Gears Coupling Efforts

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Abstract. This study presents an analysis of the efforts involved in a flexible coupling – gears coupling - based on analytical equations. It intends to evaluate system behavior due to the rotation variation, as well as the significant parameters in comparison to the hypotheses usually adopted. The analytical equations have two different formulations for the friction force, allowing the comparison of two different models to the usual ones.

Keywords. *Flexible couplings, Newton-Raphson method, numerical simulation*

1. Introduction

The coupling of two shafts supposed to be perfect aligned is a frequent subject in several analyses, receiving a lot of attention in the academic and in the industrial researches. There are several situations where it is necessary, such as joints between power trains and other equipments, where it is necessary to supply power from the first to the second, in the way of axis torque.

In these situations, even using the most rigorous facilities, it is impossible to reach a perfect alignment between the shafts for both the static and the dynamic conditions. Although the static alignment is very close of the ideal, the dynamic effects will take charge to put it out of this condition. A lack of mass balance, vibrations of the foundation, rotation variations and natural frequencies in the range are factors that will remove the shafts of this impossible condition of perfect alignment.

By this way, the joining points of the shafts are, usually, the most critical for the installation, being frequently the fail point of the system. There are high stresses in the shafts, flanges and bearings, when a rigid coupling is used. In these cases, generally, the coupling is designed as a mechanical fuse, what means that, in an overload, the coupling is going to fail before the other components, to avoid greater damages for the installation.

For these conditions it is very adequate to use a coupling, where some level of misalignment is allowed, without affecting the torque transmission largely. These elements are the flexible couplings.

2. Research goals

In this research, it is intended to evaluate the mechanical efforts in the gear type flexible coupling, its behavior and most important parameters.

Instead of the existence of a lot of models for the flexible coupling, the most of them takes care only about the global effects of the coupling in the system, in the direction of determine the equivalent stiffness and damping coefficients, in order to simulate the joint element in the system. For the forces, the stress inside the coupling element and the reaction in the coupled shafts there are few studies.

This work is supposed to present a computational procedure (program) to evaluate the forces and moments, using two different formulations of friction forces, based on the shafts misalignment data, theoric or experimental. Initially, theoric data calculated through a Finite Elements procedure, developed by Tadeo, will be used. In the future it is intended to use more real data, increasing the adjustment of these parameters through experimental measurements, and, maybe, using directly real measured data.

3. Gear coupling

Basically it was the most used coupling by several sections of the industry in certain moments and, it still nowadays consists broadly of a system used. According to Mancuso, it is the simplest and more commonly employee coupling.

These coupling are formed by a pair of gears with external teeth that are geared with a joining glove with internally machined teeth. The teeth can be straight or curved, as presented in the Figure 1. For the applications that demand more than 0,5º of angular displacement the inclined teeth gear presents great advantages. The flexibility of this coupling is exactly in the gaps of the gears (Backlash).

Generally they are used for equipment of medium and high load, which demand transmission of prohibitive power for all the other coupling, especially the ones with plastics components.

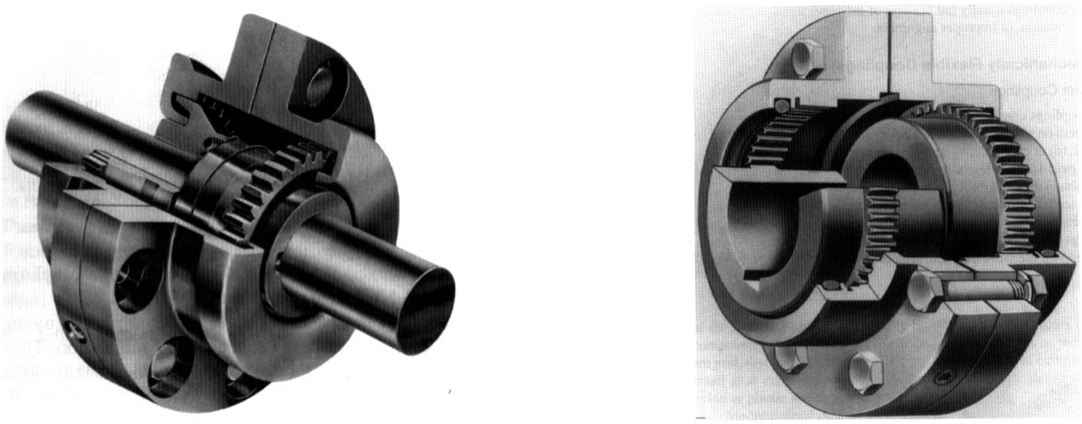

Figure 1 – Straight Teeth Gear Coupling and Curved Teeth Gear Coupling.

