-wire method (Blackwel, 1954, Log, 1993) the flash method (Parker et all, 1961), parameter estimation technique (Beck and Arnold, 1977), (Blackwel et all, 2000), impedance techniques (Guimarães et all, 1995) or correlation techniques (Lima e Silva et all., 1998). The flash method is one of the best methods to obtain the thermal diffusivity while the hot wire is suitable to estimate both properties but are employed more efficiently for thermal conductivity estimating in non metallic materials. The parameter estimation can also be used to estimate simultaneously the thermal properties for different types of material. However some efforts should be done to avoid low sensitivity regions for obtaining both properties with confidence. Lima e Silva et all (1998) and Guimarães et all (1995) have proposed the simultaneous estimation in independent ways. They use two independent objective functions for each one of the properties, thermal conductivity and thermal diffusivity, what assures the values estimated to be unique. In the first technique the properties are estimated in time domain case while the frequency domain is used in the last technique. However, both techniques have some difficulties in the metallic material application.

The additional difficulty that appears in metallic application is due to the thermal contact resistance, low sensitivity and the difficulty of obtaining two-dimensional experiments. The technique proposed here tries to avoid these problems. Two different materials (polymer and conductor) are simulated in order to demonstrate the wide applicability of the technique. Besides, the time and frequency domain are mixed to estimate the thermal properties simultaneously and in an independent way. Some advantages can also be mentioned, such as more freedom in the choice of thermocouple locations, easy and low cost apparatus, the short time duration of the heat flux input, and the good accuracy of the obtained results.

2. Theoretical fundamentals

The technique proposed here is based on the use of an input/output dynamical system (Fig. 1) that is obtained from a thermal model (Fig. 2). In Figure (1), ϕ_i represents the heat flux, T_i the temperature and i the index to describe the location of the respective heat flux and the temperature in the sample. The thermal model can be given by a one-dimensional or two-dimensional model as shown in Fig. (2). In both cases the input signal **X** and output signal **Y** must be obtained by solving the respective heat diffusion equation with the appropriate boundary conditions. The one dimensional heat diffusion solution can be analytically obtained just involving the variables ϕ_i and T_i in lower and upper surfaces (Guimarães et all, 1995). Yet, for the two-dimensional case, the lateral boundary conditions (convection) need to be known additionally. Once in this work, only simulated case are studied it will be considered that the heat transfer coefficient, h, is known and equal to 10 W/m^{2}K without generality losses. In practical measurements, this boundary condition can be evaluated by using a good insolating material in the border. It means h could be assumed zero.

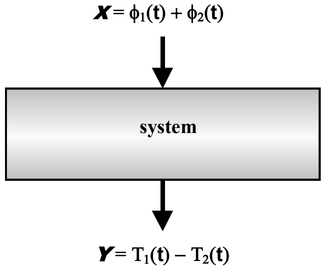


Figure 1. Input/output dynamic system

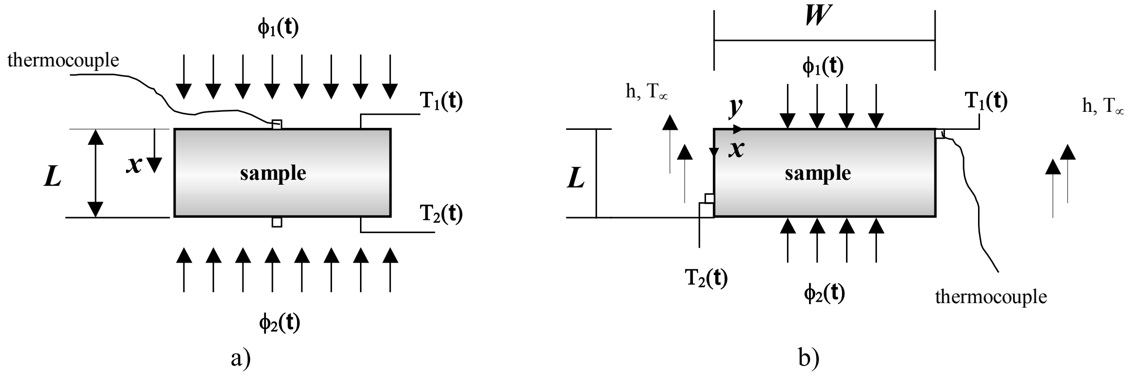


Figure 2. Thermal model scheme: a) one dimensional, b) two-dimensional

The heat conduction problem for each model (Fig. 2) can be described by the heat diffusion equation subject to the suitable boundary conditions.

