LAMINAR NATURAL CONVECTION INSIDE A CLOSED SEMI-CYLINDRICAL CAVITY

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Abstract. *In the present work we consider the laminar natural convection inside a semi-circular shaped enclosure filled with air, which is numerically analyzed in two cases: (a) heated from the top and cooled from the bottom; (b) cooled from the top and heated from the bottom. Two dimensional equations for mass, momentum and energy conservation, with the Boussinesq aproximation are numerically solved using a finite volume method. The discretized equations are obtained through the Patankar's Power-Law scheme and a fully implicit formulation. The adopted numerical procedure for pressure calculation is based on the SIMPLE algorithm. The governing parameters used are:* $10^3 \leq Gr \leq 10^8$ *; Pr=0.72. The isotherms, streamlines and profiles of velocity along the middle of the vertical and horizontal line of the cavity and the normalized average Nusselt number are presented in terms of Grashof number. When* $Gr \approx 10^3$*, in both cases the process is similar to pure conduction; when the Grashof number increases, the convection dominates the process of heat transfer. In the first case, the Nusselt number at the base of the cavity increases when the Grashof number increases up to about* $Gr=1.5x10^7$*, and decreases for Grashof numbers greater than this value. In the second case, the Nusselt number increases for values of Gr greater than* 10^3*.*

Keywords. *Natural laminar convection, semi-cylindrical cavity.*

1. Introduction

Natural convection in enclosures plays an important role in many engineering applications, such as building heating systems, electronic cooling systems, metallurgical processes and material processing. Due to its importance, it has been the study of many researchers. In addition, enclosure geometry has been widely studied in heat transfer because of its fundamental importance and its many applications. Most of the previous studies were carried out on rectangular and triangular geometry, covering wide ranges of Grashof number and aspect ratios which can be found in the literature. Manglik et al (1988) analyzed the laminar flow heat transfer in a semi-circular tube with uniform wall temperature. The theoretical study of the natural convection in an air-filled enclosure was numerically studied for U-C. Shin et al (1994). The enclosure was composed of the partial horizontal cylinder limited by a plate, and calculation was carried out for a wide range of Grashof numbers and the inclination of the enclosure, using bicylindrical coordinates. Van Dyne et al (1994) studied the natural convection heat and mass transfer in a semi-cylindrical enclosure filled with a heat generating porous media. Liaqat et al (2001) presented a numerical comparison of the conjugate and non-conjugate natural convection for internally heated semi-cylindrical cavity. Chakroun et al (2002) studied the effect of roughness on heat transfer in semi-cylindrical cavities. The present work focusses the study of natural convection in a semi-cylindrical cavity filled with air, which is numerically analyzed in two cases: (a) heated from the top and cooled from the bottom with a uniform and constant temperature; (b) cooled from the top and heated from the bottom with a uniform and constant temperature. Two dimensional equations for mass, momentum and energy conservation, with the Boussinesq aproximation are numerically solved using a finite volume method. The discretized equations are obtained through the Patankar's Power-Law scheme and a fully implicit formulation. The adopted numerical procedure for pressure calculation is based on the SIMPLE algorithm. The governing parameters used are: $10^3 \leq Gr \leq 10^8$; Pr=0.72. In order to define the semi-cilyndrical solution problem domain, the blocking-off method also suggested by Patankar (1980) is used.

2. Mathematical Modeling

The considered geometry is shown in Fig. (1). The gravity vector is normal to the base of the cavity. R is the radius of the cavity. Two cases are studied: Case I - heated from the top and cooled from the bottom; Case II - cooled from the top and heated from the bottom. In order to formulate the fluid convection in the cavity, the following assumptions were considered: the flow is two-dimensional and laminar; the temperature gradients are moderate for which the Boussinesq approximation is valid; viscous dissipation and the work done by compression forces are negligible.