4. Equations’ overview

The first system of equations used in this work has been proposed in Mancuso’s report (1971). In this report he presents an analytical model for the forces and moments in a gear coupling in comparison with experimental results the results of that work indicate a good correlation between the analytical and the experimental models.

In this initial model the friction forces are evaluated as a friction coefficient multiplied by the force normal to the contact surface. In addition a second model has been implemented considering the friction as a dumping coefficient multiplied by the first differential of the misalignment angle. We changed the whole system of equations for this second model in order to compare different formulations of the friction forces.

The gear coupling can deal only with one type of misalignment: the angular misalignment. For situations where the parallel misalignment is significant it is necessary to split the coupling into two joints. These joints can both be gears coupling, but it demands an extra shaft element between the couplings.

By this way, it is possible to suppose this kind of coupling to be subject only to angular misalignment, representing the misalignment as an angle between the shafts. This angle is indicated by φ in the equations and in the next picture, and is supposed to be the external angle between the center lines of the axes. The perfect alignment condition is reached when this angle is reduced to zero.

The main parameters of the equations system are the geometry of the coupling, the torque transmitted, estimated values for the frictional coefficient and, the most important, and the misalignment angle.

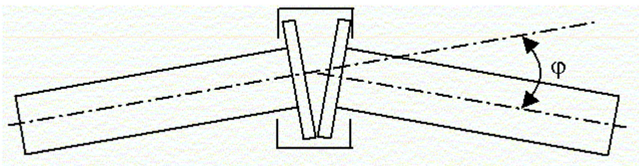

Figure 2 – Schematic representation of the coupled pair of shafts and the misalignment angle.

The analytical equations, basically, try to evaluate the force that should be applied to the contact teeth to put the system into the equilibrium condition. From this force the resultant moment in the coupling is calculated.

It is also considered the theoric approximation for the reaction in the shafts and the approximation to determine the contact arc of the gears. By turning in an angular condition the line of contact in the gears is not a circle.

The forces balance developed by the equations system covers, basically, the application of a force P to put the system into equilibrium. The evaluation of these forces is the base of the equations. Upon this force the resultant moments, in the two non axial perpendicular directions are evaluated.

The angular response of the shafts with the variation of the rotation shows periods where the angle is increasing and others, decreasing. In Mancuso’s equations the best results, closest to the real data, were gotten when the quasi-equilibrium, considering each rotation as a quasi-equilibrium state, forces diagram defines the friction force in the direction to bring the system back to initial state, whatever the previous rotation was.

In the second system of equations the same consideration has been done. The friction force is always in the direction to bring the system back to the perfect aligned condition. The main difference is that the friction force is not related to the normal force, but to the first differential of the misalignment angle.

Forces in the Contact Teeth considering Ffric = μ . N

$$\sum Fx' = +P\cos\Psi - m\,N - W\,sin\Psi = 0 \tag{1}$$

$$\sum Fz' = N - P\ sin\Psi - W\cos\Psi = 0 \tag{2}$$

Solving equations (1) and (2) for P results

$$P = \frac{W\left(sin\Psi + m\cos\Psi\right)}{\cos\Psi - m sin\Psi} \tag{3}$$

Forces in the Contact Teeth considering Ffric = $c.\dot{q}$

$$\sum Fx' = +P\cos\Psi - c.\dot{q} - W sin\Psi = 0 \tag{4}$$

Solving equations (1) for P results

$$P = \frac{W\left(\sin\Psi\right) + c\dot{q}}{\cos\Psi} \tag{5}$$

Moment around the axis Z-Z

$$M_P = \frac{M_t P \sqrt{R^2 - \left(\frac{R}{3}\sin\frac{A}{2}\right)^2}}{R} \tag{6}$$

Moment around the axis Y-Y

$$M_S = 2.\left(c\dot{q}.Y\cos\Psi + W.xr\right) \quad \text{where} \quad Y = \frac{R}{3} sin\frac{A}{2} \quad \text{and} \quad xr = A * sin\Psi \tag{7}$$

Resultant Moment

$$\Delta M_3 = \sqrt{\left(M_{S3}\right)^2 + \left(M_{P3}\right)^2} \tag{8}$$

At $q° = \tan^{-1}\frac{2R}{D_C}$ about Y in the plane ZY (9)

5. Input data

According the last item, the main input for the calculus procedure is the misalignment angle. To get this angle there are a lot of alternatives: experimental measurements, using instruments on an experimental set up of bearings, shafts and coupling, to vary the rotation, keeping all others conditions in constant values (but the power or the transmitted torque).

A second way, chosen for this work, is to use a theoric model of simulation, using the Finite Elements Method, and the classical models for the coupling, bearings and shafts. For the coupling there are a lot of models, as proposed by Kramer and Nelson-Crandal, to simulate the coupling in the system, without evaluating the forces in the coupling.