One-dimensional thermal model

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \tag{1a}$$

subject to the boundary conditions

$$-k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = \phi_1(t) \qquad -k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=L} = \phi_2(t) \tag{1b}$$

and initial condition

$$T(x,0) = T_0 \tag{1c}$$

Two-dimensional thermal model

$$\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T(x,y,t)}{\partial t} \tag{2a}$$

subject to the boundary conditions

$$-k\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=0} = \phi_1(t) \qquad k\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=L} = \phi_2(t) \tag{2b}$$

$$+k\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{x=0} = h\left[T(x,y,t)-T_\infty\right] \qquad -k\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{x=W} = h\left[T(x,y,t)-T_\infty\right] \tag{2c}$$

and initial conditions

$$T(x,y,0) = T_0 \tag{2d}$$

In contrast of previous work Guimarães et all (1995) or Lima e Silva et all (1998) presented an analytical solution for the one dimensional case, in this work not only has the two dimensional model performed numerically but also the one dimensional. The great advantage of this procedure is the easiness in data experimental manipulation.

3. Method proposed

As in Guimarães et all (1995) a frequency response, H, can be defined from the input/output dynamic system, as shown in Fig. (1). In this case, it can be written

$$H(f) = \frac{\mathbf{Y}(f)}{\mathbf{X}(f)} = \frac{\phi_1 + \phi_2}{\theta_1 - \theta_2} \tag{3}$$

In this domain, frequency variable f replaces the independent variable t. The frequency response, $\mathbf{X}(f)$ and $\mathbf{Y}(f)$ are obtained by applying the Fourier transform to $X(t)$ and $Y(t)$ signals, it means,

$$\mathbf{Y}(f) = \int_0^\infty \boldsymbol{Y}(t) e^{-j2\pi f t} dt \tag{4}$$

$$\mathbf{X}(f) = \int_0^\infty \boldsymbol{X}(t) e^{-j2\pi f t} dt \tag{5}$$

where $j=\sqrt{-1}$ is the imaginary unit.

The response frequency can, then, be obtained by multiplying Eq. (3) by the complex conjugate $\mathbf{X}(f)$,

$$Z_e(f) = \frac{\mathbf{X}^*(f)\mathbf{Y}(f)}{\mathbf{X}^*(f)\mathbf{X}(f)} = \frac{S_{xy}(f)}{S_{xx}(f)} \tag{6}$$

Fourier Transforms of $\boldsymbol{X}(t)$ and $\boldsymbol{Y}(t)$ are performed numerically by using the Cooley-Tukey algorithms (Discrete Fast Fourier Transform), where S_{XY} is the cross-spectral density of $\boldsymbol{X}(t)$ and $\boldsymbol{Y}(t)$ and S_{XX} is the auto-spectral density of $\boldsymbol{X}(t)$ (Bendat and Piersol, 1986). Equation (6) is more suitable to calculate $H(f)$ due to the more stable behavior of the spectral density, with f. In polar form, $H(f)$ can be written as:

$$H(f) = |H(f)| e^{-j\varphi(f)} \tag{7}$$

where, $H(f) = |S_{xy}(f)| / S_{xx}(f)$ is the modulus of $H(f)$, and $\varphi(f) = \varphi_{xy}(f)$ is the phase angle. In the same way, $|S_{xy}|$ and φ_{xy}, are the modulus and phase of S_{xy}, respectively. The auto-spectral density function S_{xx} is a real function. The transformed frequency response in the f-x plane, is, thus, a complex variable, with a modulus $|H|$ and a phase factor φ.

3.1. Thermal diffusivity estimation: frequency domain

The great convenience of working in the frequency domain is the fact that phase angle is a function that depend only on the thermal diffusivity α (Guimarães et all, 1995). The basic idea here is the observation that the delay between the experimental and theoretical temperature is an exclusive function of α. So, the minimization of an objective function, S_p, based on the difference between of the experimental and calculated values of the phase is the way to determine the thermal diffusivity. This function can, then, be written by

$$S_p = \sum \left(|\varphi_e| - |\varphi_t|\right)^2 \tag{8}$$

where φ_e and φ_t are the experimental and calculated values of the phase H respectively. The theoretical value is calculated by using the mathematical model, ***i.e.***, using the solution of the heat diffusion problem to one or two dimensional model, Eq.(1) or Eq.(2). In this case the phase angle can be obtained by

$$\varphi_t = \arctan\left[\mathrm{Im}(H_t)/\mathrm{Re}(H_t)\right] \tag{9}$$

where *Re(H)* and *Im(H)* are, respectively, the real and imaginary parts of *H*.

