A MECHANISTIC MODEL FOR HORIZONTAL GRAVEL PACK DISPLACEMENT

André Leibsohn Martins
CENPES/PETROBRAS – Av. Hum, Quadra 7, Ilha do Fundão, Rio de Janeiro, RJ, ZIP: 21494-900
aleibsohn@cenpes.petrobras.com.br

João Vicente M. de Magalhães
CENPES/PETROBRAS – Av. Hum, Quadra 7, Ilha do Fundão, Rio de Janeiro, RJ, ZIP: 21494-900
jvmagalhaes@petrobras.com.br

This article presents the detailed formulation for each of the three steps of a horizontal gravel pack displacement operation, including sand injection and alpha/beta waves propagation. The main core of the model, aiming the definition of alpha wave height, is based on a well known two layer model. Initially developed for hydrotransport applications, this kind of model has been adapted by several authors for drilled cuttings transport analysis. Besides, a comparison between theoretical predictions and pumping charts from a field operation performed in Campos Basin is presented.

Gravel Pack, Modelling, Sand Control, Stratified Flows

1. Introduction

Gravel Packing is today the most frequently applied sand control technique in Campos Basin, offshore Brazil. Due to the critical conditions, such as the deep and ultra deepwaters and low frac gradients, a lot of precision is required to assure gravel packing success. Most models available in the industry for horizontal gravel pack design are essentially empirical, resulting in imprecise predictions for extrapolated conditions.

These aspects were the main motivators for the development of a mechanistic model to describe the hole operation. It is a consensus among design and operation engineers that a physically based software is a necessary rigsite tool for determining operational parameters, specially when last minute data have to be considered.

Several authors present experimental results of horizontal gravel packing performed in test facilities: Forrest (1990) presents a correlation to estimate pack length limits in highly inclined wells based on 45 and 100 ft model tests with viscous fluids and water.

Penberthy *et al.* (1996) presents several field tests in a 1,500 ft long simulator to identify the main variables which govern the phenomenon. Extensive field-scale testing has aided in the development of procedures and operational guidelines. Software has also been developed, based on empirical correlations to assist design tasks.

Sanders *et al*. (2002) presents a numerical model based on a pseudo three-dimensional approach aiming the simulation of an alternative flow path concept during the horizontal gravel pack displacement. The model solves the equations of volume and momentum conservation for the incompressible slurry in the wellbore. In order to validate the flow path concept both small-scale and large-scale experimental tests using models ranging from 5 to 1,000 ft in length were performed.

2. Brief Description of the Operation

For displacement calculation purposes, the horizontal well gravel pack operation can be divided in three different stages: the injection, the alpha wave propagation and the beta wave propagation.

The injection stage, as highlighted in Fig. (1), consists of pumping a fluid-gravel mixture (red line) through the pipe from the rig until a cross over tool, where the flow will be diverted to the open hole annulus. At this moment, there is usually a decrease in the mixture displacement velocity, due to the larger area, resulting that the force which sustains the gravel particles is not high enough to maintain them in suspension. Consequently, the solids begin to sediment in the lower portion of the annulus (Fig. 4), forming a bed that, for a given flow rate, reaches an equilibrium height (h_α). The deposited sand length will propagate till the extremity of the horizontal section, leaving a free channel between the superior wall of the well and the top of the bed. This stage is known as alpha wave propagation and is illustrated in Fig. (2).

When the alpha wave arrives at the extremity of the well, a new step, called the beta wave propagation, begins: since the sand can not flow through the screens, it will start to deposit above the sand deposited in the alpha wave stage, beginning at the extremity of the well and finishing at the crossover tool. Figure (3) highlights the process.

While during the alpha wave propagation, the fluid flow totally happens through the annular space between the screen and the open hole, in the beta wave the fluid will flow radially through the screen and then axially through the annular gap formed between the screen and the washpipe. Figure (4) shows the cross section of the horizontal wells when equipped with the gravel pack displacement columns.