It will be used in this work a computational procedure, in FORTRAN language, implemented by Tadeo (2001).

5.1. The shaft-bearing-coupling system

The mechanical system modeled consists of two flexible shafts, two rigid discs, simulating a rotor system, one flexible coupling, represented by the Second model of Kramer, and two bearings, as showed in next picture. The bending vibrations of the system have been also considered. The inertial reference system **XYZ** and the auxiliary reference **xyz** (fix in the system axis) are used to describe the system movement equations. A transversal section of the rotor in the deformed state is defined in the system **XYZ** through its translations *u*(Y,*t*), *v*(Y,*t*) in the directions X and Z, determining the position of the section center in any time instant *t*. The orientation of the section is defined through the small rotations α(Y,*t*), β(Y,*t*) around the axis X and Z, respectively.

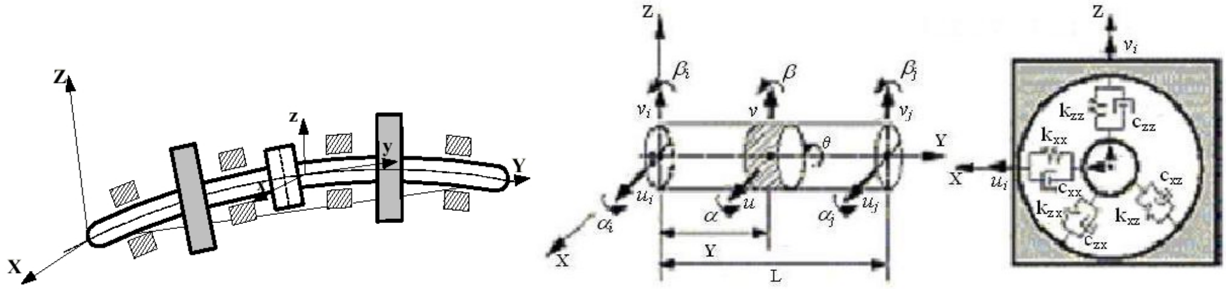

Figure 3 – Shaft-bearing-coupling system and finite elements components of the shaft (middle) and the bearing (right)

5.2. Flexible Coupling

In the literature it is a difficult task to find information about physical models for couplings. The most usual is to consider it as a rigid disc. However this hypothesis does not consider the flexibility of the coupling, its main characteristic. The procedure implemented by Tapia (2001) allows the user to choose between the second model of Kramer (1993) and the model of Nelson and Crandall (1992).

Considering only the angular misalignment is a strong initial restriction to this procedure, the coupling center points have always the same value of translation. But, for gear couplings, this consideration is very reasonable, due to the very high stiffness for translation of the coupling.

This condition agrees perfectly with the second model of Kramer, where the effect of the coupling is to restrict the degree of freedom of the translations of the node ***i***, before the coupling, and ***j***, after the coupling, making them always identical: $\boldsymbol{u_i} = \boldsymbol{u_j}$ and $\boldsymbol{v_i} = \boldsymbol{v_j}$. The model also considers a rotational stiffness $\mathbf{k_r}$ and damping $\mathbf{c_r}$ characteristics to the coupling, as showed in figure 4.

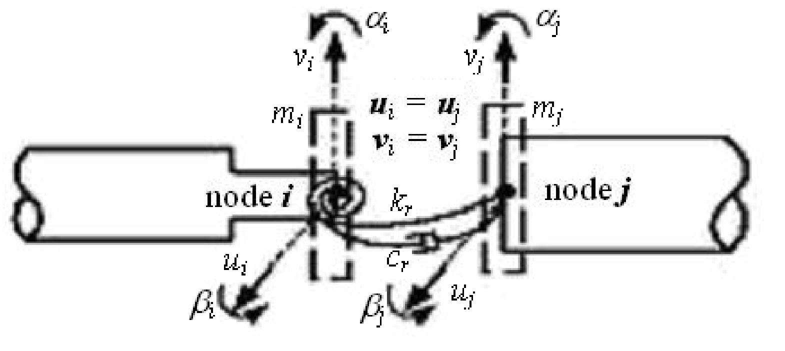

Figure 4 – Second flexible coupling model of Kramer (1993)

From the values of this simulation for the nodal rotations α and β (around the axes X and Z, respectively) of the nodes **i** and **j** of the coupling, we have determined the geometrical relations to evaluate the resultant angle that describe the misalignment, as showed next:

$$
\begin{aligned}
j_i &= \arctan\sqrt{(tga_i)^2 + (tgb_i)^2} \\
j_j &= \arctan\sqrt{(tga_j)^2 + (tgb_j)^2} \\
j &= j_i + j_j
\end{aligned}
\qquad (10)
$$

5.3. Misalignment angle behaviour in the rotation range

The values for the misalignment angle are showed in the next graph. In this graph it is clearly presented three peaks of misalignment. These peaks are very close to the normal modes of the system. It can be easily realized that this points have the maximal angle.