The values of α will be supposed to be those which minimize Eq. (8). In this work this minimization is done by using the golden section method with polynomial approximation (Vanderplaats, 1986)

3.2. Thermal conductivity estimation: time domain

Once the thermal diffusivity value is obtained, a usual objective function based on temperature error can be used to estimate the thermal conductivity. In this case, there are no identifiability problems once just one variable is being estimated. Therefore, the variable λ will be supposed to be that which minimizes the least square function, S_{mq}, based on the difference between the calculated and experimental temperature. Therefore the objective function can be written by

$$S_{mq} = \sum_{j=1}^{s} \sum_{i=1}^{n} \left[Y_j(i) - T_j(i)\right]^2 \tag{10}$$

where $Y_j(t)$ is the experimental temperature and $T_j(t)$ is the calculated temperature, n is the total number of measurements and s represents the number of sensor.

As mentioned before, the values of $T_j(t)$ come from the one of the models given by the Fig. (2) and the optimization technique used is the golden section with polynomial approximation.

4. Numerical simulation data

Experimental data are supposed to be simulated by using numerical values. The procedure used here simulates these measured temperatures introducing random errors ε to the exact temperatures as

$$Y(x_j, y_k, t) = T_{\text{exact}}(x_j, y_k, t) + \varepsilon \tag{11}$$

where the exact temperature T_{exact} is determined from the solution of the direct problem, Eq.(1) or Eq.(2), by using the thermal properties and geometrical dimensions presented in Tab. (1). The value of ε is within $\pm$ 0.3K, which corresponds to 6% of $\mathbf{Y}_{\text{max}}$.

Table 1 - Thermal properties and geometrical dimensions

Sample	Model	Dimension (mm)		$Y(x_j,y_j)$ location (mm)		time sample (s)	λ (W/mK)	α (m^2/s)
		L	W	x	y			
AISI 304	1D	20	-	0,0	17,0	1	14,9	$3{,}95\ 10^{-6}$
	2D	20	50	50,0	3,0	1		
	1D	20	-	0,0	17,0	16	0,16	$1{,}30\ 10^{-7}$
PVC	2D	20	50	50,0	3,0	16		

Figure (3) and Figure (4) present typical heat flux signals that were simulated to be imposed each sample. The time duration of heating was approximately 50 seconds. For each experiment simulated 1024 points were taken with a time interval of 1 or 16s as shown in Tab. (1) Figures (5)-(8) present the temperatures response for each sample and its respective thermal model.