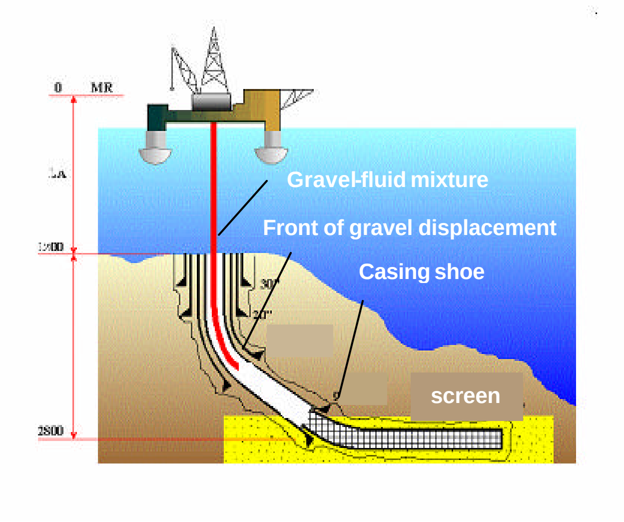

Figure 1. Injection stage.

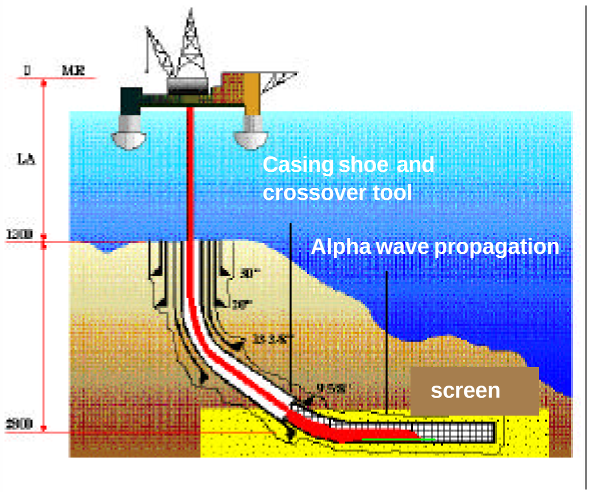

Figure 2. Alpha wave stage.

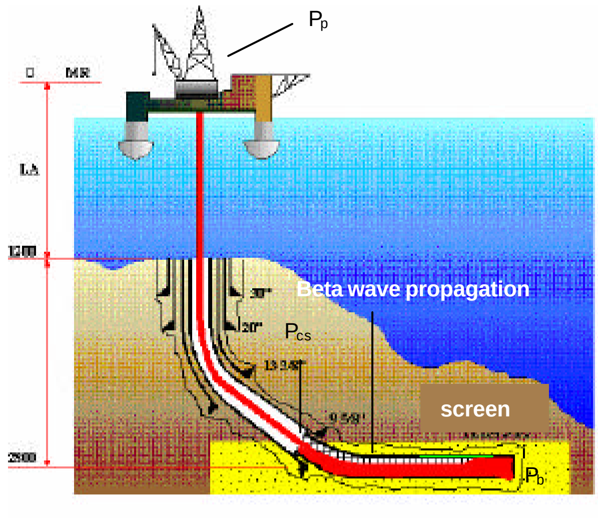

Figure 3. Beta wave stage.

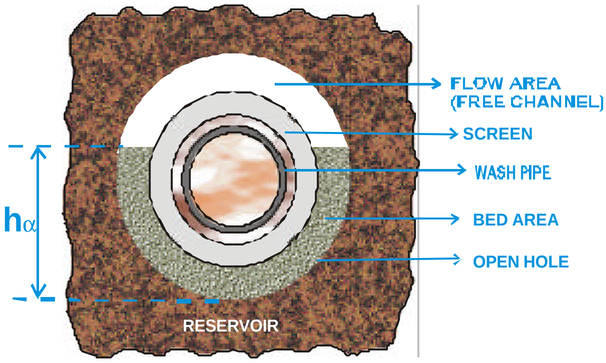

Figure 4. Cross section.

3. Calculation of the Alpha Wave Height

In order to predict alpha wave deposition heights, a two layer model was adopted. The present model is an extension, for horizontal gravel packing applications, of the model proposed by Martins (1990) for drilled cuttings transport analysis.

The following formulation was developed to describe the eccentric horizontal annular flow a solid-Newtonian fluid mixture, aiming the prediction of an equilibrium alpha wave bed height. The solids are characterized by their average diameter and sphericity.