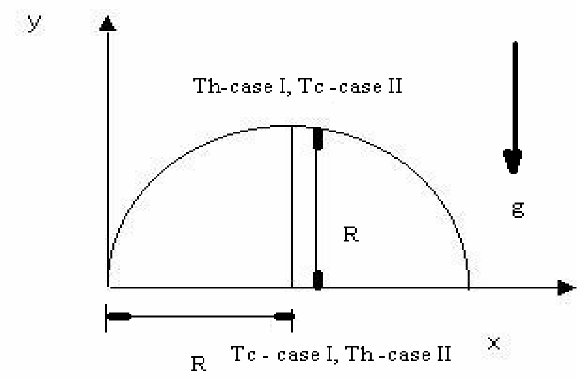

Figure 1: Semi-cylindrical cavity and the coordinate system.

The basic equations for the unsteady-state natural convection can be written in the dimensionless form as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\Pr^2 Gr\theta-\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} \tag{3}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right) \tag{4}$$

where u and v are dimensionless velocities in x and y directions; g is the acceleration due to gravity; Pr is the Prandlt number; Gr is the Grashof number; θ is the dimensionless temperature and p is the dimensionless pressure.

For the construction of dimensionless quantities the following non-dimensionless variables are defined:

$$x=\frac{x'}{R} \qquad y=\frac{y'}{R} \qquad u=\frac{u'R}{\alpha} \qquad v=\frac{v'R}{\alpha} \qquad p=\frac{p'+\rho_0 g y'}{\rho_0(\alpha/R)^2}$$

$$\theta=\frac{T-T_c}{T_h-T_c} \qquad t=\frac{\alpha}{R^2}t' \qquad Gr=\frac{g\beta R^3\left(T_h-T_c\right)}{\nu^2} \tag{5}$$

where R is the radius of the cavity; α is the thermal diffusivity of the fluid; β is the thermal expansion coefficient; ν is the cinematic viscosity, ρ is the density; T_c is the dimensional temperature of the cold wall; T_h is the dimensional temperature of the hot wall and T is the dimensional temperature inside the cavity . The upper indices ’ indicates dimensional variables.

The boundary conditions for the two cases are:
Case I:

$$\begin{aligned} &\text{for } 0\le x\le 1, && (x-1)^2+y^2=1, && u=v=0, && \theta=1 \\ &\text{for } 0\le x\le 1, && y=0, && u=v=0, && \theta=0 \end{aligned} \tag{6}$$

Case II:

$$\begin{aligned} &\text{for } 0 \le x \le 1, \quad (x-1)^2 + y^2 = 1, \quad u = v = 0, \quad \theta = 0 \\ &\text{for } 0 \le x \le 1, \quad y = 0, \quad u = v = 0, \quad \theta = 1 \end{aligned} \tag{7}$$

The boundary conditions for the velocity vector components in impermeable solid surfaces are the non-slip and non-flow conditions. As it was not possible to determine the pressure, but only pressure differences from Eq. (2) and (3), for the sake of definiteness, it will be assumed that the pressure is zero (or a reference value) at the point with the coordinates x = y = 0.

The local Nusselt number at the base of the cavity is calculated using

$$\mathrm{Nu}_L = \frac{hR}{k} \tag{8}$$

and the average Nusselt number can be obtained by integrating the local Nusselt number along the wall

$$\overline{\mathrm{Nu}} = \frac{1}{R}\int_0^R \mathrm{Nu}_L \, dR \tag{9}$$

The normalized Nusselt number is defined as

$$\mathrm{Nu} = \frac{\overline{\mathrm{Nu}}\,\Big|_{Gr}}{\overline{\mathrm{Nu}}\,\Big|_{Gr=0}} \tag{10}$$

where $\overline{\mathrm{Nu}}\,\Big|_{Gr=0}$ is for pure conduction in the same conditions.

3. Numerical Procedure

In order to perform the discretization of the governing equations, the power law scheme introduced by Patankar (1980) is adopted.

As in all control volume methods, the discretized equations are obtained by the integration of the equation over control volumes surrounding each grid point.

