Methodology Development Based on Robust Design and Statistical Optimization Techniques Applied in Mechanical Components

Silveira, Z. de C.
Department of Mechanical Design (DPM), Faculty of Mechanical Engineering (FEM) – UNICAMP
silveira@fem.unicamp.br

Cavalca, K.L.
Department of Mechanical Design (DPM), Faculty of Mechanical Engineering (FEM) - UNICAMP
katia@fem.unicamp.br

Abstract. *The current systems are designed considering multi-disciplinary aspects. Their development and analysis expose the designer to a series of unknown parameters from several sources such as material properties, environmental and operational conditions. Therefore, the qualification and quantification of these inherent sources of design uncertainties become very important in several aspects in the context of design development. In this way, a system is reliable and robust if it allows a certain range of uncertainties before the first failure occurs. In this work it is proposes the development of a methodology that can identify the sources of uncertainties and parameters that largely influence the whole design. An initial study focuses on a simple oscillatory system that consists of a mass-spring-damper system, which it is extending to study in a rotor system. The full methodology developed consists in choice of some elements of the mechanical system and after this, it is applied a specific design of experiments for identifying the critical parameters. If there were many parameters it is used a factorial planning or orthogonal arrays, which are a kind of screen experiments. Once identified the critical parameters, can be used the factorial planning to study the interactions between these parameters. Afterwards, the development carries on with polynomial models (linear and quadratic) that should fit the experimental results from the factorial design. Once the critical parameters are obtained a search process must be done to find the optimum regions maximum, minimum or singular point. The steps used in the search interval occur along maximum or minimum lines that describe a region of interest or experimentation. The connection between optimization and statistics dates back at least to the early part of the 19th century and encompasses many aspects of applied and theoretical statistics, including hypothesis testing, parameter estimation, model selection, design of experiments and control process and product. A sensitivity analysis takes place using canonical analysis and optimization methods for parameter fittings such as the Gradient Methods. In this case it is used concepts from Response Surface Methodology for estimating and quantifying the allowable ranges of variability to each parameter with respect to the critical parameters in the robust design concepts.*

Keywords: *design of experiments, robust design, optimization methods, response surface methodology, mechanical systems.*

1. Introduction

The first studies about statistics applied in engineering occurred through Fisher's work in the 1920´s and 1930´s firmly established the role of statistics in experimental design and, vice versa, the role of experimental design in statistics. Fisher was employed to analyze data from studies conducted at Rothamsted Experimental Station in England, but some important questions could not be answered because of inherent lack of robustness in the planning of many of the experiments. Box (1980) described Fisher's work on the design of experiments, and how much of it was inspired by problems of field experimentation. He developed his insights concerning randomization, blocking and replication; he invented new classes of experimental designs and he worked together with scientists who applied his ideas in their experiments. Since Fisher first introduced statistical principles of experimental design, much useful statistical research has been done. The term *robust* was introduced by Box in 1953 in the statistical literature to describe procedures that give good results even though there might be violations in the assumptions upon which these procedures are based. The examination of standard statistical techniques to determine their sensitivity to assumptions and the development of new techniques that are less sensitive have been focal points of statistical research in the last two decades. Experimental design is an area in which it is particularly compelling to investigate questions of robustness because a researcher's assumptions about the experimental process are often very important in determining the design. Moreover, the design must be chosen before the data are collected and so cannot be discarded if the data indicate that the assumptions are seriously incorrect. The assumption that underlies most research work in experimental design is that the experiment can be adequately described by an equation of the form:

$$\text{Response} = f(x) + \varepsilon \tag{1}$$

Where the model states the effect of the predictor variables on the response variable and the error describes the general form of departure from the model. In the terms of engineering applications, it is considered that the technological systems are designed to perform definite functions during the design concept. However, all of those designs either have several uncertainties from uncontrollable factors or same errors in the mathematical modeling of the system, so much so that the mathematical models of systems can be represented in two different ways: (a) Deterministic models, in which all parameters are known and; (b) Probabilistic models, in which the mathematical model explicitly incorporated. In the, 50´s an engineer and researcher called Taguchi developed the concept of robust design, whose objective was to reduce the influence of the external factors, making the product insensitive to the noise of large factors (Padke, 1985). Therefore, considering that the fundamental principle in a robust design is to improve the quality of a product/system by minimizing the effects of variation without eliminating its causes and as for the robust design problems, there are two principal goals: optimize the means and minimize the variation of performance. These goals can be reached by mathematical and statistical techniques that make possible to measure the sensitivity in relation to parameter variation. The aim of this paper is to develop a methodology based on the design of experiments and optimization techniques to identify critical parameters of design and to optimize some these parameters into a confidence range.