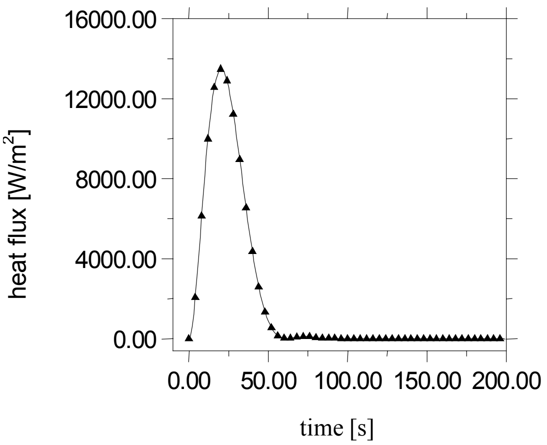


Figure 3. Heat flux evolution applied to AISI 304

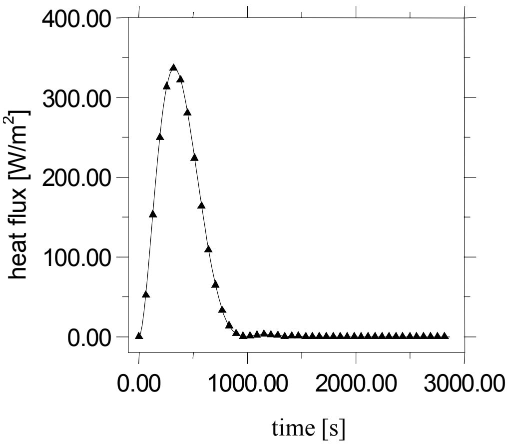


Figure 4. Heat flux evolution applied to PVC

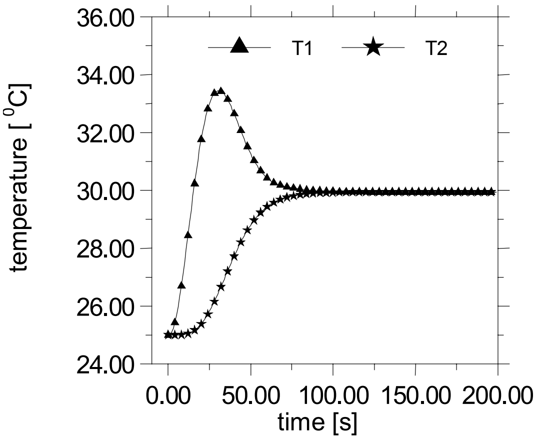


Figure 5. Temperature evolution to AIS304 sample: 1D model

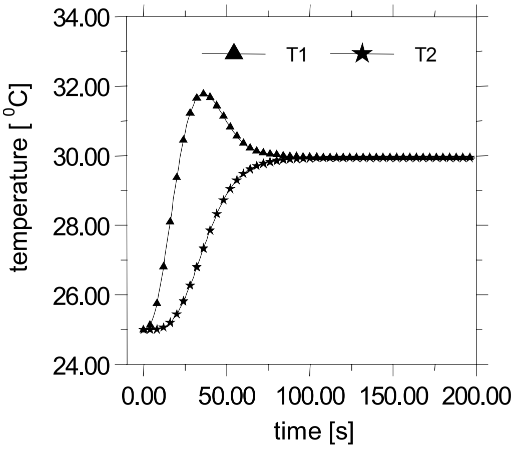


Figure 6. Temperature evolution to AIS304 sample: 2Dmodel

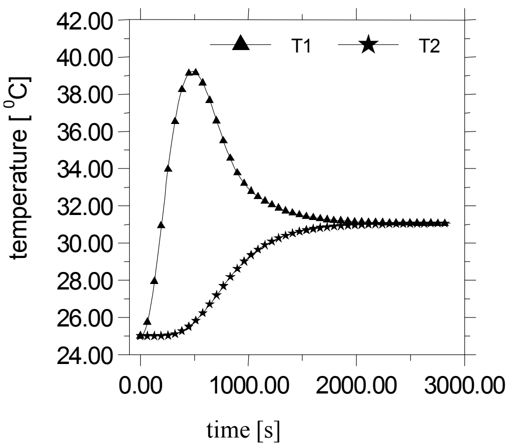


Figure 7. Temperature evolution to PVC sample: 1D model

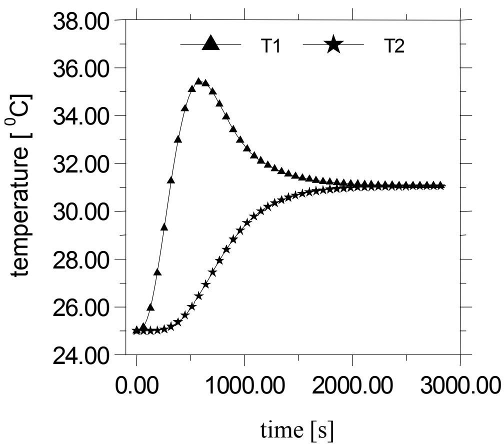
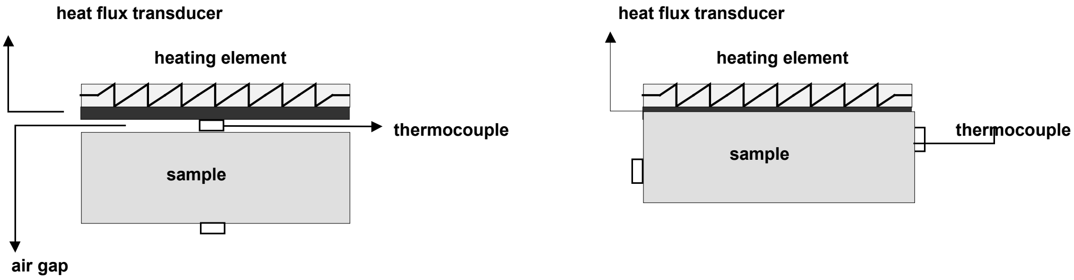


Figure 8. Temperature evolution to PVC sample: 2D model

5. Experimental considerations and sensitivity analysis

A great attention should be given when using experimental techniques in conductor materials. For example, a model based on one-dimension hypothesis is harder to execute than for non-conductors samples. In addition to the two dimensional difficulty, there is the thermal contact resistance problem. Whereas for non-metallic materials this resistance can be neglected, for conductors this problem is delicate and needs to be considered. A method that considers two-dimensional effects and has an experimental apparatus that minimizes the contact resistance can be an alternative solution. In order to illustrate these points, Figure (9) shows schematically the thermal contact problem for one-dimensional model apparatus and a possible solution given by a two-dimensional version with alternative thermocouple location.



a) b)

Figure 9. Apparatus experimental scheme: a) one-dimensional assembly; b) two-dimensional assembly

It can be observed that in the two-dimensional alternative scheme (Fig. 9b) the thermal contact problem is minimized beside the fact that this model is able to consider any lateral heat losses. To assure the success of the two-dimensional model is necessary to investigate its ability to estimate the parameters. Sensitivity coefficient analysis is one way to demonstrate this. These coefficients can be defined as the first derivative of the dependent variable ($\boldsymbol{\varphi}$ or $\boldsymbol{T}$) with respect the parameter to be estimated α or λ (Beck and Arnold, 1977).