Experiments conducted by Iyoho (1980) showed that the liquid-solid system may assume several configurations in the interior of a horizontal annular section. Four different flow patterns were identified: stationary bed, moving bed, heterogeneous and pseudo-homogeneous. The two first ones are characterized by the deposition, stationary or not, of the solid particles in the inferior part of the annulus. In the two last ones the system is completely suspended and the solid phase may present a concentration profile (heterogeneous) or be uniformelly dispersed in the annulus (pseudo-homogeneous).

The model consists of a stratified two layer configuration which allows, with an unique formulation, the simplified representation of the system in different flow patterns (stationary and moving beds). Figure (5) shows schematically the proposition.

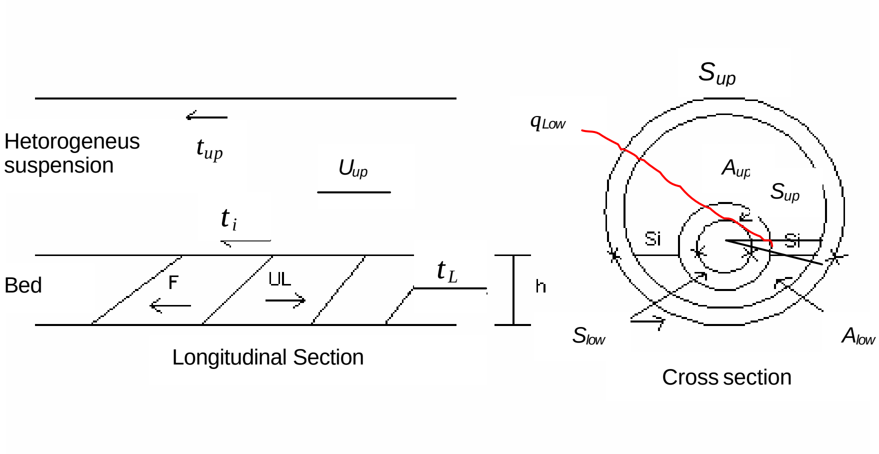

Figure 5. Two layer model.

The bottom layer represents the gravel bed which deposits in the annulus due to the action of gravitational forces. In this layer a fixed solids concentration of 52% is assumed. The top layer contains the particles which are suspended due to the action of turbulent forces plus the carrier fluid.

Other hypothesis:

- There is no slip between the solid and liquid phases in each of the layers.
- There is no mass transfer between the solid and liquid phases.
- The solid-liquid system is incompressible and its rheological parameters are the same of the fluid.
- Superficial tension effects between the layers are neglected.
- Bed height is considered constant throughout the annular section and, consequently, a hydrostatic distribution of pressure along a cross section is assumed.

The following equations represent simplified forms of the conservation laws, described by Bergles *et al.* (1981) where time-space averaged properties (velocity, concentration) are considered along a cross section. Two mass conservation equations are presented (one for each phase) and only two momentum equations (one for each layer), since the slip between the two phases of a layer was neglected. Besides the conservation laws, the proposition of Carstens (1969) to describe the mechanism of turbulent diffusion of solids particles in the top layer is considered.

Mass Conservation:

- Solid phase:

$$U_{up}C_{up}A_{up} + U_{low}C_{low}A_{low} = U_{mix}C_{mix}A_{an} \quad (1)$$

- Liquid phase:

$$U_{up}(1-C_{up})A_{up} + U_{low}(1-C_{low})A_{low} = U_{mix}(1-C_{mix})A_{an} \quad (2)$$

where:
U,C,A, are average velocity, concentration and area, respectively, of the upper layer (*up*), lower layer (*low*) and feed mixture (*mix*).

Momentum Conservation:

- Upper layer:

$$A_{up}\frac{dP}{dx} = -t_{up}S_{up} - t_iS_i \quad (3)$$

- Lower layer:

$$A_{low}\frac{dP}{dx} = -F - t_{low}S_{low} + t_iS_i \quad (4)$$

where:
dP/dx is the differential friction loss along the displacement. t_{up} and t_{low} are shear stress in the upper and lower layer walls respectively. S_{low} is the lower layer perimeter, S_{up} is the upper layer perimeter and S_i is the interface perimeter between the layers.

The shear stresses between the layers and the walls can be expressed as functions of the well known Fanning friction factor (f). Laminar flow predictions are analytical while turbulent flow friction factors are predicted by the Colebrook correlation (1939).