2. Design of Experiments

The planning or experimental delineation represents a set of tests with scientific and statistical criteria, whose objective is to determine the influence of several variables in the results of a specific system or process. According to Montgomery (1991), the experimental planning can be divided in the following way: a) determination of the variables that may influence the results more; b) attribution of values to variables of influence in order to optimize results and; c) to attribute values to these variables, minimizing the influence of uncontrollable parameters. The advantages of using statistical techniques of experimental planning are to lead to the reduction of the number of tests, without affecting the information quality. A simultaneous study of several variables can be made by separating their effects and determine the validity of the results. The research can be carried out in an iterative process of new results upgrade. The system can be represented and studied through mathematical expressions, and the elaboration of the results can also have references in qualitative analysis (Box and Draper, 1987). In mechanical engineering, more specifically in mechanical design the expanded factorial design is the most indicated, once during the study of mechanical systems there are many parameters uncertain. Factorial design first development by Fisher and Yates at Rothamsted, are one of the major contributions of statistical insight into experimental design (Box and Draper, 1987). Their essential feature, the simultaneous study of several factors, is a marked departure to the common idea that experimenters should vary only one factor at the time. As Fisher in 1926 observed, factorial designs offers many advantages: each experimental run gives information of several factors, not just one; the experiment yields as much information about each factor as though it alone had been varied; valuable additional information is available through the ability to check for possible interactions among the factors. In the event that no interactions are found, there is a much broader base for generalizing conclusions on the main effect of a factor, since the effect has been observed in a variety of experimental conditions. Finney (1945) extended the factorial design for fractional design, which allows the researcher to study main effects and low-order interactions of several factors in far fewer runs than required to complete the full factorial designs by sacrificing the ability to estimate high-order interactions. Fractional factorial designs thus offer great economy of time and resources when, as is often the case, high-order interactions are negligible. Box and Hunter in your works described in detail the theory and application of 2^{k-p} fractional factorial designs. Recent research on factorial designs has considered several problems, including incomplete factorials, weigh designs, screening designs, asymmetric factorials and blocking schemes.

3. Robust Design

Taguchi's methods have been used to assist the preliminary design of the product, to reduce the number of evaluation functions, or to get an optimized starting point, as well as the continuous improvement of different aspects of the design process. The objective is to determine control variables, parameters that satisfy the basic design functions such as cost reduction and good technical performance. Meanwhile, it makes sure that the variability in the manufacturing process, material and operational environment, has a minimum effect on the expected design performance. Thus, the design has robust functions respect to the variability of these uncontrollable perturbation factors. Critics suggest that other types of designs such as response surface designs are equally effective. Thus, Taguchi's methods attempt to identify the factors that most influence the system measurements. They do not focus on the reason why this happens. In Taguchi´s methods the main objective is locating the most significant variables that influence the controlling parameters system. On the other hand, alternative designs using fractional factorial experiments attempt to identify which components cause the variability, and how they contribute to it. Taguchi´s method studies one-factor-at-a-time, but is most efficient to estimate the effects of several variables simultaneously.

3 Optimization in Simulation

Mathematical models play an important role in the description and analysis of data. Valid inferences can be made with the help of suitable mathematical models for the phenomenon under study making the understanding of the model simulation possible. The development of a model for a given system requires the use of the available knowledge of the system. When modeling the system, a mathematical language is normally used and its performance of the proposed model is studied through simulations. Therefore, all information available from the system under study is used to validate the proposed model that is showed in Figure 1. Optimization techniques are required in the validation of models. This is the reason why they are used in the specification of models. So, the optimization is basically dependent on the criteria used in a given situation. The same design may lead to different solutions depending upon the criteria of optimality utilized, which in some cases are mathematical solution.

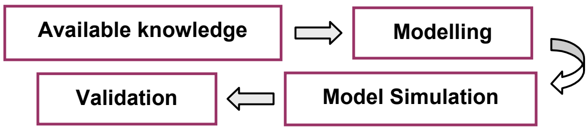


Figure 1- Information by optimization in simulation.

In addition, the modern theory of experimental design is being used simulation experiments. Classical designs such as those of randomized block, Latin square, fraction factorials and factorial design were developed for applications to agricultural, biological and industrial problems. However, they can be used in system simulation experiments, as well, but in this case we are concerned with designs for regression experiments. Suppose a response y depends on a variable x that can have levels under the control of the experimenter. If the number of observations is fixed, the experimenter in knowing based on specific optimality criteria. Where the response y is to be observed at a level x. Depending on the objective of the experimentation, these criteria will be different. A large number of optimality criteria have been developed for the regression experiments. For example, suppose the experimenter observes y_i, x_{1i}... x_{ki}, for a given subject i, i=1,2,..., n, where y_i denotes some kind of response and x_{1i},..., x_{ki} denote k levels of independent variables x_i. For this set of observations, we may assume the linear model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki} + \varepsilon_i \tag{2}$$

Where, β_1, β_2,..., β_k are unknown parameters and ε_i are random errors assumed uncorrelated, but having the same variance σ^2 . Let:

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ y_n \end{pmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \qquad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix} \tag{3}$$

In matrix notation, the linear model (2) is: $Y = X\beta + \varepsilon$.