Sensitivity coefficients are very important because they indicate the magnitude of change of the response the function of interest due to perturbations. These coefficients play an important rule in the identifiability problem. Several examples can be given for which one cannot uniquely estimate all the parameter involved. One identifiability criterion given by Beck and Arnold (1977) says that *parameter can be estimated if the sensitivity coefficient over the range of the observation are not linearly dependent.* Besides this behavior, the sensitivity coefficient should have larger values as possible. This criterion is used to determine if the parameter can be simultaneously estimated without ambiguity. In this case, these coefficients can be given by:

$$\overline{X}_{T,\alpha} = \frac{\partial T}{\partial \alpha} \tag{12}$$

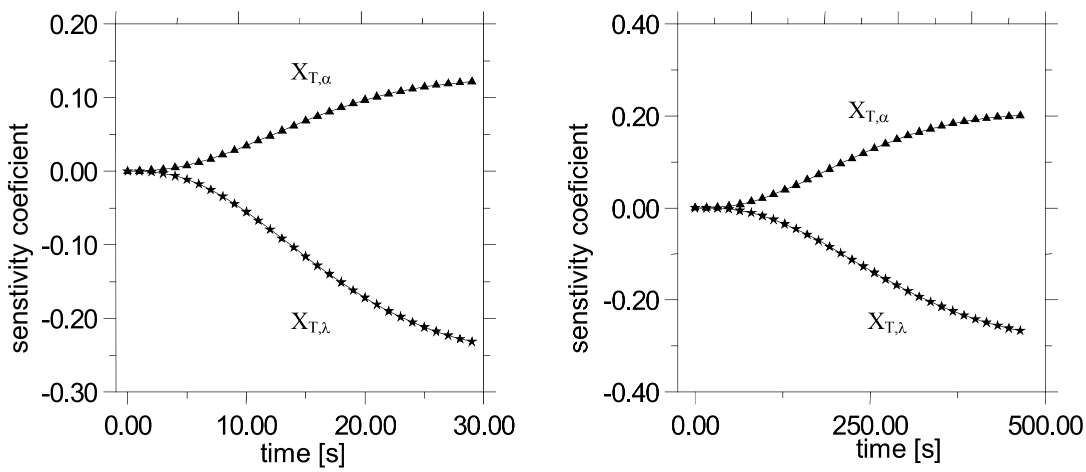
$$\overline{X}_{T,\lambda} = \frac{\partial T}{\partial \lambda} \tag{13}$$

where $\overline{X}_{T,\alpha}$ and $\overline{X}_{T,\lambda}$ represent the sensitivity coeficient of the phase angle with respect to thermal diffusivity and thermal conductivity respectively in time domain. In order to obtain a better comparison these coeeficients are normalized as

$$X_{T,\alpha} = \frac{\alpha}{T}\frac{\partial T}{\partial \alpha} \tag{14}$$

$$X_{T,\lambda} = \frac{\lambda}{T}\frac{\partial T}{\partial \lambda} \tag{15}$$

where $X_{\varphi,\alpha}$ and $X_{\varphi,\lambda}$ represent the normalized version of $\overline{X}_{T,\alpha}$ and $\overline{X}_{T,\lambda}$, respectively. The behavior of these sensitivity coefficients are shown in Fig. (10).



a) b)

Figure 10. Sensitivity coefficients, $X_{T,\alpha}$ and $X_{T,\lambda}$ a) 2D thermal model: AISI304 b)2D thermal model PVC

It can be noted in Fig. (10) that the sensitivity coefficients, $X_{T,\alpha}$ and $X_{T,\lambda}$ seem to be correlated once the shape of both are nearly symmetrical. This fact indicates that α and λ can not be estimated simultaneously if just the range of temperature in time domain is used. This behavior occurs for both samples (PVC and AIS304) and thermal model (1D and 2D). The comparison between $X^{T}{}_{\alpha}$ and $X^{T}{}_{\lambda}$ to the 1D thermal model is omitted here due the space limitation.

The behavior of $X_{T,\alpha}$ and $X_{T,\lambda}$ in time domain is the main motivation to propose the thermal properties estimation in two independent ways: frequency domain to estimate α and time domain to estimate λ. A new sensitivity coefficient can, now, be defined, as the coefficient of phase angle with respect to the thermal diffusivity. This coefficient can be written by

$$\overline{X}_{\varphi,\alpha} = \frac{\partial \psi}{\partial \alpha} \tag{16}$$

or in a normalized version as

$$X_{\varphi,\alpha} = \frac{\alpha}{\psi}\frac{\partial \psi}{\partial \alpha} \tag{17}$$

Figure (11) and Figure (12) present the behavior of coefficients $X_{\varphi,\alpha}$ and $X_{T,\lambda}$ for both samples simulated. It should be mentioned that if only one variable for each minimization is necessary, the process of estimation is independent and therefore there is no limitation related to the behavior of the coefficients be or not to be proportional. Another information can also be inferred from Fig. (11) and Fig. (12). Once $X_{\varphi,\alpha}$ has larger values than $X_{T,\lambda}$ the thermal conductivity estimation in time domain is expected to be more sensitive to measurement uncertainties. The exclusive dependence of thermal diffusivity in phase angle was proved by Guimarães et all (1995) and will not be presented here.