The shear stress in the layer interface may be calculated by:

$$t_i = \frac{1}{2} f_i \, r_{mix.up} (U_{up} - U_{low})^2 \tag{5}$$

Where the friction factor in the interface (f_i), according to Televantos *et al.* (1979), is given by:

$$\frac{1}{\sqrt{2 f_i}} = -0.88 \ln \left[\frac{\frac{D_p}{D_{h.up}} + 2.51}{3.7 \, \mathrm{Re}_{up} \sqrt{2 f_i}} \right] \tag{6}$$

The static force F (lb.ft/s^2), present in Eq. (4) is due to the contact between the particles of the bed and the well wall. Its maximum value, when the cuttings bed is prior to moving, is proportional to the resultant of the normal forces exerted by the cuttings bed. The maximum static force (F_{max}) can be estimated, according to Bagnold (1954), by:

$$F_{\max} = h \left[(r_p - r_f) g C_{low} \, sen \, q A_{low} + \frac{t_i S_i}{\tan j} \right] \tag{7}$$

where:
$r_{mix.up}$, r_p , r_f are the densities of the mixture phase in upper layer, particle and fluid, respectively. D_p is the particle diameter. $D_{h.up}$ is the hydraulic diameter in the upper layer. Re_{up} is the Reynolds number in the upper layer. h is the slip coefficient. g is the gravity acceleration. q is the well inclination in radians. j is the internal friction angle (rad).

Diffusion Equation. The solution of the diffusion equation gives the following concentration profile in the upper layer of an inclined annulus:

$$C_{up}(y) = C_{low} \exp\left[-\frac{w}{e} (y - h) senq \right] \tag{8}$$

The terminal velocity of non-spheric particles in Newtonian fluids can be estimated using the procedure proposed by Santana *et al.* (1991), where population and wall effects are considered.

Finally, the integration of Eq. (8) in the upper layer gives:

$$C_{up} = \frac{C_{low} D_{well}^2 I}{2 A_{up}} \tag{9}$$

where:

$$I = \int_{q_{low}}^{p/2} \exp\left[\frac{-w D_{well}}{2e} (seng - senq_{low}) senq \right] \cos^2 g \, dg \tag{10}$$

where:
q_{low} is defined in Fig. (5). w is the fall velocity of solid particles. e is the diffusion coefficient. x and y are, respectively, the longitudinal and vertical positions in the annulus space. D_{well} is the well diameter. h is the alpha wave height. g is the integration variable.

Solution. The bed height, the average velocities of the layers, the average solids concentration in the upper layer and the friction losses, unknowns of the problem, may be obtained through the solution of the system of equations (1), (2), (3), (4) and (9). The form of solution for the system will depend on the flow pattern. As maps describing the regions of occurrence of each flow pattern are not available, two strategies are required: the successive solution of the system for the two stratified patterns and the definition of mechanisms or criteria for the transition line between them. The following procedure was adopted:

- Stationary Bed – in this case U_{low} is zero and the direct solution of Eqs. (1) and (2) gives values of U_{up} and C_{up}. The iterative solution of Eq. (9) gives values of heights. Finally, the friction losses are calculated by Eq. (3).
- Stationary Bed-Moving Bed Transition – the force F is calculated by Eq. (4) not used in the previous item. If this value is inferior to F_{max} calculated by Eq. (7), the system is closed. If not, the stationary bed flow pattern does not satisfy the set of independent variables.
- Moving Bed – here $F=F_{max}$ and the five equation system must be solved simultaneously using iterative methods.

4. Predicting Pressures During the Displacement

A computer simulator for gravel pack displacement aims the prediction of pressures at relevant points during the open hole displacement process. The three major points of interest for pressure evaluation and monitoring are:

- P_p, the pumping pressure, to adequate the project to the pumping unit.
- P_{cs}, at the casing shoe, to guarantee the integrity of the formation during whole pumping.
- P_b, at the bottom of the well.