The least square criteria for estimating β requires that we find: $\min_{\beta}(Y - X\beta)^T(Y - X\beta)$,

That can be estimated the parameters of polynomial function:

$$\beta = S^{-1}XY \tag{4}$$

Where, $S = X^T X$, and S are assumed to be nonsingular. The estimate of σ^2 is usually taken as:

$$\hat{\sigma} = \frac{1}{n-k-1}(Y - X\hat{\beta})^T(Y - X\hat{\beta}) \tag{5}$$

Note that for a given: $\{x_0\}^T = \{x_{10} \quad x_{20} \quad . \quad . \quad x_{k0}\}$ and, the predicted y is:

$$\hat{y} = (1, x_{10}, x_{20}, \ldots, x_{k0})\hat{\beta} \tag{6}$$

4. Response Surface Methodology (RSM)

The method (RSM) originally developed by Box and Wilson (1951) *apud* Box and Draper [1987] attempts to determine the shape of the response function and its sensitivity to the parameter design factors. It determines maximum or minimum points and the region where they occur in the design parameter space. The advantage of these response surface methods is that it helps us to understand the nature of the response function and thereby the process. The knowledge of the model is quite important when relating the physical response or statistical response performance to the design factors and/or their functions. Initially, the Response Surface Methodology (RSM) was developed to solve problems in the chemical and biological industry. It was used to determine of determining levels of input variables so as to optimize a certain response. The set of input variables, say $x = (x_1, x_2, ..., x_k)^T$,are under the control of the experimenter. Suppose the mean response (η) depends on the variables x through a function ϕ: $\eta = \phi(x)$. The subset of R^k to which x belongs is known as the *factor space*. Suppose the variance of the response does not depend on x. The basic problem addressed in response surface methodology is to find the smallest number of experiments so as to optimize η. Usually, the response surface η is not known by experimenters, and it must be estimated. The problem of experimental determination of a maximum was introduced statistically by Hotelling in 1941, and later developed by Friedman and Savage in 1947. Box and Wilson (1951) introduced the basic framework for developing response surface designs[14]. The technique has found many research and industrial applications. In particular, it is used in process and product optimization, where the designed experiments are used. Consider the problem of finding x ∈ R^k so as to maximize: $\phi(x) - \phi(0)$.

Subject to: $\sum_{i=1}^{k} x_i^2 = r^2$ with given r. This can be accomplished by maximizing the Lagrange:

$$\phi(x) - \phi(0) - \frac{\lambda}{2}\left(\sum_{i=1}^{k} x_i^2 - r^2\right) \quad (7)$$

The stationary solution is given by the partial derivative equation of (7) with respect x_i to zero. That is, we obtain: $\lambda x_i = \frac{\partial \phi(x)}{\partial x_i}, \quad i = 1,2,...,k.$ Squaring and summing from i = 1,2... k, we have:

$\lambda^2 r^2 = \sum_{i=1}^{k}\left(\frac{\partial \phi(x)}{\partial x_i}\right)^2$, So that the point giving the optimal is: $x_i = r\left(\frac{\partial \phi(x)}{\partial x_i} \Big/ \sqrt{\sum_{i=1}^{k}\left(\frac{\partial \phi(x)}{\partial x_i}\right)^2}\right)$

Clearly, it can be seen that the direction of Steepest Ascent is provided by the partial derivatives of the response function. Suppose the conditions of Taylor's expansion for ϕ(x) in the neighborhood of the original hold. Then ϕ(x) can be expanded to linear, quadratic or even higher order terms. If we assume that second and higher order terms in the expansion of ϕ are zero, then ϕ(x) is approximated by a linear function of the following type: $\frac{\partial \phi(x)}{\partial x_i} = \beta_i, \quad i = 1,2,...,k.$ Then, $\phi(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k$, the optimal x_i are proportional to the regression coefficients β_i. Similarly, expressions involving coefficients of linear and quadratic equations can be obtained if the Taylor´s expansion of ϕ(x) does not contain third or higher order terms. The move along the derivatives of the response function gives the steepest ascent approach to a maximum. In this work has been used the Steepest Ascent that is the same of Steepest Descent optimization method, but with search inverted direction. These methods are a kind of gradient method. Basically the method is defined by iterative algorithm:

$$x_{k+1} = x_k - \alpha_k g_k \quad (8)$$

where, α_k is a nonnegative scalar maximizing $f(x_k - \alpha g_k)$. From the point x_k its search along the direction of the negative gradient $-g_k$ to a minimal point on this line; this minimum point is taken to be x_{k+1}.