The solution of the momentum equations requires a procedure to calculate the pressure fields. Since there is no specific equation for the pressure, the SIMPLE algorithm developed by Patankar, using the mass conservation equation to obtain an equation in terms of the pressure in the grid points, is considered. To solve the set of algebraic equations an interactive line-by-line process is used. This process is a convenient combination of a direct method TDMA for one dimensional situation and the Gauss Siedel method. The construction of the discretized equations was done considering a fully implicit method.

As shown by Patankar, to avoid non realistic solutions, independent grids for the variables u, v and p are required. The use of the staggered grids was adopted here. This process facilitates the application of the boundary conditions. Other advantage is that the u and v velocity components coincide with the faces of control volume for pressure and temperature. On the other hand, it imposes the necessity of using interpolation schemes for the u and v grids.

It is very difficult to guarantee the convergence of a non linear system of equation. To achieve the convergence, under-relaxation factors are applied in the solution procedure to avoid large corrections in one step of iteration, which may cause the divergence of the process.

The convergence must be verified in each iteration, following a predetermined criterion. In this work we selected the criterion which considers the average error for each control volume, and the convergence parameter could be different for each variable depending on the order of magnitude of the variable in study. Here we set the convergence parameters as 10^{-5} to all variables.

In order to obtain grid independent results, grids with 1624, 2964, 3784 and 4704 uniform volumes were tested. The differences between the 3784 and 4704 internal volume grids were minimal for all the values of Gr used. 4704 volume grids were used for all situations. In order to define the semi-cilyndrical solution problem domain, the blocking-off method suggested by Patankar was used. It consists in considering inactive the control volumes of a regular grid, so that the active volumes make the desired domain. This is got by considering the velocities components equal zero in the inactive region, and fixing a constant known value for the temperature in the inactive region. For example, the velocities in the inactive region can be considered equal zero by the use of a great value of the viscosity in this region and the value zero for the velocities in the nominal border.