The pressures in these notable points, illustrated in Fig. (3) can be calculated by:

$$P_p = P_{ret.} + \Delta P_{kc} + \Delta P_{an.cas.} + \Delta P_{wp} + P_{hyd.an.} \tag{11}$$

$$P_{cs} = P_p + \Delta P_{oh} + \Delta P_{tools} \tag{12}$$

$$P_b = P_{cs} + \Delta P_{col} - P_{hyd.col} \tag{13}$$

where:

- $P_{ret.}$ refers to the return pressure, which normally is the atmospheric pressure.
- ΔP_{kc} refers to the friction losses through the kill and choke lines, when the return flow is diverted this way.
- $\Delta P_{an.cs.}$ refers to the friction losses in the annulus formed by the casing and the column. The fluid goes through this section after returning through the washpipe and the gravel pack tool.
- $\Delta P_{tools.}$ refers to the friction loss generated in the existent contractions in the gravel pack tool (or crossover tool).
- ΔP_{wp} refers to the friction loss in Wash Pipe, path which the fluid flows when coming back from the end of the well, after having crossed the screens.
- $P_{hyd.an.}$ refers to the hydrostatic pressure in the annulus.
- ΔP_{oh} refers to the friction loss in the openhole, when the mixture flows through the space between the formation and the screens.
- $\Delta P_{col.}$ refers to the friction loss generated by the flow of the mixture fluid-gravel, through the column.
- $P_{hyd.col.}$ refers to the hydrostatic pressure in the column.

The calculation of each of the terms where the mixture flows will be different for each stage of the operation (injection, alpha or beta waves) and, consequently, these terms will be time dependent. On the other hand, the terms where only water flows will be constant with time. They are the return pressure ($P_{ret.}$), the friction losses in the Kill and Choke lines and in Wash Pipe (ΔP_{kc}, ΔP_{wp}) and the hydrostatic pressure in the annulus ($P_{hyd.an.}$). In the next items all the relevant calculations for each phase are described.

1st Stage: Injection

Propagation front:

$$L_{inj.} = \frac{4Q_p t}{pD^2_{\text{int}.col.}} \tag{14}$$

Friction loss in the open hole:

$$\Delta P_{oh.inj.} = \frac{2 fr_f Q_p^2 L_{oh}}{A^2_{an.} D_{h.}} \tag{15}$$

Friction loss inside the column:

$$\Delta P_{col.inj.} = \frac{32 fr_{mix} Q_p^2 L_{inj}(t)}{p^2 D^5_{\text{int}.col.}} + \frac{32 fr_f Q_p^2 (L_{cs.} - L_{inj}(t))}{p^2 D^5_{\text{int}.col.}} \tag{16}$$

Hydrostatic pressure inside the column:

$$P_{hyd.col.inj.} = r_{mix.} g h_{inj.} + r_f g (h_{cs} - h_{inj.}) \tag{17}$$

where, h_{inj} and h_{cs}, refer to the vertical depths of the injection front and casing shoe, respectively.

2nd Stage: Alpha wave

Alpha Wave Propagation:

$$L_a = \frac{Q_p C_{mix} t}{A_{low}(1 - f)} \tag{18}$$

Friction loss inside the open hole:

$$\Delta P_{oh.a} = \frac{2 fr_{mix.} Q_p^2 L_a(t)}{A^2_{up} D_{h.up}} + \frac{2 fr_f Q_p^2 (L_{oh} - L_a(t))}{A^2_{an.} D_{h.an.}} \tag{19}$$

Friction loss inside the column:

$$\Delta P_{col.a} = \frac{32 fr_{mix.} Q_p^2 L_{cs}}{p^2 D^5_{\text{int}.col.}} \tag{20}$$

Hydrostatic pressure inside the column:

$$P_{hyd.col.a} = r_{mix.} g h_{cs.} \tag{21}$$

3rd Stage: Beta wave

Beta Wave Propagation:

$$L_b = \frac{Q_p C_{mix} t}{A_{up}(1 - f)} \tag{22}$$

Friction loss inside the open hole:

$$\Delta P_{oh.b} = \frac{2\, fr_{mix.} Q_p^2 (L_{oh} - L_b(t))}{A_{up}^2 D_{h.up}} + \frac{32\, fr_f Q_p^2 L_b(t)}{p^2 \sqrt{2/3}(D_{\text{int}.scr.}^2 - D_{ext.wp.}^2)^2 (D_{\text{int}.scr} - D_{ext.wp.})} \tag{23}$$

Friction loss inside the column:

$$\Delta P_{col.b} = \frac{32\, fr_{mix.} Q_p^2 L_{cs}}{p^2 D_{\text{int}.col.}^5} \tag{24}$$