5 Methodology Proposed

In the industrial world, there are many situations where several input variables may influence some performance measure, product quality characteristics or a process. This performance response or quality characteristic is called the *response*. It is typically measured on a continuous scale, although ranks, attribute responses, and sensory responses are not unusual responses. The input variables are sometimes called *independent variables*, and they are subject to the control of the engineer or scientist, at least for experimental or test purposes. The Figure 2, shows a contour plot between variables ξ_1 and ξ_2 against response *y* that has been studied.

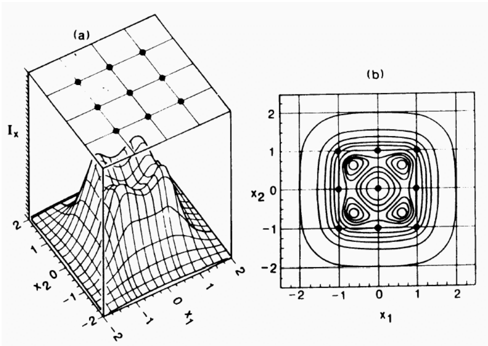


Figure 2 - (a) Example of a surface from factorial planning with central point used as second-order designs; (b) Contours plot.

The Figure 2 shows a contour plot between variables ξ_1 and ξ_2 against response *y* that has been studied. In this presentation we look down at the ξ_1 -ξ_2 plane. We also connect all the points that have the same response (y) in order to produce contour lines of constant responses this type of display is called a *contour plot,* and it is obtained from the response surface methodology that studies the relationship between the *independent variables* and the *responses*. In summary, the field of the response surface methodology consists of the experimental strategy used to explore the process space, the design or the independent variables ($\xi_{1,}\, \xi_{1,\ldots}\xi_k$). The empirical statistical model develops an approximating relationship between the response and the independent variables, and it assembles a set of variables for the design. Optimization methods are used to find the levels of the responses (maximizing or minimizing responses).

6 Case Study

The proposed problem is a study of a mass-spring damping system, because this system can be extended to a foundation or a rotor analysis. In this first stage a factorial planning (3^3) is made with three trials at the intermediate level. The independent variables (factors) are: Mass (*M*), Stiffness (*K*) and Damping (*C*) and the response is the natural frequency (w_d) of the system for the range that was initially established in the design. For initial runs, the following equations were used:

$$w_n = \sqrt{\frac{k}{m}} \quad \xi = \frac{c}{c_{crit}} \quad C_{crit} = 2m * w_n \quad w_d = \sqrt{1-\xi^2} * w_n,$$

So, the damped frequency is related with mass, stiffness and damping: $w_d = \frac{1}{2m}\sqrt{4mk - c^2}$

The Figure 3 proposed by Phadke (1989) *apud* Taguchi, to assist in the identification of influence factors in system, product or process and it is represented in *P-Diagram*:

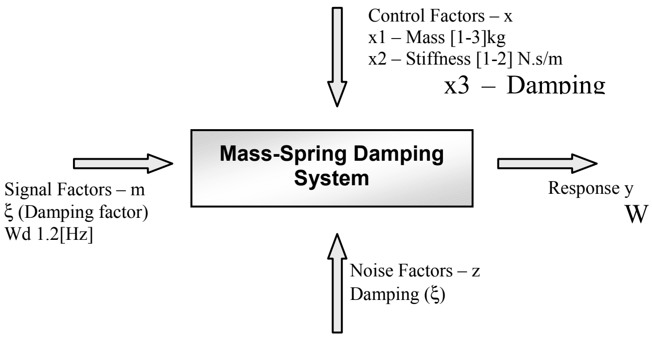


Figure 3- Classification of design parameters.

6.1. The initial levels to independent variables

Mass 1-3 [kg] ; *Stiffness* 1-2 [N.s/m]; *Damping* 0.25-0.75 [N/m]. So, it is choose maximum and minimum values to experiment and these values are changed in coded variables:

$x_{i1} = M - 2$ $\quad x_{i2} = \frac{K - 1.5}{0.5}$ $\quad x_{i3} = \frac{C - 0.5}{0.25}$

6.1.1 Design of experiments

Table 1- Factorial planning (2^3) with three trials at intermediate points.