4. Results and Discussion

4.1. Case I - Cavity heated from the top and cooled from the bottom

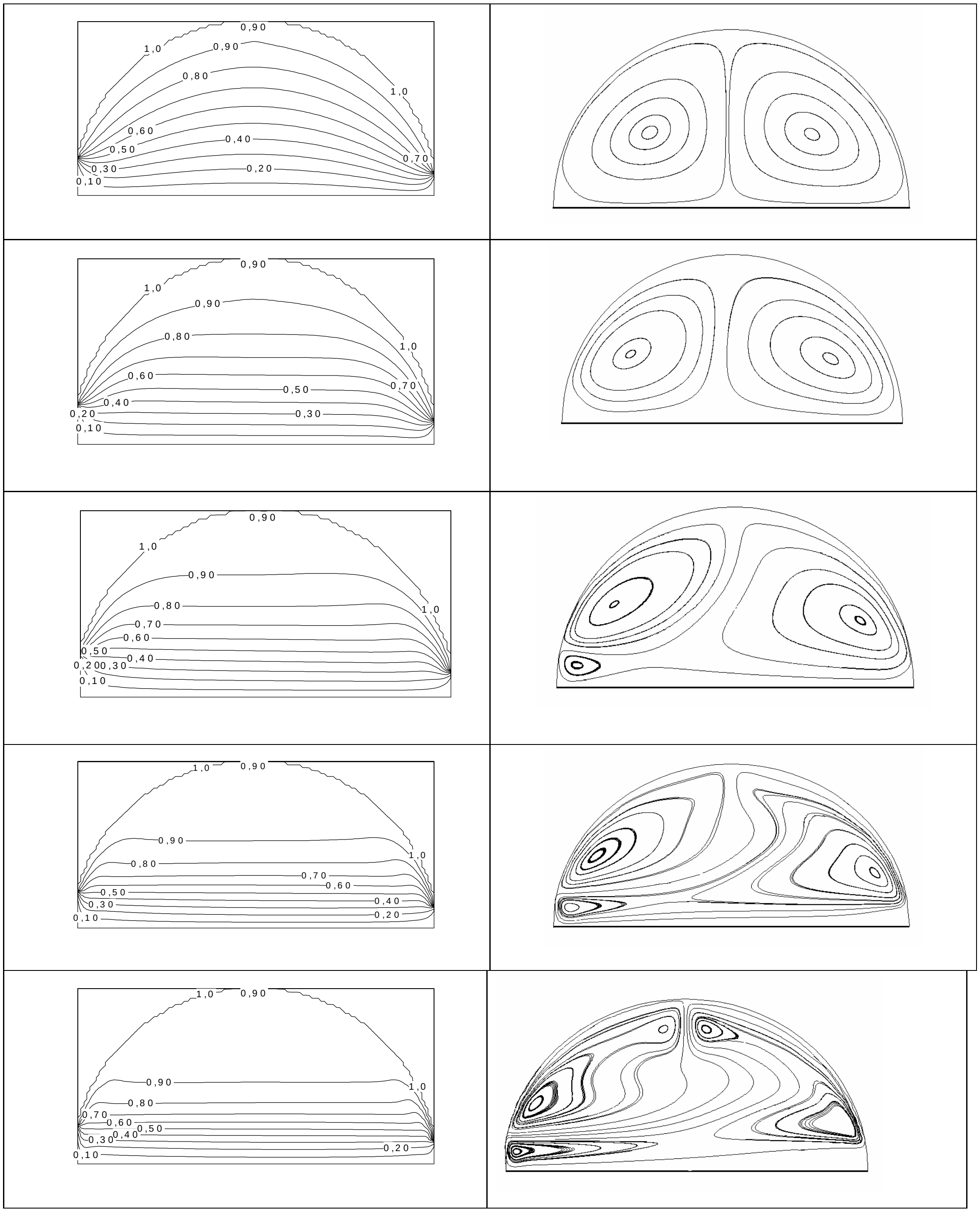

Figure 2. Isotherms (left) and streamlines (right) for the steady state flow Gr = 1.0×10^3, Gr = 1.0×10^5, Gr = 1.0×10^6, Gr = 1.0×10^7, Gr = 5.0×10^7.

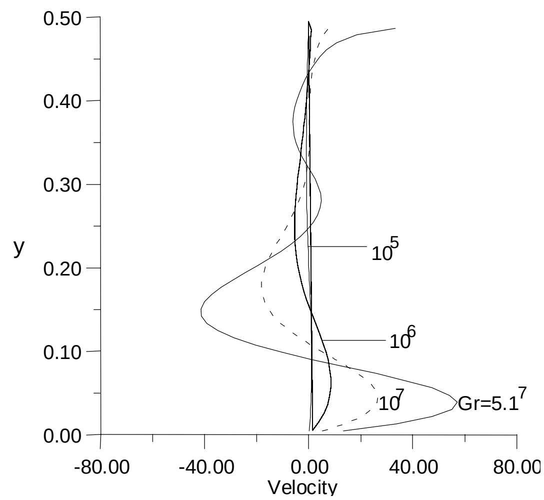

Figure 3. Horizontal velocity component profile along the middle vertical line of the cavity.

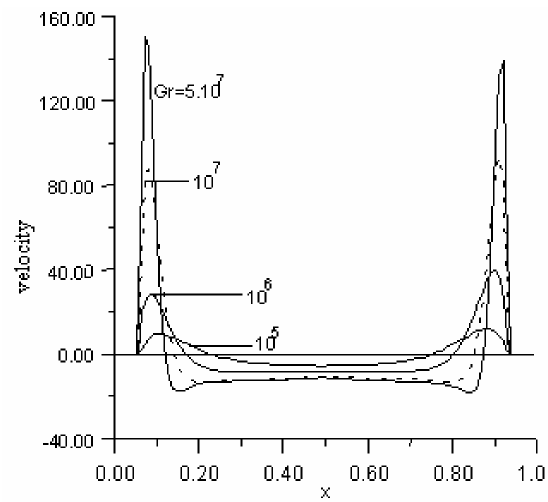

Figure 4. Vertical velocity component profile along the middle horizontal line of the cavity.