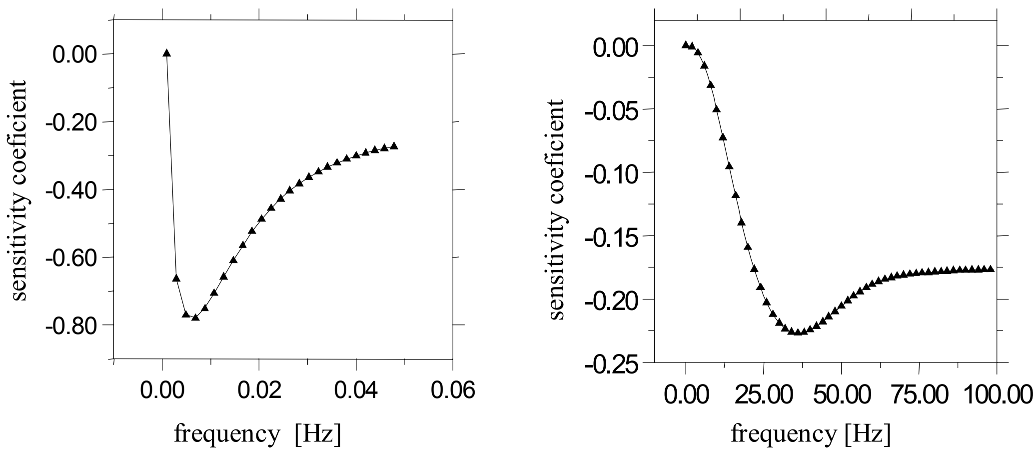


Figure 11. Sensitivity coefficients to AIS304 sample and 2D thermal model a) $X^{\varphi}{}_{\alpha}$ b) $X^{T}{}_{\lambda}$

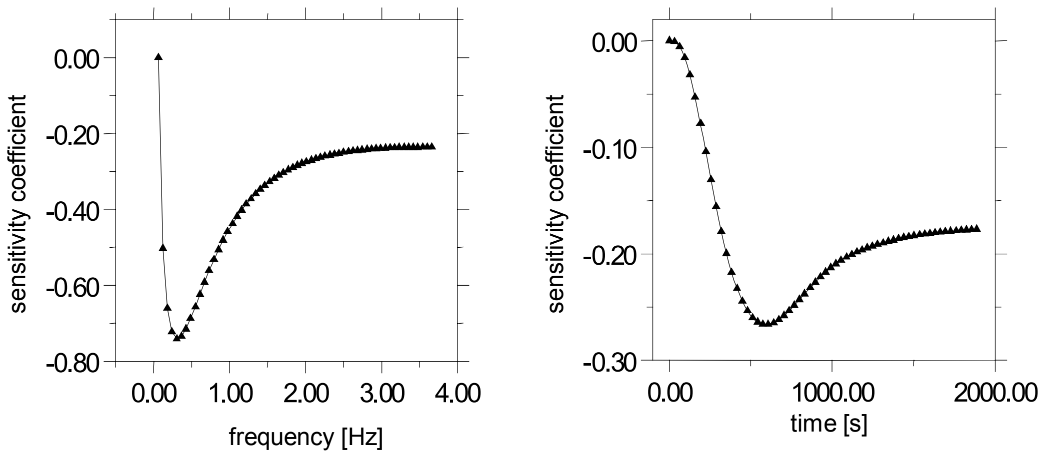
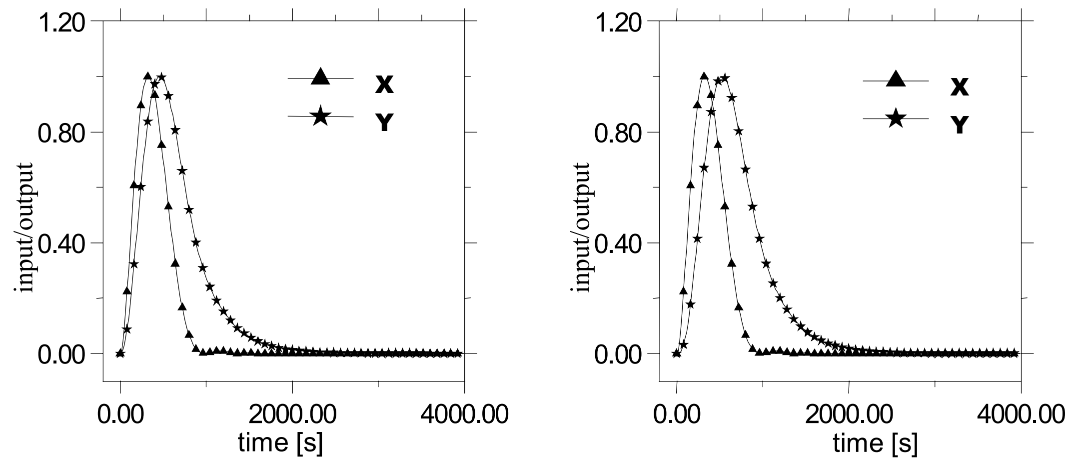