No. Exp.	Control Parameters			Coded Variables			Responses		
	M(ξ1)	K(ξ2)	C(ξ3)	x_1	x_2	x_3	w_n	ξ	w_d
1	1	1	0.25	-1	-1	-1	1.000	0.125	1.004
2	3	1	0.25	1	-1	-1	0.577	0.072	0.646
3	1	2	0.25	-1	1	-1	1.414	0.089	1.439
4	3	2	0.25	1	1	-1	0.817	0.051	0.839
5	1	1	0.75	-1	-1	1	1.000	0.375	0.949
6	3	1	0.75	1	-1	1	0.577	0.217	0.589
7	1	2	0.75	-1	1	1	1.414	0.265	1.391
8	3	2	0.75	1	1	1	0.817	0.153	0.848
9	2	1.5	0.5	0	0	0	0.866	0.144	0.865
10	2	1.5	0.5	0	0	0	0.866	0.144	0.859
11	2	1.5	0.5	0	0	0	0.871	0.143	0.889

A) Estimation of Parameters: Fit through of a Linear Model

Fitting of experiment (data) to linear polynomial model of first-order, using the Least Square Method: $\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$. The estimated parameter values can be calculated by: $b = (X'X)^{-1} * X'*y$. So, in terms of coded variables, the equation is:

$\hat{y} = 0.9380 - 0.2326x_1 + 0.1661x_2 - 0.0189x_3 + \varepsilon$, That must be substituting in the natural variables

The matrix of estimated parameters that fit the polynomial model linear is:

$$\{\beta\}^T = \{0.9380 \quad -0.2326 \quad 0.1661 \quad -0.0189\}$$

The table 2 shows the errors between the theoretical and experimental responses obtained after the fitting of linear model parameters.

Table 2 - Fitting of response and residue values for first-order model:

Runs	y_i ***(wd)***	y_{ajust}	**e_i (Residue)**
1	1.004	1.023	**0.019**
2	0.646	0.527	**0.089**
3	1.439	1.355	**0.084**
4	0.839	0.889	**0.05**
5	0.949	0.985	**0.036**
6	0.589	0.519	**0.07**
7	1.391	1.317	**0.074**
8	0.848	0.851	**0.003**
9	0.865	0.937	**0.072**
10	0.859	0.937	**0.078**
11	0.889	0.937	**0.048**

Once obtained, we made the variance analysis (ANOVA) for the first model (first-order) in order to verify the fitting of the models. The ANOVA is used in multiple regression problems and is based on the variability distributed over the response variable. In this case, it is assumed that the distribution will be normal.

Table 3 (a) - Analysis of variance (ANOVA) to First-order model.

Variation Source (FV)	Square Sum	Degree of freedom	Square Mean	Ratio F (Snedecor)
Regression (SS_R)	*0.69*	*3*	*0.230 (MS_R)*	
Error or residual (SS_E)	*0.03*	*7*	*0.0043(MS_E)*	*35.91*
Total (S_{yy})	*0.72*	*10*		

The hypothesis significant test for regression is: $\begin{cases} H_0 : \beta_1 = \beta_2 = ... = \beta_k = 0 \\ H_1 : \beta j \neq 0, \text{ at least one j} \end{cases}$

The values obtained for the F distribution (theoretical and experimental) with $\alpha = 0.05$, were: $F_{\alpha, k, n-k-1} = F_{0.05, 3,7} = 4.35$ and $F_{exp} = 35.91$ As $\underline{F_{\alpha, k, n-k-1} \leq F_{exp}}$ rejects the H_0, then at least one factor significantly contributes to model, there is least interaction between factors (M, K and C) on response y (w_d).
The next step is to determine the multiple coefficient R^2, defined as: $R^2 = 1 - \left(SS_E / S_{yy}\right)$.The coefficient R^2 is a measure of the amount of reduction in the variability of y obtained by using the repressor variables (x_1, x_2, ..., x_n) in the model (in this case, first-order model). The coefficient is a kind of correlation coefficient. So, the first-order model explains about *95,69%* of the variability observed in damped frequency.

B) Estimation of Parameter: Fitting through a Bi-Linear Model

If the first-order model does not fit the experimental results, a bi-linear or a quadratic model could be used. In this case, a bi-linear model is sufficient for fitting the experimental data, where:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 + \varepsilon$$

The matrix of parameters is:

$$\{\beta\}^T = \{0.9380 \quad -0.2329 \quad 0.1664 \quad -0.0186 \quad -0.0531 \quad 0.0091 \quad 0.0071\}$$

In the natural variables:

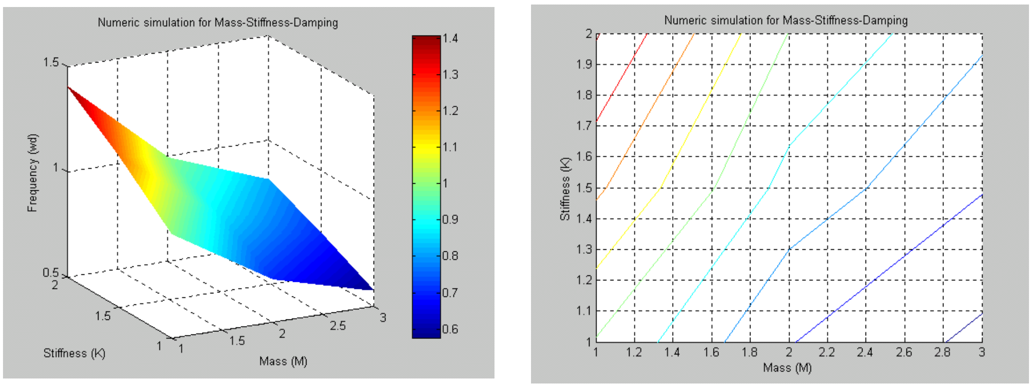
$$\hat{y} = 0.6202 - 0.0448\,\xi_1 + 0.566\,\xi_2 - 0.0684\,\xi_3 - 0.135\,\xi_1\xi_2 + 0.0576\,\xi_1\xi_3 + 0.0576\,\xi_2\xi_3 - 0.0408\,\xi_1\xi_2\xi_3 + \varepsilon$$

As made in the first-order model, the *response y* is fitted, substituting the values coded for variables (M, C and K) in the polynomial of natural variables and the residue is the difference between the experimental response (y) and the fitted response (y′) of damped frequency. The residue is obtained from difference y and y′. Once obtained the polynomial model, can be made the ANOVA for the new model.

Table 3 (b) - Analysis of variance (ANOVA) to Bi-Linear Model.

Variation Source (F.V)	Square Sum	Degree of freedom	Square Mean	Ratio F (Snedecor)
Regression (SS_R)	*0.681*	*7*	*0.097*	
Error or residual (SS_E)	*0.02*	*3*	*0.0067*	*14.48*
Total(S_{yy})	*0.70*	*10*		

Again, is applied the hypothesis significant test for regression, in this case the values obtained for the F distribution (theoretical and experimental) with confidence interval of 95%, were: $F_{\alpha, k, n-k-1} = F_{0.05, 7,3} = 8.89$ and $F_{exp} = 14.48$. As $\underline{F_{\alpha,k,n-k-1} \leq F_{exp}}$ rejects the H_0, then at least one factor significantly contributes to model, have been at least interaction between factors (M, K and C) on response y (w_d). The coefficient multiple R^2 to the Bi-linear model explains about *97,30%* of the variability observed in damped frequency, with a improvement in relation at first-order model *93,80%.* The response surface for factors mass, stiffness and damping fitting can be compared with a numeric response of system with the response (damped frequency – w_d), fitting in the Figure 5, through of three factors can be fixed: mass, stiffness and damping, to analysis the interactions between them. These graphics allow a first study about the factors that are the interactions or not and the sensitivity with the response (damped frequency). For example, the damping can be fixed in three levels and mass and stiffness versus damped frequency plotted were the mass and stiffness can be fixed in levels to analysis the damping with the damped frequency.



(a) **(b)**

Figure 5 (a) - Fitted surface with a Bi-linear model (with damping fixed) and (b) contour plot of response surface.

In, summary the first and second step of the methodology is to apply the response surface analysis to fit a surface, where the fitted surface is an adequate approximation of the real response function. Then, the analysis of the fitted surface will be approximately equivalent to the analysis of the studied system. Thus, the third step will be to find the optimum region for operating conditions of the system. Once the region of the optimum has been found, a more elaborate model such as a second-order model may be employed to locate the optimum point between the factors and is used the canonical analysis can be used to determine the sensitivity between these three factors after the optimisation.

7. Development of a program based on design of experiments and statistical optimization of the mechanical components

In this work a program has been developed that including the theory of design of experiments (Factorial Designs (2 levels and expanded), Fractional Factorial designs, Taguchi Methodology), where are considered the Analysis of Variance (ANOVA) for the contrasts between parameters and validation of linear or non-linear models; Response Surface Methodology (RSM) and optimization techniques to find the optimum values to critical parameters. The Figures 6, 7 and 8 show some stages of this program:

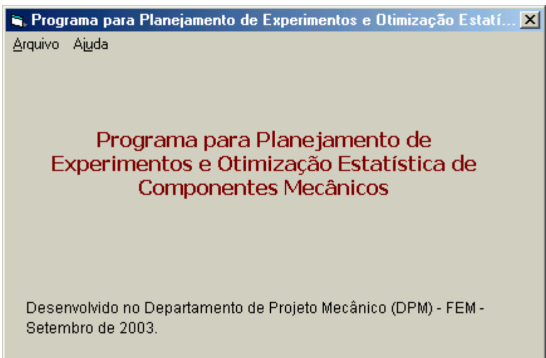


Figure 6 - Initial screen of the Program that has been developed for the study of the parametric sensitivity in mechanical components.