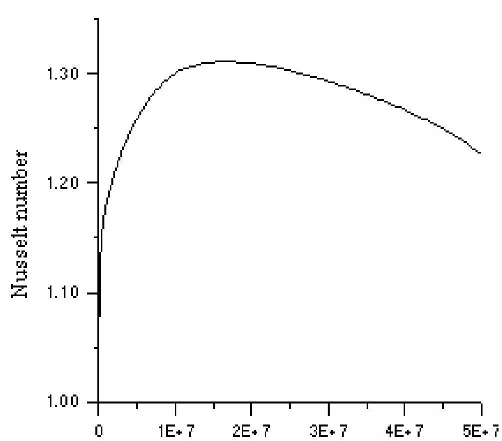

Figure 5. Average normalized Nusselt number in the base as a function of the Grashof number.

Figure (2) shows the streamlines and isotherms considering different Grashof numbers for the case in which the cavity is heated from the top and cooled from the bottom. When $Gr \approx 10^3$, the isotherms almost take a position indicating that conduction is the main process occurring. With the increase in the Grashof number, the convection process is much more intense and there is a greater circulation of the fluid shown through the streamlines; the isotherms are influenced by the convection process and show a stratified profile.

Figure (3) shows the horizontal velocity component profile along the middle vertical line of the cavity for some values of Grashof number. We can notice that for lower Grashof number, the horizontal velocity tends to zero; and when the Grashof number increases, there is also an increase in the velocity, indicating a more intense fluid movement due to convection currents.

Figure (4) shows the vertical velocity component profile along the middle horizontal line of the cavity for some values of Grashof number. The same effect is observed, when the Grashof number increases, there is also an increase in the velocity.

Figure (5) presents the average normalized Nusselt number at the cavity bottom as a function of the Grashof number. As it can be verified, the Nusselt number increases when the Grashof number increases up to about $Gr=1.5x10^7$, and presents a slight decrease for Grashof numbers greater than this value.