Figure 12. Sensitivity coefficients to PVC sample and 2D thermal model a) $X^{\varphi}{}_{\alpha}$ b) $X^{T}{}_{\lambda}$

6. Results

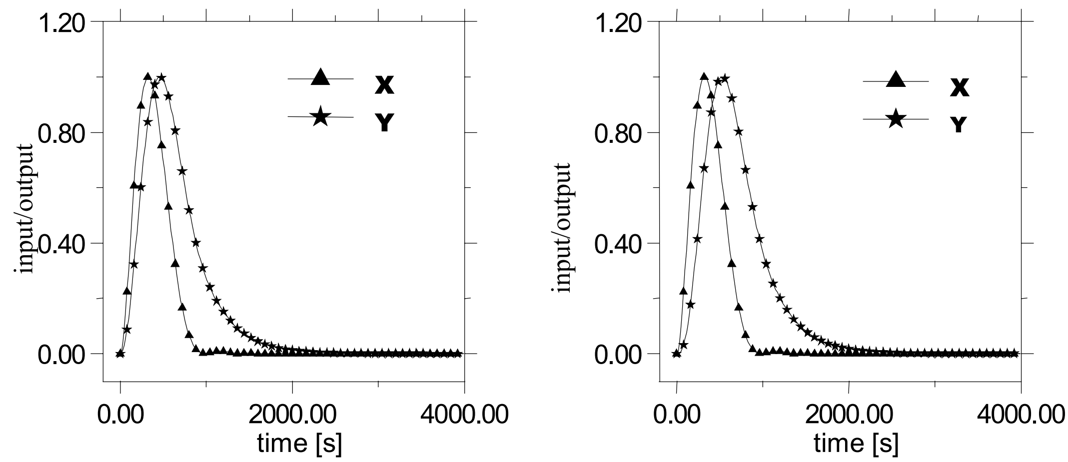
Typical input and output signals are shown in Fig. (13) and Fig. (14) and Tab. (1) for both samples simulated. One and two dimensional model normalized results are shown in the same figure to allow a better comparison. The signals have dimensionless values and are defined by

$\mathbf{X} = \dfrac{\mathbf{X}}{\mathbf{X}_{max}}$ and $\mathbf{Y} = \dfrac{\mathbf{Y}}{\mathbf{Y}_{max}}$ where $\mathbf{X}_{max}$ and $\mathbf{Y}_{max}$ represent the maxim values to input/output signals.



a) b)

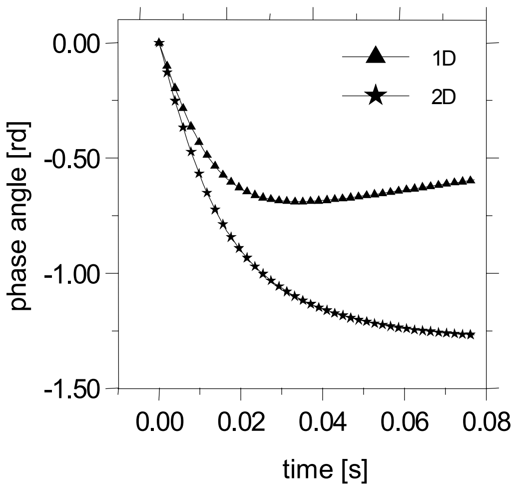
Figure 13. Input/output signals to AISI 304 sample a) 1D thermal model b) 2D thermal model



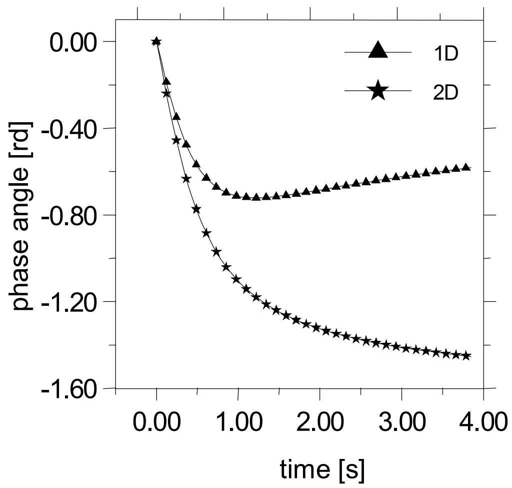
a) b)