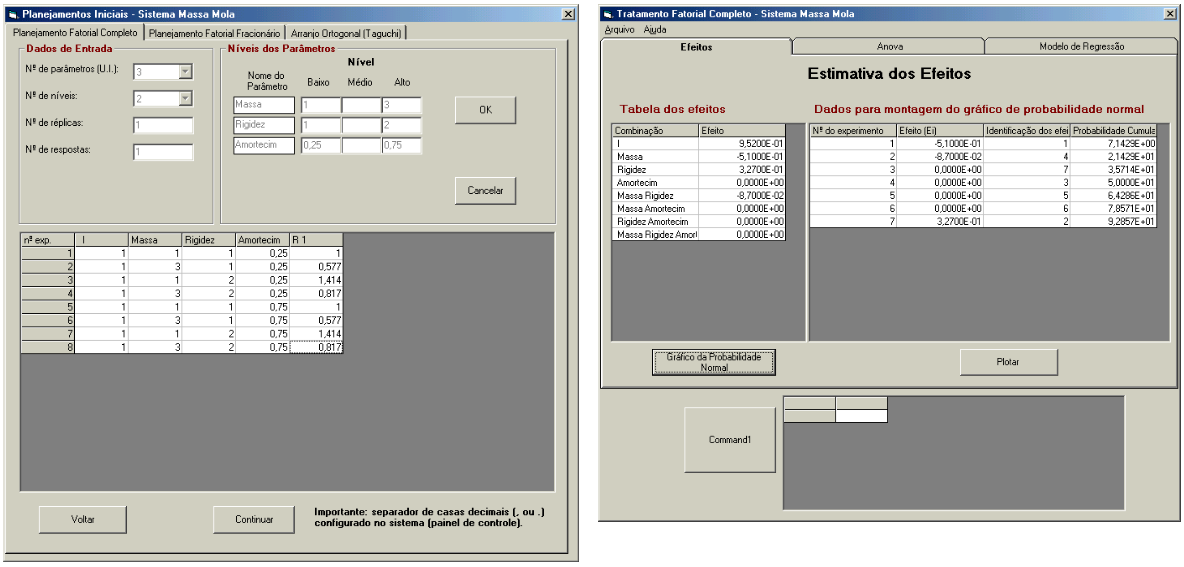


Figure 7 - Example of the design of experiments implemented (Factorial Design) with levels and experimental matrix and interactions between factors (parameters) respectively.

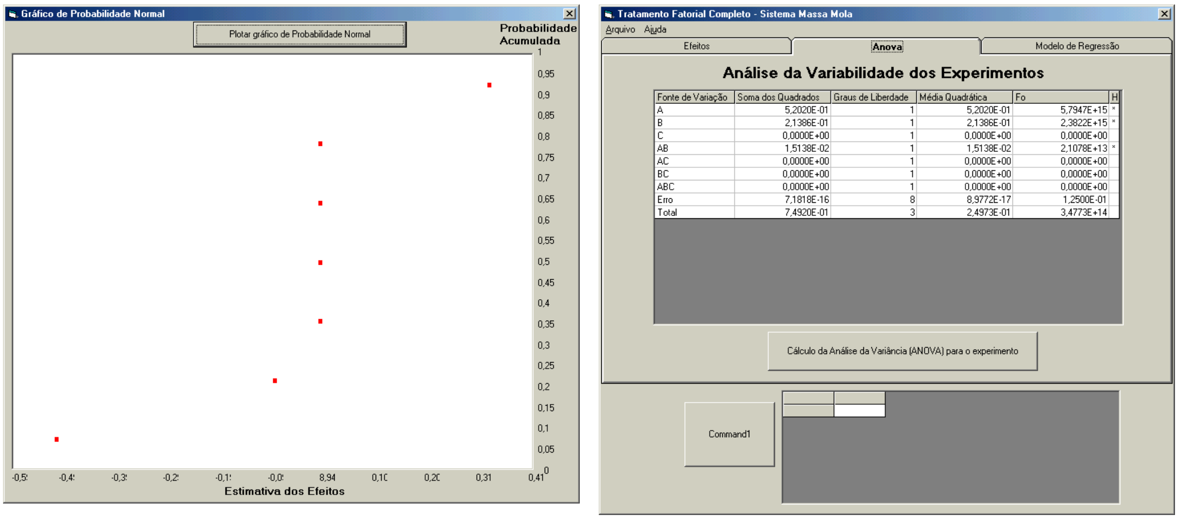


Figure 8 – Normal Probability Graphics from experimental matrix and its ANOVA respectively.