4.2. Case II -Cavity cooled from the top and heated from the bottom.

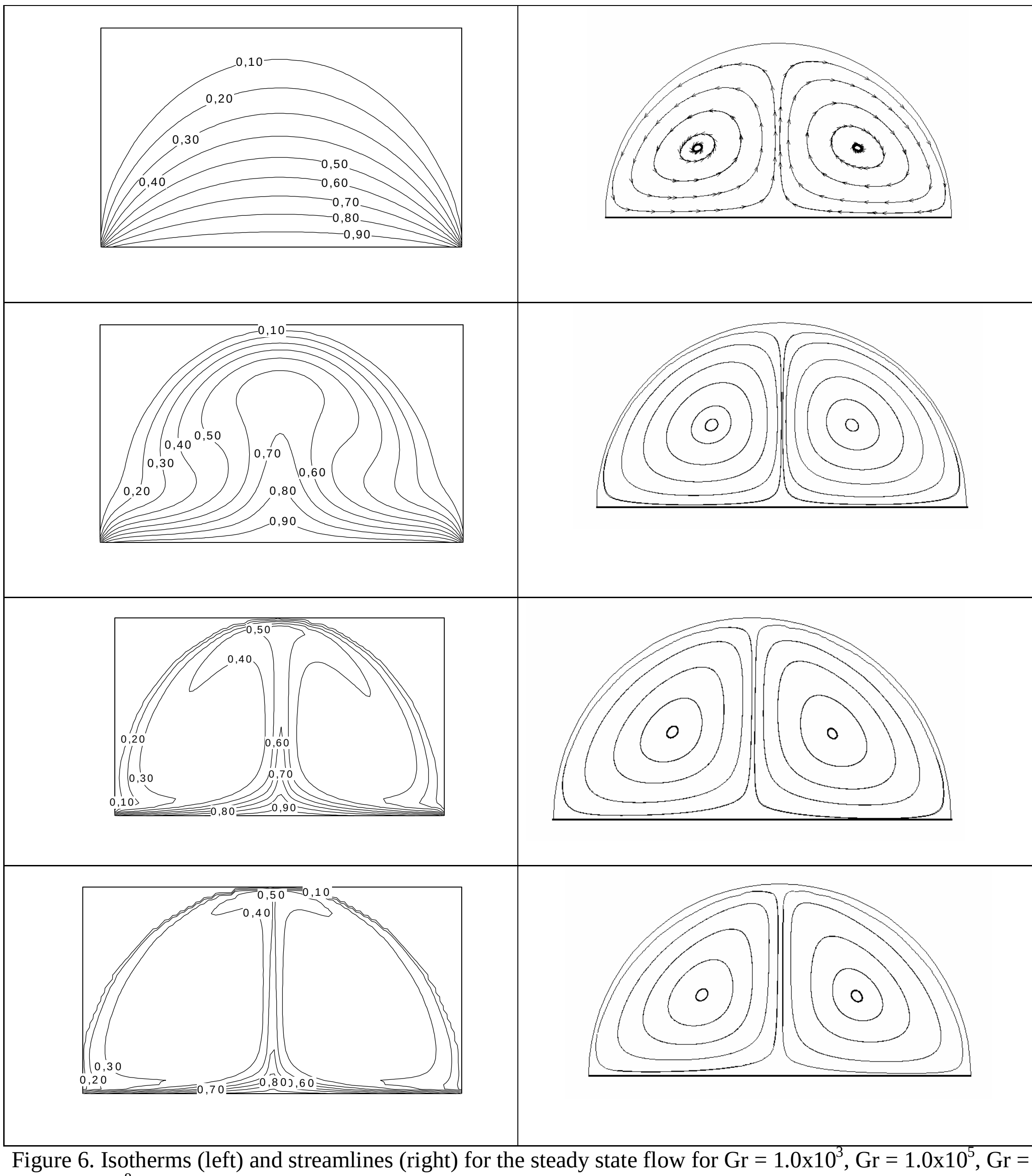

Figure 6. Isotherms (left) and streamlines (right) for the steady state flow for $Gr = 1.0x10^3$, $Gr = 1.0x10^5$, $Gr = 1.0x10^7$, $Gr = 1.0x10^8$.

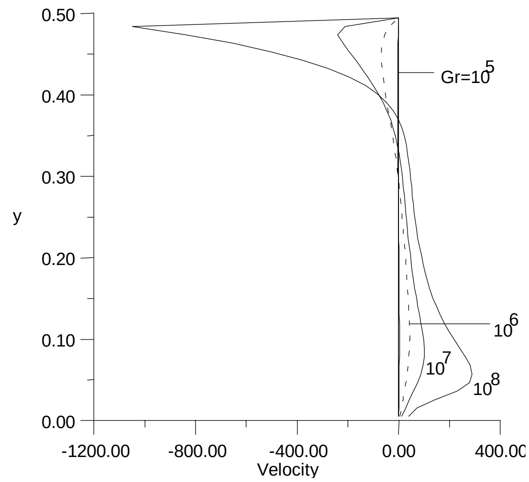

Figure 7. Horizontal velocity component profile along the middle vertical line of the cavity.

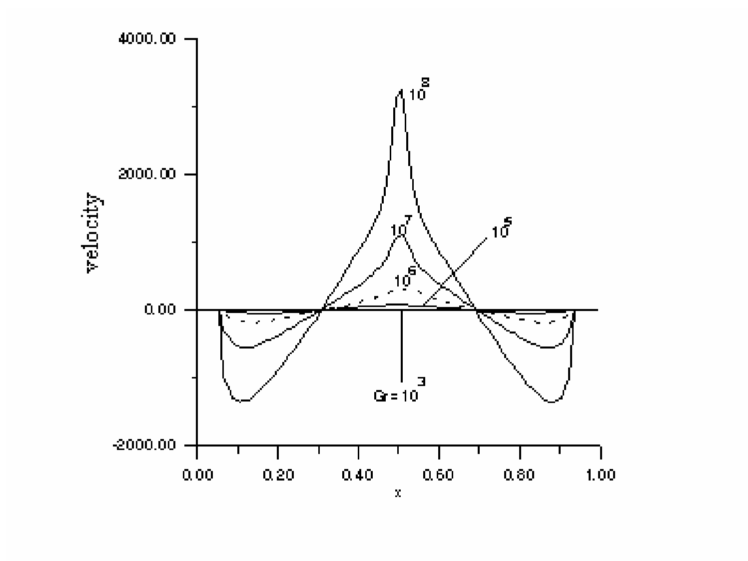

Figure 8. Vertical velocity component profile along the middle horizontal line of the cavity.