Hydrostatic pressure inside the column:

$$P_{hyd.col.b} = r_{mix.} g h_{cs.} \tag{25}$$

where:

$$D_{h.up} = \frac{4A_{up}}{S_{up} + S_i} \tag{26}$$

$$D_{h.an} = \frac{4A_{an}}{S_{tot}} \tag{27}$$

$$r_{mix} = r_p C_s + r_f (1 - C_s) \tag{28}$$

where:
L_{inj}, L_a, L_b, L_{oh} and L_{cs}, are the measured depth of the injection front, alpha wave front, beta wave front, open hole and casing shoe, respectively. Q_p is the pump flow rate. t is the time. f is the porosity. A_{an} is the area of the annular space between openhole-screen. $D_{int.col}$ and $D_{int.scr}$ are the internal diameters of the column and screen, respectively. $D_{ext.wp}$ is the wash pipe external diameter.

5. Model Implementation

The proposed modelling was implemented in a computer code for use in projects and during the gravel-pack operations in horizontal wells. This code was written in PASCAL language for DELPHI environment. Input data for the simulator are:

- Open hole diameter and length.
- Rathole diameter and length.
- Casing internal diameter.
- Screen diameters (OD and ID).
- Wash Pipe and gravel diameters.
- Well path.
- Injection and return flow rates.
- Particle and fluid densities.
- Frac gradient.

6. Sensibility Analysis

6.1. Effect of Particle Size and Concentration

Figures (6) and (7) illustrate the effect of sand particle size and concentration on alpha wave heights. Figures (8) and (9) show the same effects on pumping and casing shoe pressures. Although both factors presented some effect on alpha wave heights, negligible effects on pumping and casing shoe pressures are observed.

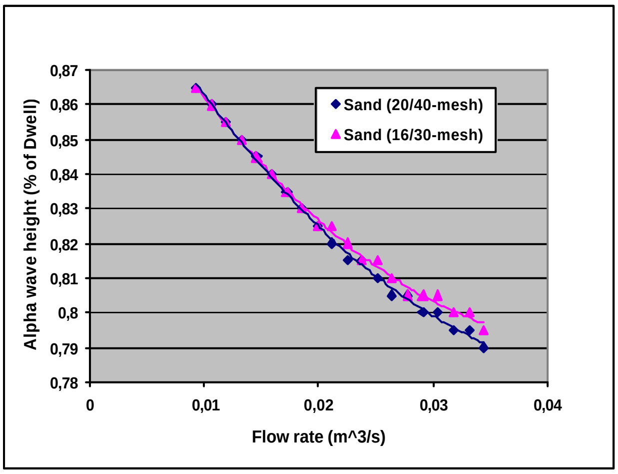

Figure 6. Effect of particle size on alpha wave height.

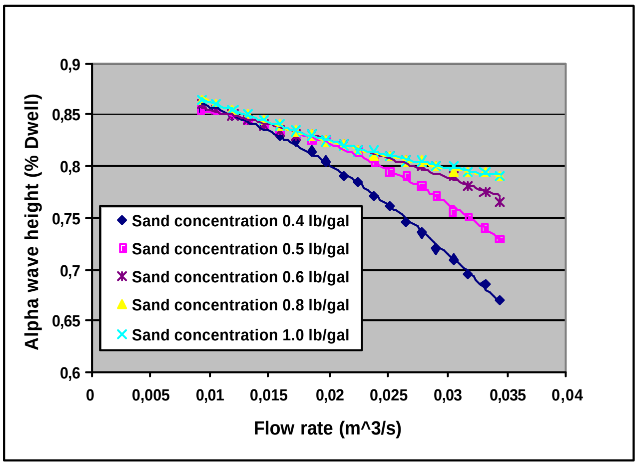

Figure 7. Effect of particle concentration on alpha wave height.

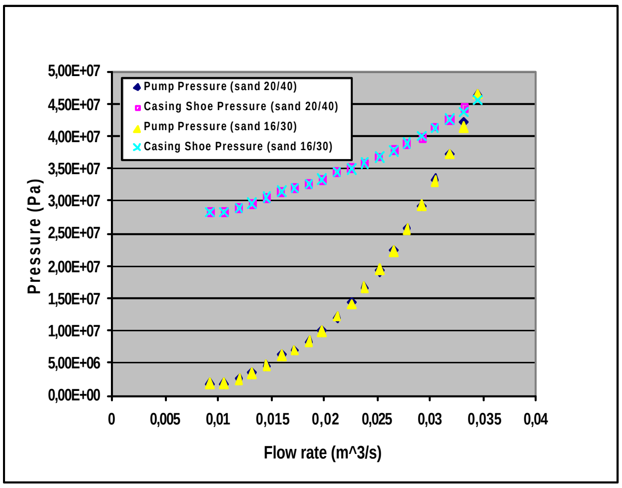

Figure 8. Effect of particle size on pump and casing shoe pressures.