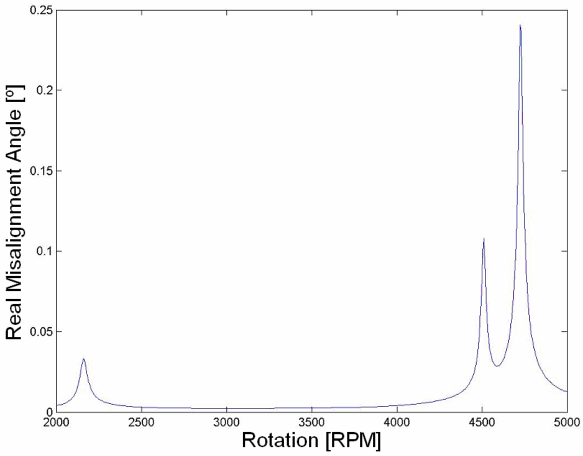

Figure 5 – Resultant misalignment angle due to the shaft rotation

6. Computational Procedure developed

Having as input data the nodal rotations at the coupling nodes, calculated through the Finite Elements Method, the equations for the calculation of the moments in the coupling have been implemented as a computational program, in Pascal language.

This program has 2 procedures, one for the calculations of the forces and moments for the friction proportional to Normal force, and another considering the d***fi***/dt. Furthermore, there is a group of procedures and functions that are a modified version of the Newton-Raphson algorithm, developed by Figueiredo.

The Newton-Raphson procedure is used for the numerical calculation of the inverse functions of the system of equations, what means, fixing some conditions for the stiffness coefficient of the coupling, the program will start an iterative calculation to find out the behavior of the input data, adjusting them.

The independent parameter for Newton-Raphson optimization, is the value of the force transmitted through the coupling, function of the torque. The optimization condition is the value of the stiffness coefficient, supposed to be constant and equal to the input in the FEM procedure, for the Kramer model. For these conditions the iterative procedure has been executed for each rotation of the range, for different values of frictional coefficient, determining for each condition the force that would have to be transmitted by the coupling.

This analysis has been executed for five conditions: no friction, and four frictional coefficient, μ = 0,004; μ = 0,01; μ = 0,03 and μ = 0,04 for the first equation system and six for the second: C_{fi} = 11,3; C_{fi} = 31,3; C_{fi} = 51,3; C_{fi} = 71,3; C_{fi} = 91,3 and C_{fi} = 111,3.

Analyzing the behavior of the force in the gear tooth submitted to the condition of constant stiffness coefficient, another analysis has been executed: keep the value of this force constant to evaluate the behavior of the stiffness coefficient. These analyses have been executed also for all the conditions given previously.

7. Newton-Raphson

Basically it is a method of numeric resolution that, starting from a relatively close initial estimative of the solution of the system of equations, it converges for the solution according a square power. In this algorithm, the equations to be solved have to be written in the form of an expression and a residue (to be minimized). For the system of equations the Jacobian matrix is calculated. The Jacobian matrix defines the direction where the variables should be increased, from the initial estimative, and their new values. It is supposed that the value of the residue is reduced after each iteration. For the cases when it doesn't happen, there is a procedure of residue control. This procedure is very good to prevent the divergence, even for badly conditioned systems.

Despite the algorithm of Newton-Raphson is already quite known, in this case, we have two basic modifications. The first is the procedure for residue control, essential for problems not so well conditioned and the other is the use of a substitutional system to reduce the number of effective variables in the problem.

The Newton-Raphson procedure is described in details in Figueiredo's work (2002), This method has been applied, initially, for the resolution of thermodynamic systems and, for the first time, it is being used for resolution of problems in mechanical systems modeling.

8. Results

8.1. Stiffeners coefficient constant

Mancuso mentions in his work that, for the range of rotations tested (2000 RPM to 5000 RPM), the effect of the friction force is neglectable. Using a null frictional coefficient, in order to neglect the effect of the frictional force, it was imposed the condition that the stiffness coefficient would be kept constant, adjusting the value of the force transmitted.

When considering the friction coefficient as null, for both equations systems, the result for the contact teeth force is the same: a constant value. So, not considering the friction force, the result behavior of the stiffness coefficient is exactly as proposed by Kramer (1993), constant for all rotation, as showed in next pictures. The relation between the total moment and the misalignment angle is a straight line.