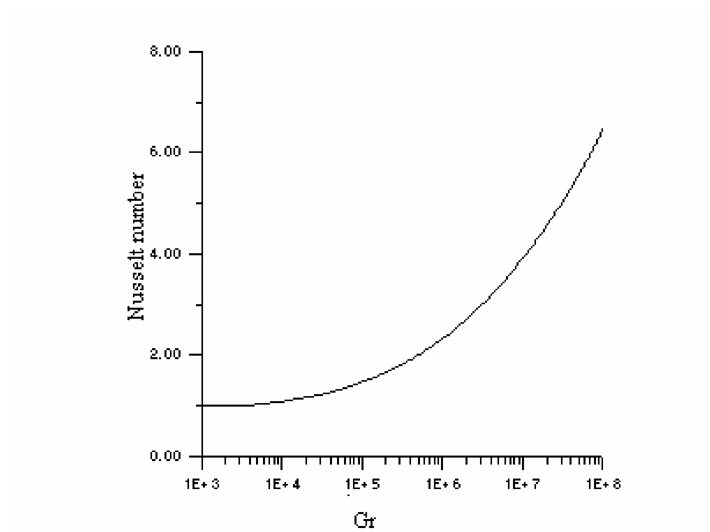

Figure 9. Average normalized Nusselt number in the base as a function of the Grashof number.

Figure (6) shows the streamlines and isotherms considering different Grashof numbers for the case in which the cavity is cooled from the top and heated from the bottom. We can notice that the isotherms and the streamlines are symmetric for all Grashof number values; these results can be compared qualitatively with the results of Shin et al (1994).

Figure (7) shows the horizontal velocity component profile along the middle vertical line of the cavity for some values of Grashof number. We can notice that for Grashof number up to 1.0 $x10^5$, the horizontal velocity is almost equal zero; and when the Grashof number increases, there is an increase in velocity, indicating a more intense fluid movement due to convection currents mainly in the upper part of the cavity.

Figure (8) shows the vertical velocity component profile along the middle horizontal line of the cavity for some values of Grashof number. The same effect is observed; when the Grashof number increases, there is also an increase in velocity, and the velocity profile is symmetric.

Figure (9) presents the average normalized Nusselt number at the cavity bottom as a function of the Grashof number. One can observe that when the Grashof number increases, the convection process dominates the heat transfer, producing greater Nusselt numbers. These results can also be compared qualitatively with the results of Shin et al (1994).

5. Conclusion

This work presented a study about laminar natural convection inside a cavity of semi-circular shape filled with air. The governing parameters used were: $10^3 \le Gr \le 10^8$ and Pr=0.72. Two cases were analyzed. The first case considered the cavity heated from the top and cooled from the bottom, with a uniform and constant temperature. It was observed that for $\mathrm{Gr} \approx 10^3$, the process is similar to pure conduction; when the Grashof number increases, the convection dominates the process of heat transfer. Based on the presented results we can note that the Nusselt number increases when the Grashof number increases up to about $\mathrm{Gr}=1.5x10^7$, and presents a slight decrease for Grashof numbers greater than this value. The second case studied was the cavity cooled from the top and heated from the bottom with a uniform and constant temperature. In this case it was also observed the same effect in relation to the increase in the Grashof number: when the Grashof number increases, the convection dominates the process of heat transfer. It was noted that the streamlines and isotherms are symmetric. The results showed that the increase in the Grashof number also increases the Nusselt number, indicating that the convection process dominates the heat transfer. Finally, the blocking-off method by Patankar (1980) seems to produce consistent results.

6. References

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