Figure 14. Input/output signals to PVC 304 sample a) 1D thermal model b) 2D thermal model

Phase angle for both samples and models are shown in Fig. (15). It can be noted that the phase angle has a larger range in thermal diffusivity dependence than one-dimensional model what assures better results of estimation. The results of thermal diffusivity estimation process are presented in Tab. (2) and Tab. (3).



a)



b)

Figure 15. Phase angle a) AISI 304 sample b) PVC sample

Table 2 presents results for α estimation without adding random errors. It can be noted that despite the large range of search interval or initial guess the estimated value to the thermal diffusivity corresponds exactly to the expected value (Tab. 1). The same behavior can be noted for thermal conductivity estimation without random errors, Tab. (3). This procedure indicates the ability to estimate both properties simultaneously.

Table 2. Thermal diffusivity estimated process whitout adding errors

Sample	model	search interval	iteration	objective function [rd^2] initial	final	α (m^2/s)
AISI 304	1D	10^{-8} to 10^{-4}	30	0.4913608	0.0	3,95 10^{-6}
	2D			0.7271634	0.0	
PVC	1D	10^{-8} to 10^{-6}	30	11.49199	0.0	1,30 10^{-7}
	2D			1.050394	0.0	

Table 3. Thermal conductivity estimated process whitout adding random errors

Sample	model	Search interval	Iteration	Objective function [$^0C^2$] initial	final	λ (m^2/s)
AISI 304	1D	0,1 to 100	30	78532.04	0.0	14,9
	2D			371643.3	0.0	
PVC	1D	0,1 to 100	30	0.1123090	0.0	0,16
	2D			2.185268	0.0	

Before to deal with a real case, an error analysis should be done. So as discussed before, the influence of uncertainty measurement can be verified by adding random noise to the exact temperature calculated from the model, Eq. (11). Table (4) and Table (5) present the results for two-dimensional model. The thermal model is proposed by Fig. (2) and simulated thermocouple locations are given in Table (1).

Table 4. Thermal diffusivity estimated process with noise errors addition

sample	Search interval	Iteration	objective function [rd^2] initial	final	α (m^2/s)	error (%)
AISI 304	10^{-8} to 10^{-4}	30	0.4850698	1,44 x 10^{-10}	3,8745 10^{-6}	1.91
PVC	10^{-8} to 10^{-6}	30	11.49744	7,255869 x10^{-9}	1,296185 10^{-7}	0.29

It can be observed in Tab. (4) that the addition of random errors has resulted in an uncertainty of order of 2% in the estimated value to thermal diffusivity for AISI 304 and 0.29% for PVC sample (Tab. 4). Table 5 presents the estimation process to the thermal conductivity. The initial tentative to estimate λ couldn't be considered with success once the value obtained presented up 25% of uncertainty. The difference in the estimation behavior can be credited to the procedure of estimation and random error characteristics. The random error has more effect to higher frequency while the thermal diffusivity is determined in lower frequency. However, the effect of uncertainty is over all data in time domain. Once the temperature difference is used to estimate the thermal conductivity the noise of a single experiment can produce serious consequences. These effects can be minimized in a real process trough raising the experiment number. These procedure reduce the influence of random error once additive errors, in contrast to multiplicative errors, do not vary greatly with the independent variables and its value can be considered zero on the average. Fitting a curve adjusts in simulated data errors given by Eq. (11) here simulates increasing of number of experiment. The results are shown in Tab. (5). As discussed, the adjustment procedure doesn't affect the thermal diffusivity estimation due to the frequency range of estimation.

Table 5 - Thermal conductivity estimated process with noise errors and fitting adjustment

sample	Search interval	Iteration	Objective function [$^0C^2$] initial	final	λ (W/mK)	error (%)
AISI 304	0,1 to 100	30	9,311774 10^6	0.131979	14,70823	1,28
PVC	0,1 to 100	30	905,1586	0.039147	0,159821	0,11

7. Conclusion

The development of a thermal identification technique to be applied in different materials (conductor and non-conductor) is presented. The technique based on signal treatment in both time and frequency domain of a same data has shown excellent results considering simulated data. The independent process for the thermal properties identification assures the obtained values of α and λ to be unique. The dynamic model used allows one or two-dimensional treatment and the use of optimal sensor locations. The new technique has presented good results for two different simulated materials: a stainless steel (AISI304) and a polymer (PVC).

8. Acknowledgement

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