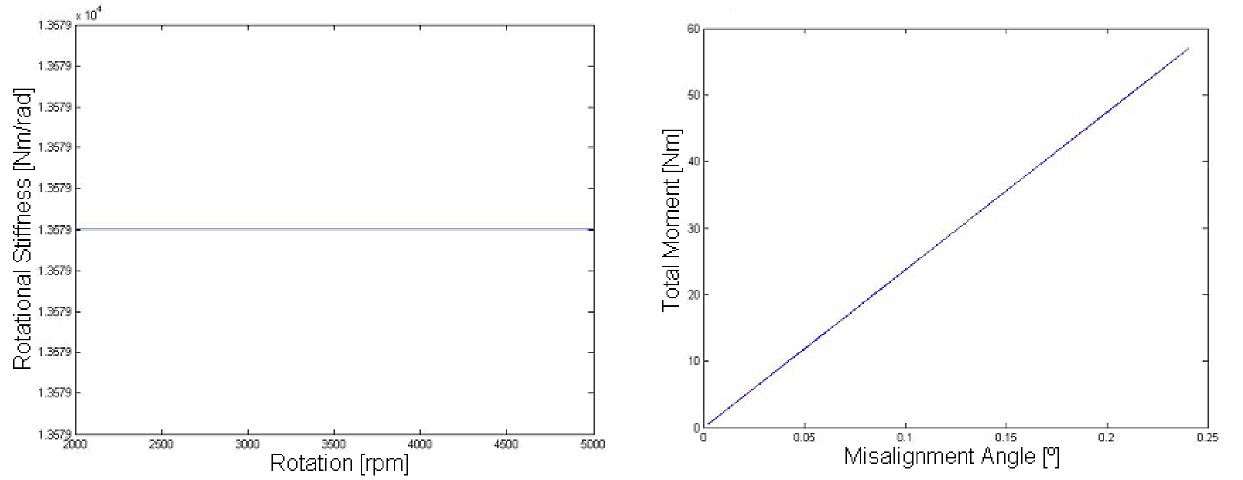

Figure 6 – Stiffness coefficient behavior in the rotation range and total moment by the misalignment angle.

But when frictional force is involved in the equations, the behavior of the transmitted force is deeply changed for both models. It is understood as the transmitted force the force responsible by the gear torque. This force is the independenty parameter for the iterative procedure, so, its behaviour will be a result from the imposed conditions. By this way, when the frriction is considered this force is not constant anymore, and it starts to assume a behavior quite similar to misalignment angles (predictable behaviour, due to the fact the rotational stiffening is assumed as constant). In the next graphs, it is showed the behavior of the force in the contact tooth for various values of friction coefficient (mi) and damping (c). The graphs are quite similar on the general behavior, but it is important to notice that the variation of the friction coefficient and the damping affects the force on different directions. For the first model, when the friction coefficient is increased the force decreases and for the second model the increase of the damping results in neglectable changes on the force. This can be understod through the equations: in the first case the directions normal the theet and tangential to theet are coupled to the resultant moment through the friction force, so the force will be related with the sine and the cosine projections; for the second model these coupling efect disappears, and the force is related only with the sine. As the angle is always quite small, the sine is very close to zero, and the cosine close to one.

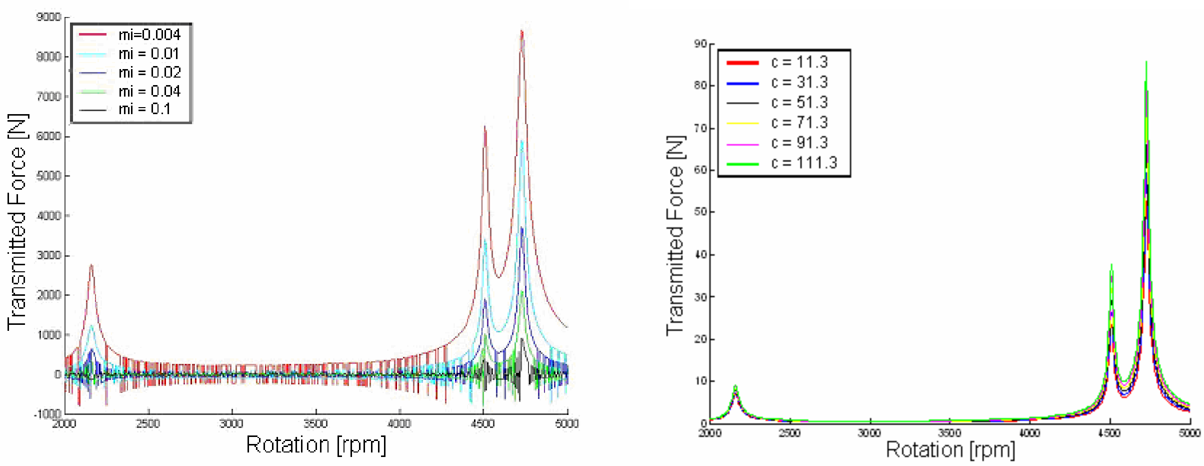

Figure 7 – Force on teeth behavior in the rotation range for both models.