8. Conclusions

During the design of mechanical systems, there are a series of parameters that generate uncertainties in its components. Therefore, this work develops a methodology that link reliability with design of experiments and statistical and numerical optimization to obtain a study of the parametric sensitivity in mechanical components. The main contribution of the methodology is the identification of the most significant variables influence and their eventual interactions on the specific design parameters. The applicability is stronger in complex systems with non-modelled phenomena, where the mathematical model is not accurate enough. The methodology can be take part in the design process during the product concept as well as during the useful life. The mass-spring-damper system was used in order to validate the proposed procedure, once that is a known example and to test the software that has been developed during this thesis. An random noise was simulated and applied to each parameter of mass, spring and damper. A first response surface was plotted from the fitted parameters of the polynomial function obtained by Response Surface Methodology. The natural frequency was the robust response in this case. An optimal set of parameters was evaluated and selected and it fitted quite well with the expected result, as shown in Figure 5 (a) by a first-order polynomial model. It is observed that the natural frequency is mainly influenced by parameters of mass and stiffness (spring) which is completely reasonable with the theory. The damper presents a less influence in a linear model (small vibrations amplitudes). Otherwise, it is quite important to verify the methodology to more complex systems as well as experimentally in order to analyse the robustness of the entire methodology, that is been applied in a hydrodynamic bearings.

9. Acknowledgement

The authors would like to thank to CNPq and FAPESP for the financial support to this work.

10. References

Achcar, J. A; Rodrigues, J. 2000, "Introdução à Estatística para ciências e tecnologia".Apostila. ICMC-USP, São Carlos, SP. São Carlos. 191p.

Barker, T.B., 1985, "Quality by experimental design." New York: Marcel Dekker.

Bisgaard S. ; Ankenman, B., 1996, "Standard errors for the eigenvalues in second-order response surface models." Technometrics. n.3, v.38, August, pp. 238-246.

Bisgaard, S.; Graves, S.; Shin, G., 2000, "Tolerancing mechanical assemblies with CAD and DOE. Journal of quality technology." pp. 231-240. v.32, July. n3.

Box, G.E.P.; Hunter, W. G.; Hunter, J.S., 1978, "Statistics for experimenters: an introduction to design, analysis and model building."John Wiley & Sons. pp. 1-14; 291-344; 510-552.

Box, G.E.P.; Draper, N.R., 1987, "Empirical model-building and response surfaces." John Wiley & Sons. 667p.

Box, G. E.P. Hunter, J.S., 2001, "The 2^{k-p} fractional factorial designs". Part 1. Technometrics. v. 42, n.1, Feb. pp. 28-47.

Bruns R. E. , *et all* ,2001, "Como fazer experimentos: Pesquisa e desenvolvimento na ciência e na indústria." Campinas: Editora da Unicamp, pp. 401.

Button, S.T. 2000, Metodologia para planejamento experimental e análise de resultados (apostila) FEM-UNICAMP, Setembro.70p.

Cavalca, K.L; Cavalcante, P.F., 2000, "Modelagem e análise de máquinas rotativas e estruturas de suporte".Apostila do Curso. DPM, FEM, Unicamp.

Chen, W. *et all.* 1996, "A procedure for robust design: minimizing variations caused by noise factors and control factors." Transactions of the ASME: Journal of Mechanical Design. v. 118, pp. 478-485. December.

Kackar, R.N., 1985, " Off-line quality control, parameter design and Taguchi Method" Journal of Quality Technology, v. 17, n.4, October.

Montgomery, D.C; Runger, G.C., 1996, "Applied and probability for engineers." John Wiley & Sons. pp. 687-800.

Myers, R.H.; Montgomery, D.C., 1995, "Response surface methodology: Process and product optimization using designed experiments."New York: John Wiley & Sons, Inc., 699p.

Myers, R.H. ; Kim, Y.; Griffiths, K.L., 1997, "Response surface methods and the use of noise variables." Journal of quality technology. v 29, n.4. October. p.429-440.

Neto Barros, B. de B.; Scarminio, I. S; Bruns, R. E., 1995, "Planejamento e otimização de experimentos." Editora da Unicamp, 2ª edição. 299p.

Norton, R.L., 2000, "Machine Design (An integrated approach)." Prentice Hall: New Jersey. pp.3-11; pp.22-30; pp.619- 677.

Oliveira Neto, P.L., 1997, "Estatística." São Paulo: Edgard Blücher, pp. 1-33; 84-106; 133-222.

Padke, M. S. "Quality engineering using robust design." AT&T Bell Laboratories Prentice Hall. 334p. 1989.

Silveira, Z.C. Cavalca, K.L., 2003, "A Methodology Based on Robust Design and Optimization Statistics for Fitting of Parameters in Mechanical Systems." Proceedings of the 11th World Congress in Mechanism and Machine Sciences. IFToMM 2003, August 18-21, Tianjin, China.

Silveira, Z.C. Cavalca, K.L., 2001, "Development of a robust reliability methodology to mechanical systems parameter fitting." 20th International Conference on Offshore Mechanics and Artic Engineering Section: Safety and Reliability, OMAE-ASME. UFRJ-COPPE. Rio de Janeiro.