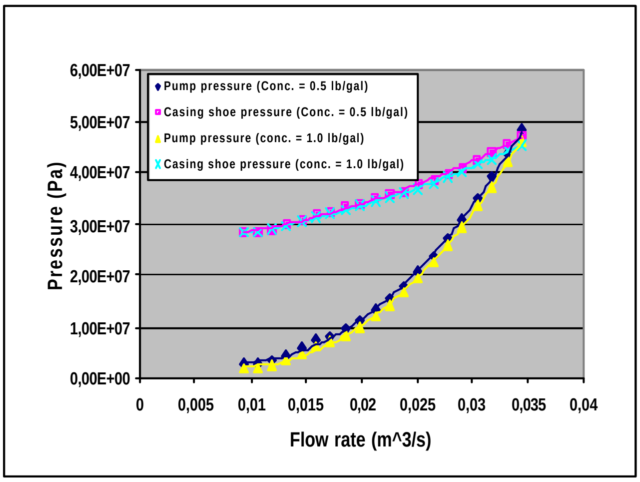

Figure 9. Effect of particle concentration on pump and casing shoe pressures.

6.2. Effect of Well path and Return flow rate

Figure (10) shows the effects of the build up ratio on the pump and casing shoe pressures for a same final vertical depth and horizontal section length. There is a significative difference in the casing shoe pressure curves, indicating that a proper well design can minimize downhole pressures. Figure (11) highlights the impact of return flow rates in the pumping pressure. The increase of the return flow/pumping flow ratio leads to a increase in both casing shoe and pump pressures.

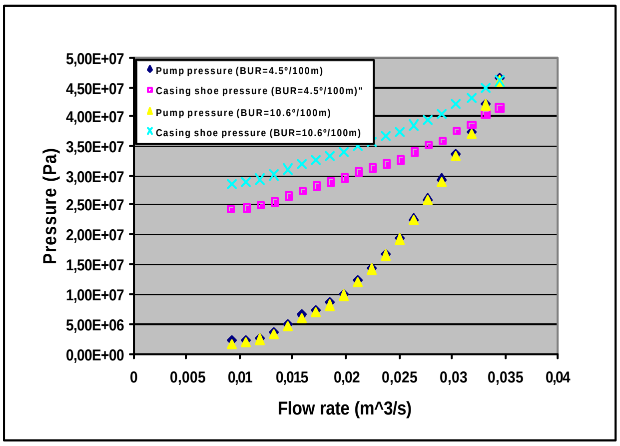

Figure 10. Effect of well path on pump and casing shoe pressures.

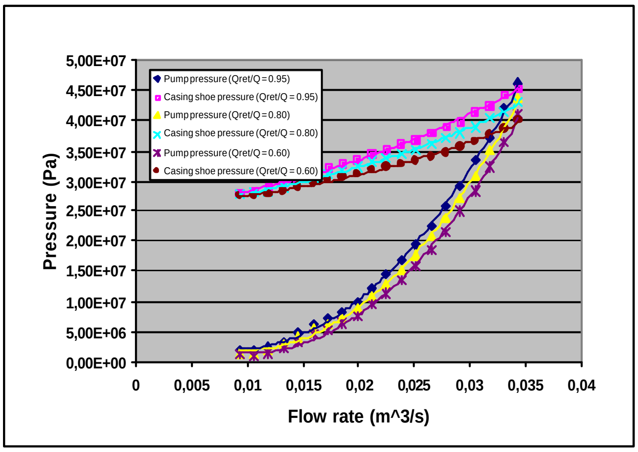

Figure 11. Effect of return rate on pump and casing shoe pressures.

7. Model Validation

Table (1) illustrates the input data for the simulation of a typical operation run in Barracuda field, Campos Basin. Figure (12) shows the comparison of the results obtained by the numerical simulator with the pump pressure registered during the operation.