8.2. Force in contact tooth constant

Considering the force in the contact tooth as constant, what means, solving the equations in direct order, we have been into results quite interesting. The first result presented is about the variation of the stiffness coefficient with the rotation. While most of the models for flexible couplings consider a constant stiffness, the results presented previously already depose against this hypothesis. The next results still reinforce more this contradiction, for both models for friction.

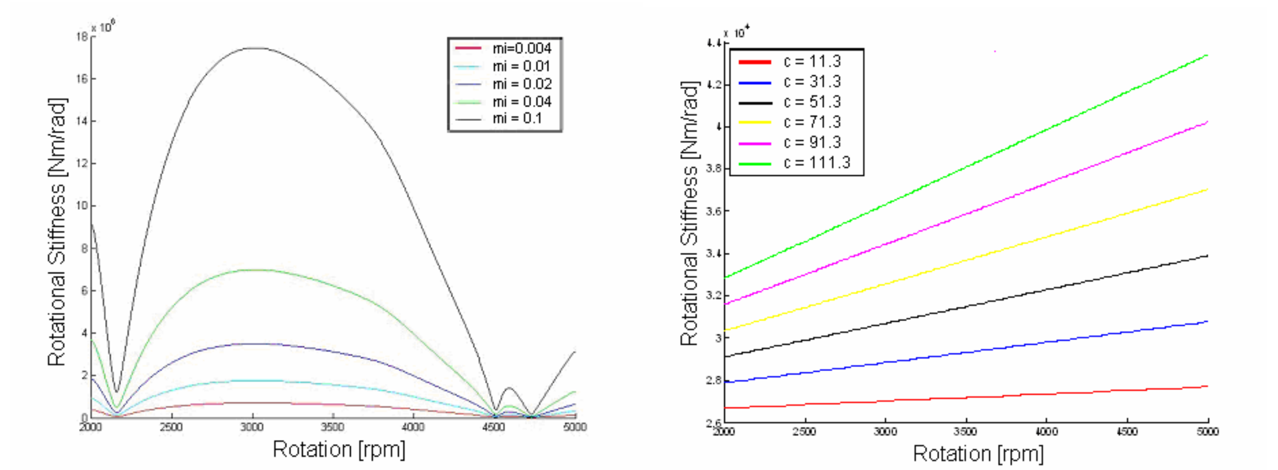

Figure 8 – Comparison of the values of stiffness coefficient in function of the rotation for different friction coefficients, on left, and for different damping values, on right.

In these graphs is showed another important factor. For the first model the values of the stiffness coefficient along the rotation range is strongly non-linear, while for the second model the relation is totally linear. This difference is not so unpredictable. For the first model the friction is modeled as a contact force, and, instead of being a linear relation, it is related to the normal force. This formulation drives the coefficient to a non-linear behavior, in the same way it has been explained for the previous graphs.

For the second model the friction force is determined by a damping coefficient multiplied by the first differential of the misalignment angle. This relation is a linearization of the friction force, driving the results to a linear relation between the coupling K to the rotation. But it is important to notice that in both cases the value changes a lot for the rotation.

The next graphs show a comparison between the total moment behaviors along the rotation range for both friction formulations. In third graphs such a curious effect can be noticed: for the first formulation the increase of the friction coefficient drives the system to a vertical translation of the total moment curve, at the same time the size of the peaks decreases. For the second formulation the increase of the damping coefficient drives the system to a increase of the total moment, but without a translation, as a linear variation.