7.1. Injection

During this stage, both the friction losses and the hydrostatic pressure increase with the progress of the water-sand mixture injection front along the column. For usual configurations (water depths and diameters previously described), the hydrostatic term prevails resulting in pump pressure linear decrease.

7.2. Alpha Wave Propagation

In this stage the curve of pump pressure grows smoothly due to the partial restriction of the annulus in consequence of sand deposition.

Table 1. Operational data.

Operational data	values
Openhole diameter *(D_{oh})*	9.0”
Last casing diameter *(D_{cas})*	9 5/8”
Casing weight	47 lb/ft
Kill and Choke lines diameter *(D_{kc})*	3”
Hole Enlargement Diameter	12 ½”
Water depth *(SD)*	800 m
Casing Shoe Measured depth *(L_{cs})*	2857 m
Open Hole Measured depth *(L_{oh})*	3227 m
String Diameter *(D_{col})*	5”
Column Weight	19.5 lb/ft
Wash pipe *(OD)*	4.0”
Wash pipe *(ID)*	3.476”
Flow rate *(Q_p)*	6.2 bpm
Return flow rate *(Q_{ret})*	6.0 bpm
Solids Concentration *(C_s)*	1.0 lb/gal
Fluid Density *(r_f)*	9.1 lb/gal
Gravel type: Sand *(mesh)*	20/40
"Bulk" density *(r_{bulk})*	13.36 lb/gal
Return pressure *(P_{ret})*	atmospheric
Screen Length inside the casing	6,22 m

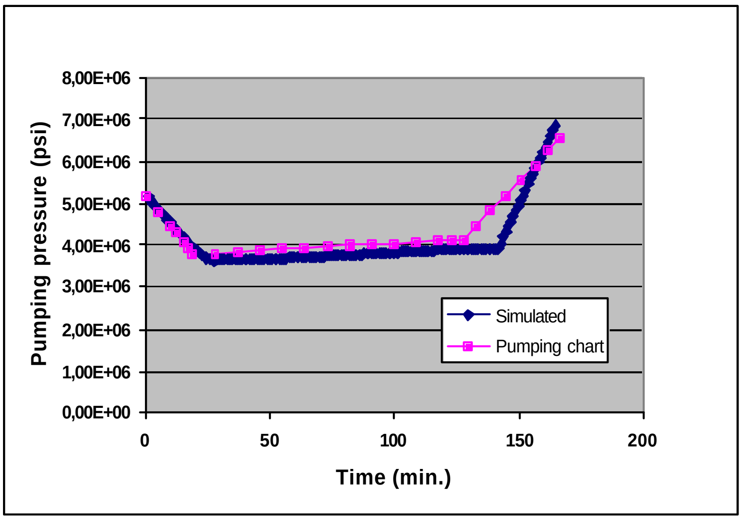

Figure 12. Comparison of the results obtained by the numerical simulator with the pressures registered during the operation.

7.3. Beta Wave Propagation

Here an accentuated growth of the pump pressure is observed, since the fluid tends to cross the screen and flows through the annulus space screen-wash pipe (with small cross sectional area), generating a bigger friction loss.

8. Final Remarks

The sensibility analysis performed highlights the potential of a mechanistic model in optimizing a horizontal gravel pack design. Sensibility results seem to be coherent and physically based.

The proposed model, implemented as a computer code, showed good predictions for pumping pressures during injection and alpha wave propagation, while there was a tendency for pressure overestimation during beta wave propagation.

A probable reason for this fact is that two empirical coefficients in the model (slip coefficient - h, and internal friction angle, tan j) have not been experimentally measured for the conditions which characterize the horizontal gravel pack displacement process. These parameters affect directly alpha wave height and, consequently friction losses in the open hole, specialy during the beta wave propagation. An experimental campaign on a large scale test loop is scheduled to evaluate these parameters. Specific friction loss correlations for the screen-wash pipe annulus available in the literature will be incorporated into the model.

Future modeling steps include also the incorporation of a fluid loss model in the open hole and of zonal isolation devices (ECP's and diverter valve), Machado *et al*. (2001).

9. Future Work

- Experimental evaluation of the heights during the alpha wave deposition.
- Experimental evaluation of the friction losses in the annular space screen-wash pipe.
- Inclusion in the mathematical model of the phenomenon of fluid loss from the open hole to the producing formation.

10. Acknowledg ment

The authors would like to thank Petrobras S.A. for permission to publish this paper.

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