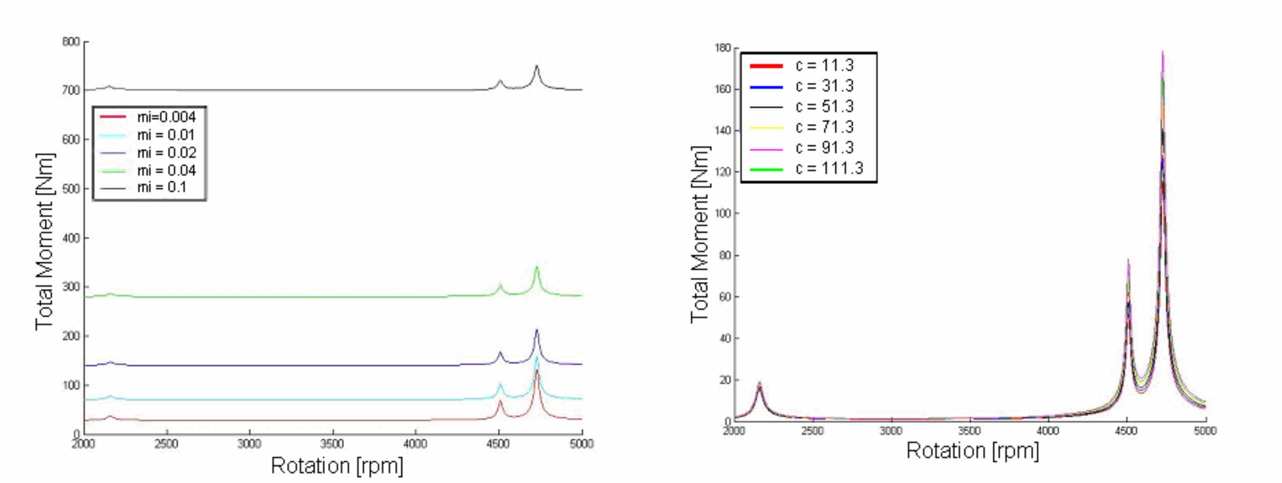

Figure 9 – Comparison of the values of total moment in function of the rotation for different friction coefficients, on left, and for different damping values, on right.

8.3. Transmitted power constant

Considering a real system the maybe the most natural configuration could be having a transmitted power constant, as a condition using an electrical motor with varied rotations. For this case the equations system is also solved on the direct mode, without using the iterative procedure. Next graphs shows the results reached with this configuration. For this case the transmitted force is showed on next picture.

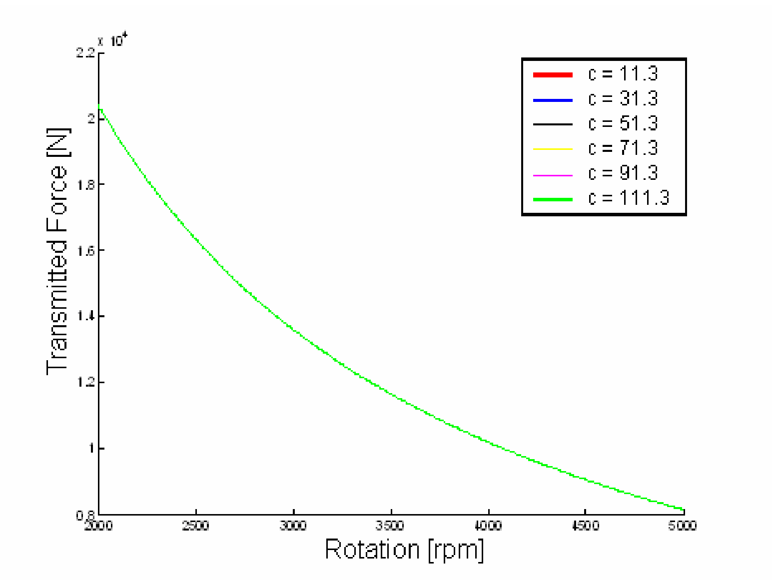

Figure 10 – For the both formulations the transmitted force is the same, as it does not depend on the coupling equations, defined directly from the constant power.

Changing the transmitted force from a constant value to a curve the other parameters also change a lot. Next picture shows the behavior of the stiffness coefficient on the rotation range. For the friction as a contact force the behavior is strongly non-linear, as in the previous cases, while for the damping formulation the change is only from a linear to a curve, following the change of the inputted transmitted force.

For these graphs it can be noticed that the curve for the first formulation seams to be deformed to the left side, consequence of the highest force on the lower rotations. In the second case the general behavior of the curves are similar to the previously showed, but it is not linear.

It is important to notice that for all cases the stiffness coefficient changes with the rotation, whatever linearly or not.

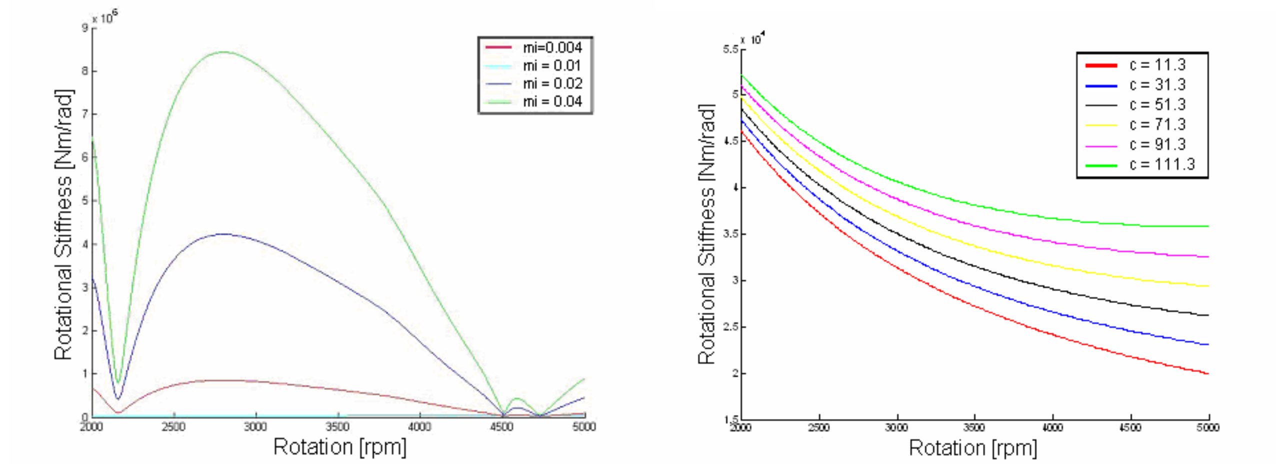

Figure 11 – Comparison of the values of stiffness coefficient in function of the rotation for different friction coefficients, on left, and for different damping values, on right.

The total moment also is affected by the changes in the force behavior. The behavior is totally predictable: for the first formulation there is a translation of the curves, increased in the beginning and decreasing on the end. Another effect is the reduction of the peaks, specially the first one, that almost disappear for the highest friction.

For the second formulation it is difficult to analyze the differences, due to the proximity of the curves, but the general behavior has not been changed.

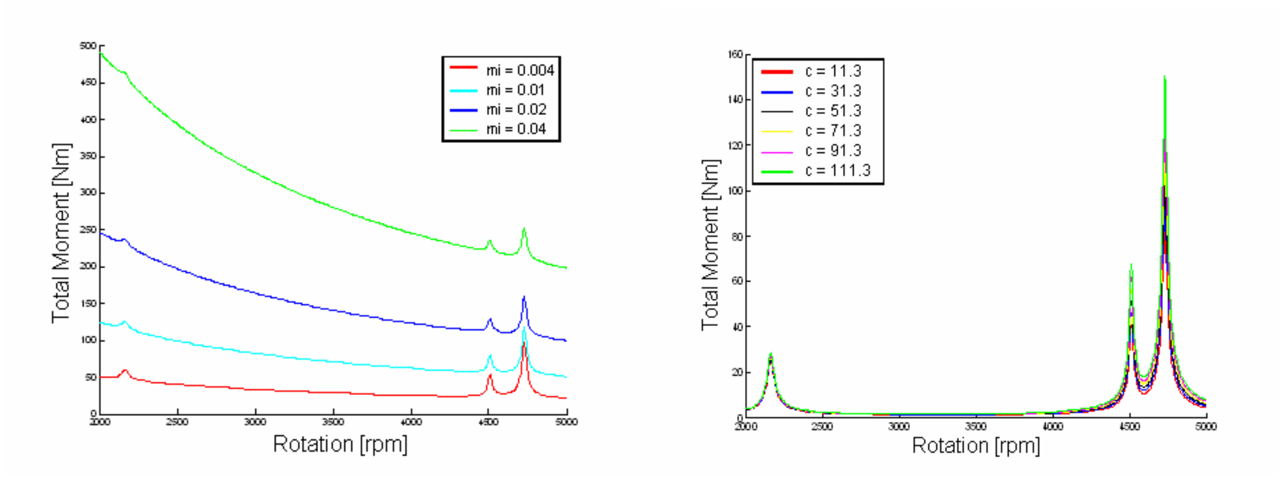

Figure 12 – Comparison of the values of total moment in function of the rotation for different friction coefficients, on left, and for different damping values, on right.

9. Conclusions

The main idea of this work is to study the flexible couplings without using the conventional finite elements models, but using analytical equations, and compare the results of these alternative formulations to the models. The first model constructed uses the formulation of the friction as a coefficient multiplied by the normal force, what means a contact force. This leads the results to strongly non-linear relations.

For the second model a linear formulation for he friction has been defined, using a damping coefficient multiplied by the first differential of the misalignment angle. This formulation leads the results to a linear behavior.

But, in both cases, when any type the friction is considered, the stiffness coefficient of the coupling is not constant. The usual assumption for the coupling's models is that the coefficient of stiffness and damping are constant. Some models consider the differences in the formulation of the coupling, that form a non symmetrical stiffness matrix, but the most of them considers these values constants.

The next steps of this work are exactly considering the non symmetrical matrix formed for coupling equations. Searching for a good formulation for the stiffness coefficient for flexible couplings.

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